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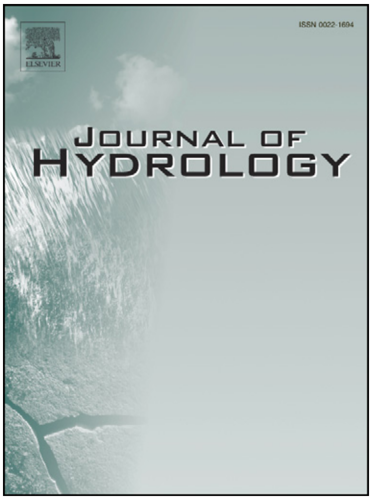

Discussion

Comment on “Experimental observations of saltwater up-coning”

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Comment on "Experimental observations of saltwater up-coning"

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Werner et al. (2009) presented 2D experimental data on time-dependent saltwater up-coning using controlled sand-tank experiments in which freshwater overlying a saltwater layer was pumped at a single extraction point, leading to saltwater up-coning. The experimental set-up imposed constant head boundary conditions for both fresh- and saltwater. The experimental results were compared to a sharp-interface perturbation-based approximate analytical solution to the governing model (Dagan and Bear, 1968). This approximation was derived assuming that: (i) the fresh-salt water interface is sharp, (ii) the interface extends to ±infinity, where it remains undisturbed, and (iii) it applies to any pumping rate, whether it is subcritical, critical or supercritical. Werner et al. (2009) noted that few analytical solutions are available for up-coning, and so they used the approximation of Dagan and Bear (1968) to compare with their experimental data although the experimental conditions and model assumptions do not coincide exactly. The accuracy of the analytical approximation of Dagan and Bear (1968) is dependent on the movement of the freshwater-saltwater interface relative to the initial height, d, of the withdrawal point above the interface. Their approximation is considered reasonable for interface movement of up to about $d/3$.

The need to place the boundary condition at ±infinity was relaxed in a series of analytic and numerical analyses of up-coning and down-coning (Zhang and Hocking, 1996; Zhang et al., 1997, 1999, 2009). In these studies, an impermeable boundary was placed symmetrically at a fixed distance, $\pm x_L$, from the pumping well, a situation that is closer to the experimental setting of Werner et al. (2009) than the model of Dagan and Bear (1968). At these boundaries, the interface position is fixed. In brief, Zhang and Hocking (1996) provided the analytical solution for steady critical and subcritical withdrawal when the pump is located at the top impermeable boundary of the flow domain; Zhang et al. (1997) found the analytic solution for steady critical withdrawal for various pump locations; Zhang et al. (1999) solved the time-dependent interface response using the boundary element method; while Zhang et al. (2009) provided the analytical solution for steady supercritical withdrawal from two layered fluids.

Table 1 gives the relevant experimental and dimensionless parameters, the latter indicated by a superscripted asterisk. An important parameter is the critical pumping rate, which is defined as the rate for which the saltwater up-coning will just reach the extraction point. Supercritical flow rates, i.e., those greater than the critical flow rate, always result in saltwater breakthrough into the extracted water. For subcritical flow rates, saltwater never reaches the extraction point. Of course, the sharp interface assumption ignores mixing across the interface but nevertheless the computed critical flow rate has obvious practical value.

The critical pumping rate was discussed by Werner et al. (2009). They observed for all their experiments that “according to the definitions of Bear (1979) and Bear and Dagan (1964), initial up-coning plumes were expected to have a convex shape (near the plume apex) and stable plumes were expected to develop. However, up-coning proceeded until the interface intercepted the well in the experiments of this study, and therefore the steady-state conditions of criticality that others have reported (e.g. Bower et al., 1999) do not appear to be transferable to the current analysis.” That is, the experiments of Werner et al. (2009) do not have convex saltwater up-coning shapes for most of their experiments. For example, for Experiment 1, their Fig. 3f shows saltwater breakthrough into the extraction well. Their Fig. 4f shows the same behaviour for Experiment 2. For both Experiments 1 and 2, the breakthrough shape was similar, as noted by Werner et al. (2009). Interestingly, their Fig. 5f shows the saltwater cone is extended, with a long “tail” reaching the extraction point, suggesting that saltwater breakthrough into the extraction well is minimal. This is confirmed by their Fig. 9a, which shows only a small increase in salinity in the pumped water. By contrast, for Experiment 4, their Fig. 6f shows that the pumping rate is clearly subcritical in that the peak of the saltwater mound shows a convex shape. This figure and their Fig. 9b show, however, that some saltwater reaches the pumping well.

In Table 1, q_{cr}^{*} is the scaled dimensionless critical flow rate, which was computed for a given impermeable boundary location, and a given pump location, h_{s}^{*}. Comparison of q^{*} and q_{cr}^{*} in Table 1 shows that the pumping rates in Experiments 1 and 2 were both supercritical, that for Experiment 3 was also supercritical, but close to critical, while for Experiment 4 the pumping rate was clearly subcritical. These results are all consistent with the experimental results shown in Figs. 3-6 of Werner et al. (2009), as discussed in the foregoing paragraph.

Because Experiment 3 of Werner et al. (2009) is close to the critical pumping rate, it is possible to check further the steady-state analytical solution given by Zhang et al. (1997), which was derived for this case. For the above given apparatus dimensions and water depths, the critical interface shape for $h_{s}^{*} = 0.43$ and $x_{L}^{*} = 0.61$ were computed using Eqs. (3.8) and (3.9) of Zhang et al. (1997), giving the interface shape plotted in Fig. 1. A few interface locations in Fig. 5f (right side) of Werner et al. (2009) were traced by hand and compared with the calculated interface shape using the model of Zhang et al. (1997). Fig. 1 shows close agreement between the experimental data and model predictions, although the analytical solution consistently over-predicts the data. It is possible that this is due to the difference in boundary conditions in the model and experiment. In the model, the interface is fixed at $\pm x_L$, but the flow above and below the interface comes from $\pm\infty$. This situation is probably a reasonable approximation for an experiment with fixed head conditions at $\pm x_L$. In the experiments, Werner et al. (2009) noted that the conditions at the sides of the experimental apparatus “were head-dependent flux conditions”. Such a condition would provide more resistance to flow than a fixed-head condition, and is consistent with the over-prediction evident in the model’s predictions.

In Fig. 1, the dimensionless crest height (i.e., actual crest height scaled by *a*) given by the model is $h_c^* = 0.36$. The model also predicts the dimensionless half plume width (actual half width scaled by *a*) as $w^* = 0.12$ at $z^* = h_s^*/2 = 0.21$. At this same location, the corresponding data for Experiment 3 are $h_c^* \approx 0.35$ ($h_c \approx 33$ cm, the last height measurement before breakthrough to the extraction, as read from Fig. 7 of Werner et al., 2009) and $w^* \approx 0.07$ ($w = 7$ cm from Table 4 in Werner et al., 2009,). Note that in Fig. 2. of Werner et al. (2009), $W(t)$ is defined as the full width of up-coning. However, a close examination of the up-coning plume in Fig. 5f and the data in Table 4 suggest that half-widths are listed in the latter, i.e., $w = W/2$ rather than W is given in Table 4. The model’s estimate of w^* over-predicts the experimental measurement, consistent with the over-prediction of the interface evident in Fig. 1. Overall, given the uncertainty in the experimental boundary condition noted by Werner et al. (2009), it is evident that the model predictions are in good agreement with the experimental data, and are far superior than the predictions of the perturbation approximation of Dagan and Bear (1968), not surprisingly since the range of application of the latter approximation is limited.

Previous studies (Zhang and Hocking, 1996; Zhang et al., 1997, 1999, 2009) have shown that the boundary location has a significant effect on up-coning predictions in two-layer, sharp interface models. Additionally, as noted above, there is a discrepancy between the boundary conditions used in the analytical solution of Zhang et al. (1997) and the experiments reported by Werner et al. (2009). Despite the difference in boundary conditions, the above analysis had led to a characterisation of the experiments into subcritical, critical and supercritical withdrawal cases that accord with the images given by Werner et al. (2009). Furthermore, for the critical withdrawal case, the location of the steady interface and interface characteristics predicted by the up-coning model of Zhang et al. (1997) match well the experimental data of Werner et al. (2009). The over-prediction of the model is most likely due to the boundary flux condition of the experiments rather than a fixed head condition. The boundary flux condition implies the presence of a resistance to flow at the boundary. Even so, we conclude from the comparison with the experimental data that the mathematical conditions imposed in obtaining the solution are a reasonable approximation to Werner et al. (2009)'s experimental setting.

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Table 1

Summary of the experimental parameters (Zhang et al., 1997; Werner et al., 2009).

Experiment Number	Q (m^3/s)	d (m)	a (m)	ρ_s (kg/m^3)	K (m/s)	q^*	h_s^*	x_L^*	q_{cr}^*
1	3.80×10^{-6}	0.43	0.97	1011	1.62×10^{-5}	1.46	0.44	0.61	0.36
2	3.90×10^{-6}	0.40	0.95	1025	3.68×10^{-5}	0.67	0.42	0.62	0.36
3	2.20×10^{-6}	0.41	0.96	1025	3.68×10^{-5}	0.37	0.43	0.61	0.36
4	5.30×10^{-7}	0.38	0.93	1096	1.41×10^{-4}	0.02	0.41	0.63	0.36

Notation: Q is the pumping rate; d is the pump location above the initial interface position; a is the freshwater layer thickness; ρ_s is the saltwater density; $K = kg(\rho_s - \rho_0)/\mu$ is the relative hydraulic conductivity; ρ_0 is the freshwater density; μ is the water viscosity; g is the magnitude of gravitational acceleration; $q^* = Q/(\pi KaB)$ is the scaled non-dimensional pumping rate; B is the thickness of the sand tank; $x_L^* = x_L/a$ is the non-dimensional location of the boundary (where the interface position is fixed); $h_s^* = d/a$ is the dimensionless vertical distance between the pumping well and the initial interface location and q_{cr}^* is the critical pumping rate, i.e., the scaled rate for which the saltwater will just reach the extraction point .

Figure Caption

Fig. 1. The modelled and measured interface shape comparison for the critical flow case (Experiment 3) of Werner et al. (2009). The dimensionless horizontal distance from the pumping location is $x^* = x/a$, where x is the actual horizontal distance. The corresponding vertical distance, measured from the point on the interface directly below the pump is $z^* = z/a$, where z is the actual vertical distance.

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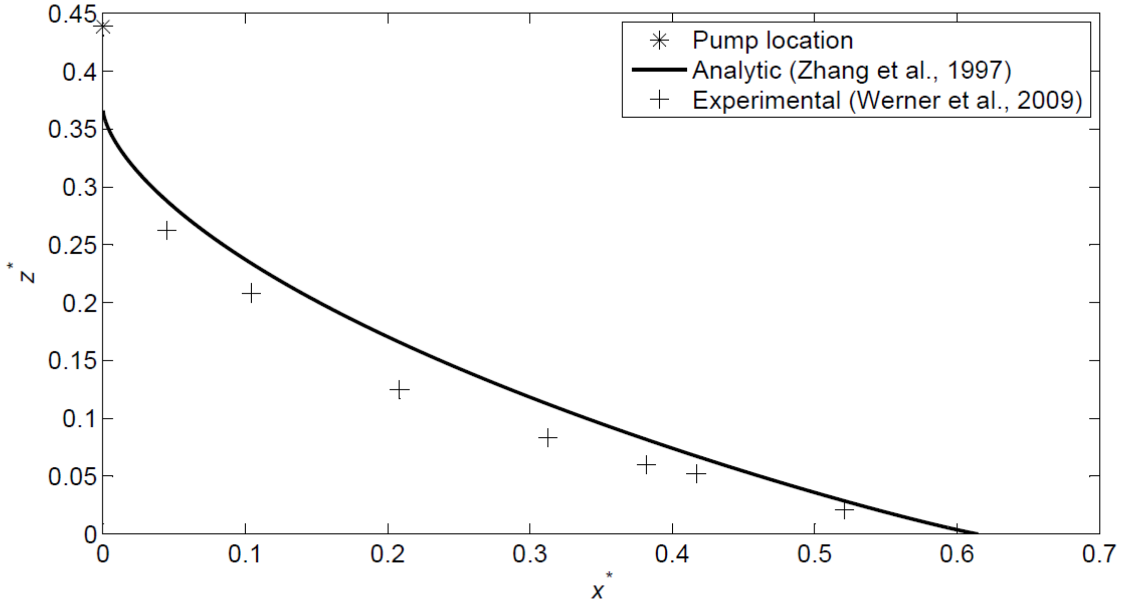

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163 **Fig. 1.**

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