Graphics calculator use in examinations: accident or design?*

Marian Kemp, Academic Services Unit
Barry Kissane, School of Education
Jen Bradley, School of Physical Sciences, Engineering and Technology

Murdoch University

ABSTRACT: As graphics calculators become more available, interest will focus on how to incorporate them appropriately into curriculum structures, and particularly into examinations. We describe and exemplify a typology of use of graphics calculators in mathematics examinations, from the perspective of people designing examinations, together with some principles for the awarding of partial credit to student responses. This typology can be used to help design examinations in which students are permitted to use graphics calculators as well as to interrogate existing examination practice.

Introduction

The last few decades have seen considerable changes in the technologies available to support mathematical work, and a corresponding change in the technologies regarded as appropriate for mathematics examinations at the undergraduate and senior secondary school levels. For many years, the only aid regarded as legitimate was the table book, which gave students access to trigonometric functions, exponential functions and logarithms (which were needed for complex calculations). Later versions of table books also provided tables of squares, square roots and reciprocals, to avoid the tedium of such common calculations using logarithms. The next innovation was the slide rule, which served the purpose of automating logarithmic calculations, albeit with some loss of accuracy. Following this, in the mid-1970s, came the electronic calculator, which eliminated the need for calculations with logarithms, and shortly afterwards the scientific calculator, which also eliminated the need for students to use a table book for trigonometric functions. In more recent times, the available functions on scientific calculators have expanded, and now often include bivariate statistical calculations, normal probability functions and the solution of systems of linear equations; at the same time, table books have expanded to include mathematical formulae of various kinds, standard integrals, and other notes intended to allow students to focus their attention on doing mathematics rather than remembering formulae.

With each of these developments, mathematics curricula were adjusted to accommodate the new opportunities provided. Part of the adjustment, of course, has been to reconsider the nature of mathematics examinations. The development of graphics calculators needs to be seen in this context of increasing access to technology for mathematics (Kemp, Kissane & Bradley, 1995). This paper provides an analysis of the relationships between graphics calculators and mathematics examinations, with a view to offering some help for designing examinations sensitive to the imperatives and opportunities created by these newer technologies. As the title of the paper suggests, without the benefit of such analyses, there is a risk that graphics calculators will influence examinations in a haphazard way.

Background

At the start of 1995 we were awarded a CAUT (Committee for the Advancement of University Teaching) National Teaching Development Grant for *Developing an assessment programme incorporating graphics calculators*. This enabled us to build upon the work done in previous years and look specifically at the design of assessment tasks. Through the support of the grant we were able to trial some of the questions before using them in the formal assessment.

Whilst graphics calculators have been used at Murdoch University since 1993 in a first year service course including algebra and introductory calculus, students were not allowed to use them in the final examination until first semester 1995. As in previous years, students were given access to a Texas Instruments TI-82 calculator, both within the tutorials and outside of formal teaching hours (Bradley, Kemp & Kissane, 1994). Students were also provided with tuition in using specific functions of the calculator. The assessment for the course included an in-class test, for which graphics calculators were used. (A description of such a test is given in Kissane, Bradley & Kemp (1994).) Before the semester started, arrangements had been made for sufficient calculators to be available for the final examination. Consequently, for the first time, the examination paper for the course was designed with the knowledge that all students would have a graphics calculator for the full three hours of the examination.

As in previous years, just over three quarters of the way through the course the students were surveyed (through the University's Independent Research and Evaluation Section) as to their thoughts on the usefulness of the calculators in their learning processes. They were also asked whether the calculators should be allowed/required to be used in the assessment components. In 1994 the students gave very positive feedback (Kissane, Kemp & Bradley, 1995) on their use. However, this attitude was coloured by the fact that calculators were not to be allowed in the final examination, with some students indicating that they were not sure about the effort considering the restriction. In 1995 the students knew from the start that they would be able to use the calculators not only in the weekly assignments but also in the final examination. The subsequent survey indicated an even more positive feedback on the calculator use. Whilst we as teachers may have become more expert in our teaching with the calculators and thus influenced the students' reactions, our feeling is that it was the knowledge that the calculators could be used throughout the entire course that was the main influence.

During 1995 both Victoria and Western Australia have made the decision to allow students to use graphics calculators in the public examinations used for tertiary entrance. This will be effective from 1997 for Victorian students and from 1998 for students in WA. As a consequence the next couple of years will see a marked increase in the use of graphics calculators in the class room and in school based assessment tasks. Anyone setting examinations in mathematics will need to consider different styles of questions from those previously set, taking into account the students' access to graphics calculators.

A typology

This paper explores and illustrates the range of ways in which the design of examination tasks can be influenced by the graphics calculator. Figure 1 gives an outline of the typology we have found to be useful, and which is exemplified in the next section of this paper.

Our concern is with features of graphics calculators that are not available on most scientific calculators. Some of these features are graphical (such as producing and analysing graphs of functions), while others are numerical (such as numerical calculus, matrix manipulation and solution of equations).

Graphics calculators are expected to be used

1 Students are explicitly advised or even told to use graphics calculators
2 Alternatives to graphics calculator use are very inefficient
3 Graphics calculators are used as scientific calculators only

Graphics calculators are expected to be used by some students but not by others

4 Use and non-use of graphics calculators are both suitable

Graphics calculators are not expected to be used

5 Exact answers are required
6 Symbolic answers are required
7 Written explanations of reasoning are required
8 Task involves extracting the mathematics from a situation or representing a situation mathematically
9 Graphics calculator use is inefficient
10 Task requires that a representation of a graphics calculator screen will be interpreted

Figure 1 Expected usage of graphics calculators and examinations

There are contexts for student assessment other than examinations. A common example is the regular assignment, which is frequently used as a form of assessment, but which also serves a learning purpose, with students expected to engage in activities designed to help them learn some aspects of mathematics. For the latter kinds of tasks, students may well use graphics calculators under specific direction, to help them exploit the advantages of the calculators for learning, or to help them acquire skills for calculator use, and probably both of these.

Even though an examination task may require student responses to be exact answers, we are not uncomfortable with students using graphics calculators to support their work. Indeed, they may even be encouraged to use calculators in such situations too. A common use of graphics calculators is to check results obtained in another way. Such use will not always be visible, as students may decide not to report it. Checking may well not be detectable, unless a student's answer does not pass the check. A signal that checking is involved is given when a correct answer is accompanied by incorrect working, however.

Similarly, sometimes, it will be intended that students *not* use graphics calculators, but sophisticated students may nonetheless do so to help their thinking. Three examples of this are:

squaring a numerical result to see if the answer is a simple rational number or an integer in order to determine an exact answer (e.g., squaring a result of 0.8660254 gives 0.75, suggesting that the exact result is \r(3/4) or \r(3) /2);

substituting numerical values into expressions to determine the form of a general case;

using a graph to help thinking about a task (e.g., to see how many solutions an equation has, whether a definite integral is positive or negative or deciding whether or not there is a local minimum on a particular interval).

A different typology from that shown in Figure 1 is likely if the emphasis is on examining student responses to assessment items rather than on designing them.

Determining the ways in which graphics calculators were actually used, where the focus is on the actions of students, differs from considering issues of item design, where the focus is on the intentions of the designers. In addition, most classification schemes, including the present one, contain a measure of overlap between categories, especially when complex matters are being addressed. Leaving aside such limitations for the moment, the next section amplifies and exemplifies the categories shown in the typology.

Some examples

To illustrate the various categories, in this section of the paper we describe a few examination questions that we have used in a particular early undergraduate course. Whilst our examples are restricted to one particular course, we are confident that the typology is more generally applicable, with respect to both levels of mathematical sophistication and content. We have chosen the examples specifically to illustrate the categories: there is no suggestion that the collection of examples would comprise a suitable examination by itself.

Graphics calculators are expected to be used

When all students are required (rather than merely allowed) to have a graphics calculator at their disposal in an examination, it may be appropriate to design questions for which it is expected that the calculators will be used. Such use, of course, involves mathematical thinking as well as mechanical operation of a calculator, and some graphics calculators require substantial familiarity and expertise to use efficiently.

\x(1)Students are explicitly advised or even told to use graphics calculators

In most cases in examinations, it seems suitable to allow students to choose their own solution methods, since the purpose of a question is exactly to see how well students can think about a mathematical situation and respond appropriately. However, there are sometimes risks that students may choose very time-consuming solution methods that do not allow us to focus on the important aspects, or may take an inordinate amount of time to decide how to proceed.

It may also be suitable to instruct students to use a graphics calculator if a purpose of a question is to test how well students can use their graphics calculators. Tasks testing how well students can use their graphics calculators are appropriate when learning to do so is regarded as an important objective of a course.

> Suppose that $f(x) = -0.5x - 1$ and $g(x) = 2\sin x$. Use a calculator to help you sketch graphs of f and g on the same axes for $-2\pi < x < 2\pi$.
>
> Use your graphs to determine for $-2\pi < x < 2\pi$ how many solutions there are to the following equations:
> (a) $f(x) = 2$ (b) $g(x) = 1$ (c) $f(x) = g(x)$.

> Use your calculator to help you sketch on the given axes a graph of $f(x) = 2x^3 + 4x^2 - x - 11$, showing the main features of the graph, including any intercepts with the axes.

In the first task above, the concern is with how well students can relate functions, graphs and equations together, not with how well they can draw graphs. The concepts associated with solving equations graphically are the main concern of the task. The second task above is concerned with how well students can use their graphics calculator, since the graph drawn on a typical default interval will not show all the important features. Students will need to realise that this is the case, based on their understanding of the possible shapes for a cubic, and adjust the axes accordingly, as shown in Figure 2.

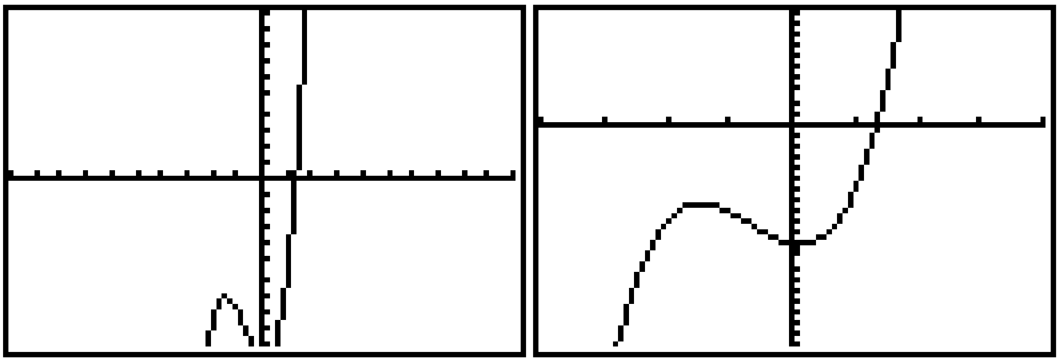


Figure 2 Two graphs of $f(x) = 2x^3 + 4x^2 - x - 11$. (Each tick represents a unit.)

Adjusting the axes and then recording both the graph and the desired points (intercepts and relative extrema) to a suitable level of accuracy all require efficient calculator operation. In designing the task, a function was deliberately chosen to ensure that students would need to engage in these kinds of activities to respond appropriately.

\x(2)Alternatives to graphics calculator use are very inefficient

A critical aspect of mathematical behaviour concerned with calculators is 'learned discretion', concerned with whether students can make sensible decisions about calculator use. Speed of course is not the only important aspect of mathematical work, although it is an important characteristic of many examinations. For some tasks, little insight is provided to students by tediously ploughing through standard solution procedures, whether or not they are in an examination. If there are good alternatives available at their fingertips, it seems important that they be expected to learn to make good use of them. Whether or not it is sensible to use a graphics calculator sometimes depends on how awkward the numbers involved are. By long tradition, examination questions are sanitised to remove computational complexity, in stark contrast to the real world.

Solve $x^3 + 4 = x$.

Solve the following system of linear equations:

$z - y - x = -5$ $\quad$ $4x + 3z = 6$ $\quad$ $x + 2y + 5z = 11$.

For each of these two questions, students were expected to realise for themselves that solution methods that did not involve calculator use are quite time-consuming. In the case of the cubic equation, for which no ready algorithm is available, and thus no exact solution possible, some form of iterative procedure is needed. Using a graphics calculator, however, students can obtain an accurate solution quickly using a variety of methods. A sketch of the pair of functions $f(x) = x^3 + 4$ and $g(x) = x$ or of the function $f(x) = x^3 - x + 4$ allows a graphical solution to be obtained efficiently. On some graphics calculators, functions can be tabulated and compared directly to approximate solutions to equations. Several graphics calculators allow a direct solve command to be used, giving the real solution $x \approx -1.7963$, while some also give the two complex solutions, $x \approx 0.8982 \pm 1.1917i$. For example, the screens in Figure 3, taken from a Casio fx-9700 graphics calculator, shows all three solutions, obtained from entering the equation coefficients into the calculator:

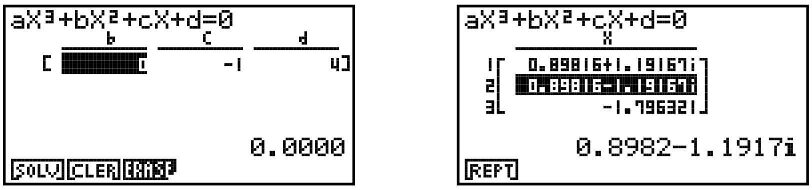


Figure 3 Solutions to $x^3 - x + 4 = 0$ on a graphics calculator

In contrast with solutions not using a graphics calculator, iterative solutions can take many minutes.

In this context, the instruction to 'solve' needs to be interpreted as 'solve approximately', and decisions about levels of accuracy and whether or not real or complex solutions are required are left to students. In many cases, but not the present one, such decisions can be made in the light of the context in which the task is set. Distinctions between 'solve', 'solve approximately' and 'solve exactly' are an integral part of *Fundamentals of Mathematics*. We have been explicit about the need to solve exactly in an examination, by using the command, 'solve exactly', as described below. At other times, we expect students to exercise their discretion.

The solution of systems of equations such as the one shown above can consume many minutes of student time, regardless of the procedure used to deal with it. A student user of a graphics calculator can solve such a system efficiently, provided they are familiar with the nuances of their calculator's operations and are careful with the order of entry of coefficients. The solution of $x = 39/31$, $y = 126/31$ and $z = 10/31$ was obtained in less than 30 seconds on a graphics calculator, rather than the several minutes that would be consumed without one. While the TI-82 requires sufficient understanding of matrices to solve linear systems, this is not the case for all calculators. Figure 4 shows the solutions to the system on a Casio fx-9700 into which the coefficients have been entered. Solutions are given both as decimals and as fractions.

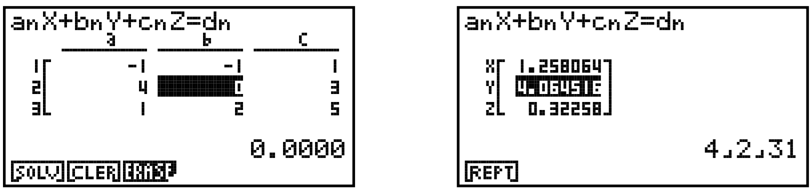


Figure 4 Solving a system of three linear equations on a graphics calculator

When coefficients are non-integral or solutions inconvenient for checking (such as the present ones), the balance swings dramatically towards calculator solution. Such factors can be taken into account in designing questions.

\x(3)Graphics calculator is used as a scientific calculator only

Any calculation that can be performed on a scientific calculator can be performed on a graphics calculator. This may seem to imply that there is no advantage in using a graphics calculator, but this is not necessarily the case. Often the extended display and easy editing available on the graphics calculator can aid a long or complex calculation.

> Solve $\sin x = 0.35$ for $0^o \le x \le 90^o$.

Use of the $\sin^{-1}$ key in the first quadrant is similar for both types of calculators. If tasks like this involved multiple solutions, however, some mathematical analysis would be necessary in order to obtain all solutions on the interval specified. In such a case, students with graphics calculators may well choose to make use of them (for example, by drawing a graph to see how many solutions were involved).

> Find the population, P, after $t = 10$ years if $P = 1909e^{--0.02t}$.

The extended display of the graphics calculator is useful, since it allows students to check that they have entered the formula correctly, and to readily make adjustments if they didn't. These lines show the use of a TI-82 with this task:

```
1909e^-.02(10)
          1562.957008
```

As this screen dump shows, the expression can be checked for correct entry before the computations are effected. If further parts of the task require evaluation at different times, the recall expression and editing facilities make this very efficient. In contrast, scientific calculators normally only display one number at a time.

Graphics calculators are expected to be used by some students but not by others

\x(4)Use and non-use of graphics calculator are both suitable

In this case we expect *suitability* to be dependent upon both the particular student and upon the task. This will depend upon the student's level of experience and confidence both with graphics calculators and with the alternate analytical methods.

> During the radioactive decay of a chemical compound the amount C (in grams) of a compound remaining after t years is given by the formula
>
> $$C = 1.40e^{--0.03t}.$$
>
> After how many years would only half of the original amount remain?

Students confident in the use of natural logarithms are likely to use an analytical method here, whilst students confident in the use of graphics calculators may use a graph or a table. If the question had been phrased in terms of *finding the half life,* students may well automatically use a standard routine taught for half lives involving logarithms, rather than think of the most appropriate (for them) method.

> Solve $|4t + 3| > 2$.

Students confident with calculator use may well use a graph to check how many intervals are required, prior to performing the analysis for determining exact endpoints.

Graphics calculators are not expected to be used

\x(5)Exact answers are required

If the main purpose of the question is to assess students’ analytical skills and concepts, then an *exact* answer may be demanded to ensure that students realise that an analytic answer is expected. Examples of *exact* answers are square roots, multiples of π, exponential or logarithmic forms, rather than just a whole number. In most cases, whilst the use of a graphics calculator will not help with the original analysis it will be possible to use the calculator to check the final answer.

Solve *exactly* $e^{x^2-5} = 1$.

The solution required is ±\r(5). This could be found by using a graph or using the solve facility and then trying to recognise the answer as a square root, log or exponential form by trial and error squaring, raising to the power or taking logs of the answer. The screen dump shown in Figure 5 shows one way in which a skilful student might use a graphics calculator to obtain the positive solution:

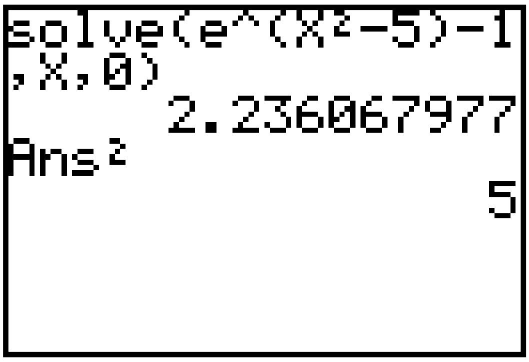


Figure 5: Using a graphics calculator to determine an exact solution

However, we would hope that this was more likely to be a *checking the answer* type of operation rather than the original working out.

\x(6)Symbolic answers are required

The main purpose of this kind of task is to assess students’ concepts and analytical skills, and a numerical answer from a graphics calculator does not provide evidence of these.

Find 2A + B for $A = \begin{pmatrix} 2 & 3 \\ -1 & \square \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix}$.

As indicated in this task, one way to do this is to include variables instead of numbers, thus indicating a necessity to solve without a graphics calculator, so that the processes by which they reach the answers can be seen more clearly. Care should be exercised in the choice of variables used in designing tasks in this way, since if students enter some common variables, such as x, y, t , the calculator will assign a numerical value to the variable. One way to overcome this would be to use symbols not normally available on calculators such as □,□ or □.

> Differentiate these functions. Do not simplify your answers.
>
> $f(t) = \frac{t^3 - 7}{10 - 3t}$ $\qquad$ $h(x) = (x^3 + 5)e^{4x}$

This task requires students to use their analytical skills and demonstrate their ability to differentiate without using a graphics calculator. For most students, tasks of this kind can only be completed efficiently using the standard symbolic manipulation skills. It should be noted, however, that more sophisticated graphics calculators with some symbolic manipulation capabilities have been available for some time, and students with such a calculator could still avoid carrying out manipulations by hand. An example is shown in Figure 5, which contains screen dumps from a Hewlett Packard HP-48 graphics calculator. The calculator version of the derivative is rather inelegant, since the machine has used the product rule rather than the quotient rule, but nonetheless the correct derivative is shown. Although the result has not been simplified, it is a common practice for such answers to be accepted for tasks of this kind, as indicated by the statement of the task itself.

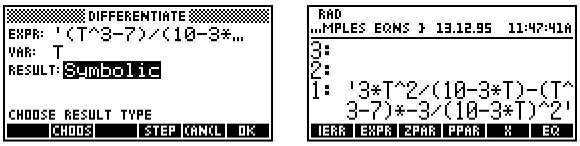


Figure 6: Using a graphics calculator for symbolic manipulation

The prospect of hand-held symbolic manipulation capabilities for students in examinations is a source of some unease to examining authorities, as discused by Bradley (1995) and Taylor (1995). While the solution of prohibiting such calculators in examinations may be an effective short term solution, it is inadequate for the longer term.

\x(7) Written explanations of reasoning are required

This requirement may be designed to elicit sufficient evidence of students' understanding of the processes of a solution, for which they may have used a graphics calculator as a tool, or to assess students' abilities to construct a mathematical argument; the context of the question should make this clear to the students.

> How many solutions are there to the equation $\frac{x+1}{2} = 2\cos x + 1$?
> Justify your answer.

Students may be advised to use a graphics calculator for the first part of the question, or the choice may be left to them. There are in fact several possibilities, described in detail by Kissane (1995). The second part illustrates a context in which students might be required to justify their answer in order to show that they understand how to solve an equation graphically or numerically; the focus is on the quality of the justification.

> Use geometry to explain why $\int_{0}^{1} \sqrt{1 - x^2}\, dx = \frac{\pi}{4}$.

Students need to recognize that the function describes the quadrant of a circle and that the integral evaluates the area of the quadrant. The focus is only on the quality of the explanation.

\x(8) Task involves extracting the mathematics from a situation or representing a situation mathematically

This kind of task is designed to ascertain students' ability to start with a contextualised problem in words from which they need to set up a mathematical model. The students may then be required to find a mathematical solution and translate the result back to the context.

> Amanda noticed that 4 jugs of beer and 3 cups of coffee cost \$28.50, while 2 jugs of beer and 5 cups of coffee cost \$19.50. Find the cost of each of a jug of beer and a cup of coffee.

Students are required to identify the relationship between the variables and set up equations, including possibly a matrix equation. The solution technique would probably depend on the complexity of the coefficients. In this case students are required to actually solve the equations; a different task would be to require students to give the equations which would be needed to find the cost of each of a jug of beer and a cup of coffee, without being asked to solve them.

> A piece of wire of length 60 cm is cut into two pieces. Each piece is bent to form the perimeter of a rectangle, one being twice as long as it is wide and the other five times as long as it is wide. Find the lengths of the two pieces of wire if the sum of the areas of the rectangles is to be a minimum.

This task uses skills similar to those in the previous task but in addition involves visualisation of the situation, and perhaps drawing a diagram before setting up any equations.

\x(9) Graphics calculator use is inefficient

Tasks in this category are designed to assess students' understanding of concepts or procedures and, indirectly, their decision whether or not to use a graphics calculator. The use of a graphics calculator would inevitably make the solution much more complicated and therefore more time-consuming.

Solve (i) $10p - 2(p - 1) = 5(3 - p)$, (ii) $4x - 7 \le 6x + 2$.

Students could use a graphics calculator to solve the equation graphically, or by using a table or a solve facility; however these are all inefficient. The inequality could be solved graphically using a graphics calculator but this also would be inefficient.

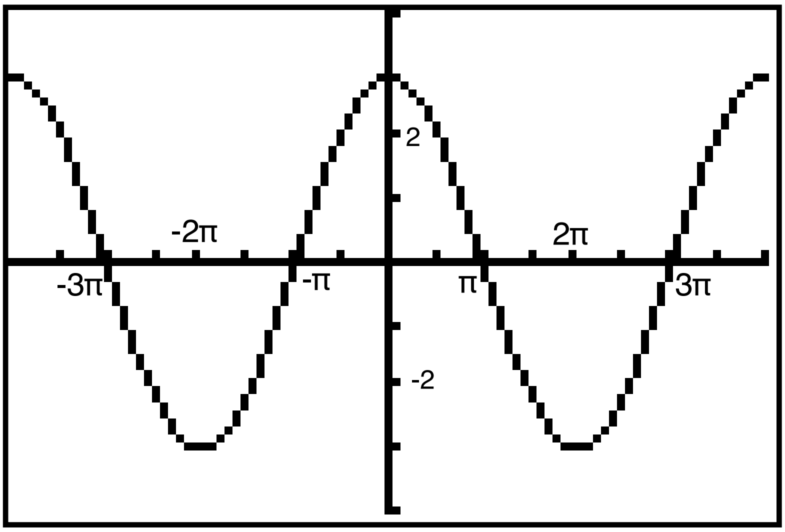
If a population of possums is undisturbed, after t years the population P is given by the growth formula $P = Ae^{kt}$. If the initial size of the population is 65 possums and the initial population doubles in two years, find the values of A and k.

The most efficient way to solve this is to use a logarithmic method. An alternative is to set up two equations in two unknowns and solve them using a graphics calculator. This would make the solution much more complicated, and therefore more time-consuming.

\x(10) Task requires that a representation of a graphics calculator screen will be interpreted

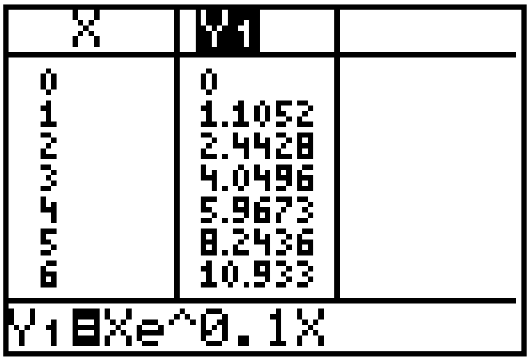
There is a difference between this kind of task where students are required to interpret a graphics calculator screen and the tasks where they actually use a graphics calculator. When using a representation of a screen it is possible to assess skills such as identifying a function, without the students knowing which function has been entered, interpretation of a matrix or table without having to set them up, thus saving time.

Find an expression for the function which is consistent with the graph.



Here students are required to interpret the screen correctly, and use their understanding of how functions are transformed in order to give the correct answer. It is important to note that in the case of a screen representation some indication of scale is necessary, whereas with a graphics calculator screen itself students adjust and interpret the scale directly.

The table below shows some values of $f(x) = xe^{0.1x}$.



For which value of x, to the nearest integer, is $f(x) = 6$?

The screen dump illustrates the table facility on a graphics calculator In this task students need to interpret the values given within the table and to accurately read off the answer.

Awarding partial credit

Long before graphics calculators were invented or their use in examinations contemplated, it has been common practice to award partial credit for partially correct responses to examination questions. The logic of this is that students ought to be given credit for what they have been able to do correctly, even if the original task is not completed in its entirety. Of course, in some cases, the 'working' is the only important part of a solution; an obvious example is any question beginning with 'Prove that ...'. But the issue is not quite as clear cut in numerical situations.

Students have frequently been encouraged to 'show their working' to maximise the likelihood that partial credit can be awarded to them, and to allow them to retrace their steps to check that their working is actually correct. While this is a perfectly sound examination strategy, it is not necessarily a good mathematical habit, however, and has long been a source of tension. Ironically, students may even have been discouraged from acquiring the fluency of their teachers by expectations to show their work in full. When most of the work is performed mentally, especially as the mathematical worker becomes more sophisticated, it is difficult to know how much to show, and in many cases students have been awarded full marks for numerically correct solutions in the absence of any working at all. In such cases, numerically incorrect answers are typically awarded no credit, so the strategy of not providing working can be a risky one in examinations. However, most mathematics teachers themselves would be irritated at being expected to 'show their working' in solving the following equations, and increasingly tolerant, as the age of their students increased, of students writing down only the numerical solutions:

$3x\square + 5 = 26$ $\quad (x + 3)(x - 5) = 0$ $\quad (x - 1)^2 = 9\log 2x = 3$ $\quad x^2 - 4 \leq 0$

When students have access to graphics calculators, a similar issue looms large. Many computational aspects can be automated on a graphics calculator, and a general instruction to students to show their working might result in students writing down detailed calculator steps to reach a solution. This would rarely be a sensible use of student time, and may not even be interpretable to the person reading it unless they happened to be familiar with the particular calculator being used by the student. It seems important to be more cautious and explicit to students in giving either advice or instructions on showing their work in an examination. We suggest that the principles for awarding partial credit shown in Figure 6 are worthy of debate:

> If working is required to be shown, it should be worth showing in its own right, and not only as a means of awarding partial credit.
>
> If only part marks are to be awarded for numerically correct answers, for which working is not provided, then this should be stated explicitly in advance.

Figure 6: Two principles for partial credit allocation

The first principle suggests that a blanket command at the front of an examination paper to 'show all your working' may not be adequate and may even be misinterpreted by students. If our advice to students to show their working is for their own protection (i.e., it will allow them to check their work in case they are uneasy about it), this too may sometimes be misguided. Students with a graphics calculator would be better advised to check their work by using a different solution method (e.g., using a graph as well as a table of values) than by retracing their steps in quest of a slip.

The reason for only allocating partial credit for a numerically correct response is presumably that the student's mathematical argument is also to be assessed. It seems important and fair to make this clear to the student, which is the main thrust of the second principle. Suitable phraseology to do this is needed.

Implications

Having a typology is one thing; deciding what to do with it is another. Two possible uses involve the design of examinations and the analysis of existing examinations. In either case, it is necessary to know very well how a particular graphics calculator works, in order to appreciate the kinds of ways in which it might be used on examination tasks.

Graphics calculators might be taken into account in designing examinations in at least three ways. One approach that has been suggested to accommodate the reality that some students do not have adequate access to graphics calculators, or that calculators differ substantially in their capabilities, is to design examinations for which the effect of the graphics calculator is minimised as far as possible. Such examinations have been referred to as *calculator neutral*, and seem to have been motivated by concerns that some students might be unfairly advantaged by differential access to graphics calculators. Since such examinations need to be designed very carefully to eliminate graphics calculator influence, they are certainly not designed oblivious to the calculator, and are quite likely to be substantially influenced by consideration of calculator capabilities. The typology presented here suggests that tasks in categories 5 to 10 should be relied upon for this purpose.

In our view, however, the use of calculator neutral examinations is an unwise long-term strategy, although it may be seen as helpful in the short term to allay concerns about disparities in student access to graphics calculators. In the long term, such a strategy would send a clear (and incorrect) signal that graphics calculators are *not* of importance in mathematics, and would discourage both students and their teachers from acquiring either hardware or expertise in its use.

Rather than designing examinations that attempt to neutralise the effects of graphics calculators, we might try to design examinations that capitalise on their capabilities, a form of *affirmative calculator action*. Part of the argument for this might be to exploit the new opportunities for mathematical work offered by the calculators. A subsidiary purpose might be to make clear to both students and their teachers that learning to use a graphics calculator well, including learning when it is not wise to use it, is a valued outcome of a mathematical education. For such an examination climate, a concentration of tasks in categories 1, 2 and 4 might be expected; it would seem unwise to *only* have tasks in these categories, however, lest undue distortion of the mathematics curriculum might result.

The third possibility falls somewhere between the first two, with a kind of *smorgasbord of calculator usage* involved. It is possible that we might use the typology to check that there are some items of each of the various kinds in the examination. Although it is debatable whether or not this is educationally sound, one argument in its favour is that it would increase the likelihood that students came to see the variety of ways in which technology of this kind is potentially involved in mathematics. It would seem on the face of it unwise to ensure that there was an even spread of kinds of tasks in an examination according to this typology; rather, it might be expected that there were at least *some* of each kind.

In analysing existing examinations using this typology, we may see our present practices in a new light. For example, it is possible that we regard an examination as forward-looking in its use of graphics calculators, but yet analysis of the items suggests that most of them are in the latter categories, in which graphics calculator use is not expected. Similarly, we may gain a better understanding of our present practices by analysing an examination that has been designed on the assumption that graphics calculators will *not* be used.

There are strategies for modifying existing examination questions to render them calculator neutral that are also exposed by this typology. One of these is to replace some numbers by algebraic variables, thus disallowing numerical solutions, and requiring general solutions. For example, rather than

Solve $(x-2)^2 = 9$.

students might be asked to

Solve exactly $(x-2)^2 = 8$.

for which graphics calculators would not be expected since this is an item of type 5. Similarly, the type 6 item below has been designed to discourage students from presenting solutions using graphics calculators.

Solve $(x-a)^2 = 9$.

A similar strategy is to require students to give an exact solution rather than a numerical approximation.

The revised versions cannot be handled directly on (most, but not all) graphics calculators, although the earlier observation that students can manipulate and interpret their calculator results to get exact answers is also pertinent here. While such changes seem to be a relatively quick fix to a substantial problem, they may share some of the other features of quick fixes, with undesirable and unintended consequences. In the first place, these (artificial) strategies may have the effect of rendering examination questions more difficult than intended (Bradley, 1995; Taylor, 1995) They may also distort the nature of mathematics to an extent, giving the mistaken impression that it is not in fact a useful tool for solving practical problems (for which the numbers are available or for which a numerical result is preferred). But the greatest concern may be that such a strategy in the long term discourages effective incorporation of graphics calculators into mathematics classrooms and curricula, in common with calculator neutral examinations generally.

A second modification strategy involves asking students for a written explanation as well as (or even instead of) numerical answers that are easily obtained on a graphics calculator.

Solve $(x-2)^2 = 9$, and explain why there are only two solutions.

Such modifications may be more defensible educationally.

Conclusion

The sensible use of new technology in mathematics education is of pressing concern, and there are many circumstances where the place of graphics calculators in examinations is of critical importance in dealing with this issue. As the title of this paper suggests, sound use of graphics calculators in examinations will not happen by accident, but rather needs to be a conscious part of the examination design process. The typology proposed and illustrated in this paper has developed from our recent practice, and we have found it useful for designing examination tasks. The development of the typology has also helped us to see relevant educational issues more clearly. We hope that the typology will prove to be a useful contribution to debate in this area, as well as providing a platform for the development of similar analyses involving other forms of technology.

Acknowledgements: The work in this paper was supported by a National Teaching Development Grant from the Committee for the Advancement of University Teaching. We would like to thank Mark Lukas of Murdoch University for his help in reacting to earlier versions of this paper.

References

Bradley, J. (1995). *Technology and assessment – An international experience*. Paper presented at the 5th Annual Australian Bridging Mathematics Network Conference, Batchelor, NT.

Bradley, J., Kemp, M., & Kissane, B. (1994). Graphics calculators: A (brief) case of technology. *Australian Senior Mathematics Journal*, *8*(2), 23-30.

Kemp, M., & Kissane, B. (1995). Integrating technology into undergraduate mathematics. In L. Summers (Ed.) *A Focus on Learning: Teaching and Learning Forum*, (pp. 130-134). Joondalup: Education Resource Unit, Edith Cowan University.

Kemp, M., Kissane, B., & Bradley, J. (1995). Graphics calculators and assessment. In A. Richards (Ed.), *FLAIR: Forging links and integrating resources*, (pp. 235-241). Darwin: The Australian Association of Mathematics Teachers.

Kissane, B., Bradley, J., & Kemp, M. (1994). Graphics calculators, equity and assessment. *Australian Senior Mathematics Journal*, *8*(2), 31-43.

Kissane, B. (1995). How to solve an equation. *Australian Mathematics Teacher*. *51*(3), 38-42.

Kissane, B., Kemp, M., & Bradley, J. (1995). Student reactions to the use of graphics calculators. In B. Atweh & S. Flavel (eds.) *Proceedings of the Mathematics Education Research Group of Australasia 18th Annual Conference*, Darwin, NT, 366-372.

Taylor, M. (1995) Calculators and computer algebra systems: Their use in mathematics examinations. *Mathematical Gazette*, 79, 484, 68-83.

* A version of this paper is published with the same authors and title in 1996 in *The Australian Senior Mathematics Journal*, 10(1), 36-50.
The paper is available on the web at http://wwwstaff.murdoch.edu.au/~kissane/papers/asmj96.pdf