Personal technology in the junior secondary school*

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Relationships between technology and mathematics education are frequent items for discussion in most countries these days, for understandable reasons. All around us, there is abundant evidence of the developments of new technologies that impact on everyday life in both developed and developing nations. In the short space of thirty years, for example, computers have changed from being very large devices available only to a privileged few in universities and research institutions to being portable devices evident in small businesses everywhere and an increasing number of homes. Mathematics itself has been significantly affected by these developments, and many university courses have been modified to account for this. The focus of this paper is on the technology available at a personal level to junior secondary school students; the emphasis is on technology for all students at this level, not technology for a more affluent or more interested minority of students.

Primary school curricula of today are increasingly likely to assume or to mandate personal student access to a basic calculator, while in the upper secondary school, students are increasingly likely to have access to a graphics calculator. (E.g., Australian Association of Mathematics Teachers, 1996) The focus of this paper is on appropriate technology for students in the lower secondary school. For almost two decades now, secondary curricula in Australia (and many other countries) have been designed on the assumption that students will have access to a scientific calculator, so that personal ownership of a scientific calculator is widespread at this level.

It seems reasonable that students need access to more sophisticated technologies for mathematics as their mathematical sophistication increases. While microcomputers offer some interesting possibilities for student learning, there are very few situations in most countries in which unrestricted personal access to a microcomputer is a realistic option for junior secondary school students, so that a concern for technology for all rapidly translates into a calculator for all. The transition from primary to secondary schooling is generally accompanied by a transition from an arithmetic calculator to a scientific calculator. A major purpose of this paper is to argue that this is no longer an optimal strategy, and that junior secondary school students ought to be provided with access to a graphics calculator rather than to a scientific calculator.

Portability is a critical aspect of technology. A graphics calculator is potentially available to students wherever they happen to be: in a mathematics class, in an examination, in a science class, at home or travelling between home and school. Accessibility has very important implications for curriculum development, which must rest on realistic assumptions of access to resources for learning. While it is most unlikely that mathematics curricula will be developed to accommodate the teaching and learning opportunities offered by computers, it is more reasonable to expect changes to accommodate the potential uses of graphics calculators.

Economic arguments

As with their predecessors, arithmetic and scientific calculators and, indeed, more powerful forms of technology such as microcomputers and graphics calculators have fallen significantly in price since their introduction more than a decade ago. Although prices have fallen in absolute terms, it is of more significance that the comparative price of graphics calculators and other forms of technology for lower secondary mathematics has changed over the recent past. At a time when there appear to be increasing pressures to provide computers in schools, partly for mathematics, it is timely to consider alternative uses to which equivalent funds for computer purchase could be put. The nature of such pressures, and responses to them, clearly differ among countries, although there is evidence of such pressures in both relatively wealthy nations like Australia and in less affluent developing nations in the Asian region.

For some time now, a school or school system could provide a class set of graphics calculators (one for each student to use) for around the price of a single microcomputer for the classroom, equipped with suitable software. The development of entry level graphics calculators, such as Casio's fx-7400G, Sharp's EL-9400 and Texas Instruments' TI-80 has rendered this comparison even more significant. Although graphics calculators are generally much less powerful than computers, either mathematically or educationally, the cost of providing *individual* access to suitable technology for learning and doing mathematics is much lower for graphics calculators than it is for computers. The notoriously difficult problems of ensuring that a single classroom computer provides educational benefits for *all* the students in a particular classroom, rather than merely a small proportion of them, render this sort of argument more persuasive.

In the late 1970's in Australia, some schools experimented with purchasing class sets of scientific calculators, but the practice quickly died out. Practical difficulties of managing such sets were partly the reason for the change, but the main reason was that personal ownership (or long-term loan) of calculators was seen to be advantageous educationally and practically. In such countries, it has been common practice for some time to ask students to purchase their own scientific calculator. Most students will probably use their scientific calculator for at most three years, however, since they will need to replace it with a graphics calculator for upper secondary school by the beginning of Year 11, at the latest; in many cases, the actual use of the scientific calculator may be as little as two years, as increasing numbers of students acquire graphics calculators in Year 10. The market for used scientific calculators does not look buoyant, so that students and their parents are unlikely to be able to recover any of their investment when they are later asked to purchase a graphics calculator. If the money presently spent on a scientific calculator were *instead* put towards a graphics calculator, some of the costs to students of owning a graphics calculator would be allayed.

Some students may need to own a more sophisticated graphics calculator than entry level machines when they move to the upper secondary school, in which case they are likely to be able to sell their original calculator second hand and trade up appropriately. The three manufacturers of inexpensive graphics calculators have all attended well to the transition from the smaller to a larger and more sophisticated calculator, so students have few difficulties adjusting to the new machines.

The calculator as a computational tool

An important reason for students to have some level of access to a calculator is that it provides them with a flexible computational tool. (Kissane, 1995a) A scientific calculator is generally recognised as a more useful computational tool than an arithmetic calculator. For example, it provides access to larger and smaller numbers (routinely using scientific notation when necessary). However, it is too little recognised that the scientific calculator is in fact remarkably limited in capabilities. Most scientific calculators are useful for three major purposes. They can handle arithmetic, they obviate the need for a table book and they can deal with numerical data analysis. None of these capabilities is of much use for learning mathematics: they provide tools for computation, but do not provide much support for learning mathematics. Ironically, most of the arithmetic that students actually encounter in lower secondary school could be handled with an arithmetic calculator, the common inbuilt tables (such as trigonometric, logarithmic and exponential) are not generally needed until fairly late in lower secondary school, and the data analysis capabilities (which are quite limited) are not really exploited until the senior secondary school. It can be argued that a slightly more sophisticated arithmetic calculator (handling fractions, for example) might serve student needs from around the middle of the primary school until they own a graphics calculator. (Kissane 1997c).

Compared with most scientific calculators, a graphics calculator is a better tool for students to use. Graphics calculators (as well as many newer scientific calculators) use standard mathematical syntax, so that functions are entered into the calculator in the same ways in which they are conventionally represented in textbooks and in handwriting. For example, $\sqrt{7}$ is evaluated by first pressing the radical sign, and sin 30 is evaluated by first pressing the sine key. Graphics calculator screens show what has been entered as well as the results of a computation, so that entries can be checked and, if necessary, edited. Several lines of calculation are shown, so that one's working can be traced to an extent. Rather than labelling calculator memories with numbers, alphabetic labelling of memories is convenient, and supports algebraic work. For example, a graphics calculator allows an expression such as $\pi R^2 H$ to be evaluated directly, after values are stored in the R and H memories; as for algebraic conventions, implicit multiplication is interpreted correctly. Generally speaking, the inbuilt mathematical capabilities of even the entry-level graphics calculators generally exceed those of scientific calculators used by students, while calculator programmability allows missing features to be added fairly easily. In summary, the graphics calculator is a much more powerful computational tool than is a scientific calculator, and is a good deal easier to use, both of which are important for junior secondary school.

The calculator as an exploratory device

While it is certainly convenient for students to have access to a computational tool, the significance of a graphics calculator for education is that it provides much more than this. Indeed, we have long recognised that efficient numerical computation is not the main purpose of school mathematics, even though popular and public conceptions of mathematics frequently seem to think that it is. The most significant educational arguments for the use of technology concern the opportunities for learning provided by student exploration. The learning opportunities provided by graphics calculators are generally not available with scientific calculators, since many of the opportunities are associated with using the calculator as an exploration device, rather than as a tool for computation, the main use of scientific calculators. A graphics calculator can provide students with opportunities to engage with mathematical ideas and concepts in an exploratory fashion, thus building a base of experience upon which concepts can be developed. (See Kissane (1995a & 1995b) for an elaboration of this argument.)

Important opportunities for learning are missed when students do not have access to appropriate personal technology. The rest of this paper suggests briefly what some of these opportunities are. Most of them start to be evident when students begin to study functions and graphs, in Australia typically around Year 9, but they are not restricted to the graphical. Indeed, like the myth that students still need a scientific calculator, the myth that graphics calculators are mainly for drawing graphs seems common, as elaborated in Kissane (1997a). These kinds of opportunities are accessible when students have their own calculator, and have become accustomed to using it; it is much less likely that they are really accessible to students reliant on a class set of calculators occasionally brought into the classroom.

Some examples

The remainder of this paper consists of some brief examples of the kinds of advantages and opportunities potentially accessible to students with a graphics calculator. These examples are not intended to be, and are not, exhaustive of all the possibilities: space constraints clearly preclude such an undertaking. Rather, they are chosen to illustrate some of the ways in which exploratory opportunities are provided by graphics calculators, but are unavailable with scientific calculators,

Understanding functions

Graphics calculators can be used to provide access to a lot of information about functions quickly; modern calculators allow symbols, graphs and tables of values associated with functions to be explored with ease and for students to move quickly from one representation to another. Without such a capability, students are restricted, as they have been in the past, to constructing tables of values by hand and then using them to sketch curves. While some activity of that kind is useful, it consumes a great deal of student time, much of which can be better spent interpreting the calculator's versions of the same things. Interpretation of graphs can be emphasised, rather than mechanics of graphing.

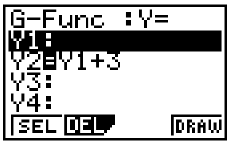
With such capabilities available to students, new learning tasks to take advantage of them become possible, such as the following extract from Lowe et al (1994), which would not be practical were students restricted to hand sketching of graphs:

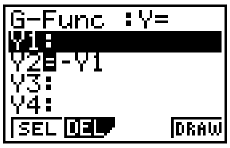
" For each of the following sets of quadratic relationships:
- Draw the four graphs on the same pair of axes.
- Discuss with your partner how the four graphs are alike and how they are different. Write a sentence or two about your conclusions.
- Write the equations of two more graphs which belong in the same group.

a	$y = p^2 y = p^2 + 1$	$y = p^2 + 2$	$y = p^2 + 3$
b	$y = p^2 y = p^2 - 1$	$y = p^2 - 2$	$y = p^2 - 3$
c	$y = p^2 y = 2p^2$	$y = 3p^2$	$y = \frac{1}{2} p^2 + 3$ "

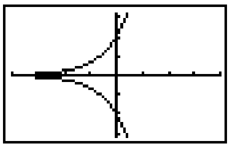
In addition, calculators can deal easily with transformations of functions, which can be used to aid understanding. The following task, from Kissane (1997b), shows how students can be provided with learning experiences of this kind:

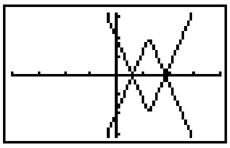
Make a *function transformer*, by defining the second function in the function list in terms of the first function in the list. The screens below shows two possibilities:

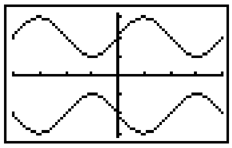




Use your function transformer with an assortment of functions, to see what effects it has. For example, here are two possibilities for the ‘opposite’ transformer on the right above:







Study the effects of one transformer on several different kinds of functions, until you know exactly what effect it has. Compare your observations and conclusions with someone else.

Equations

A calculator provides access to the conceptual links between equations, functions and graphs, through the exploration of graphs of functions and tables of values of functions. In addition, equations that could not be solved analytically can be numerically solved on a graphics calculator by relatively simple processes of successive refinement, such as tracing and zooming graphs or zooming in tables of values. Although some equations can be solved exactly (notably linear and quadratic equations), students with a graphics calculator can deal with practical situations involving many other kinds of equations, obtaining contextually appropriate numerical approximations to solutions. There are a number of examples of this in Lowe et al (1994) and Kissane (1997b). As well as finding good approximations to solutions, students can readily appreciate the importance of first deciding whether or not an equation has any solutions and, if so, how many there are likely to be within a certain interval.

Chance and data

All graphics calculators contain commands for generating uniform random numbers, as indeed do most scientific calculators. However, graphics calculators allow for random numbers to be transformed to suit a particular situation (such as the simulation of a dice roll), to be accumulated easily (such as the simulation of a 100 dice rolls) and for the results to be informally summarised and analysed (either numerically or graphically). Together, these provide opportunities for rich explorations with chance phenomena, to build a base of experience for the later formal study of ideas in probability. A range of examples of this kind is provided in Kissane (1997b; 1997d).

A critical difference between scientific and graphics calculators concerns the storage of data. In a graphics calculator, data are stored in the calculator, while for a scientific calculator, only summaries of the data are stored. Consequently, with a graphics calculator data entry can be checked, data can be edited, outliers can be deleted and data can be transformed. Together, these provide much more powerful opportunities for analysing data. For example, alternative analyses of the same data set are possible, while both numerical and graphical summaries can be chosen and studied. Data transformations can be undertaken for either practical or conceptual use. For example, data can be transformed from one unit to another or residuals from a linear model can be constructed. Large sets of simulated data can be analysed efficiently. The focus of student activity can be placed on ensuring that data are faithfully entered into the calculator, choosing an appropriate form of analysis and interpreting the results in the context of the original data collection, rather than a focus on computation and drawing, frequently observed when students analyse data with more primitive technologies at their disposal.

Personal finance

Most school curricula at this level include some mathematics associated with personal finances, such as that concerned with interest on loans and investments. Without access to technological help, students are restricted to using financial formulas and tedious procedures to compute amounts such as compound interest, and are generally not able to undertake analyses of practical situations (such as those involving reducible interest). However, the numerical capabilities of graphics calculators allow lower secondary students to solve practical problems with efficient and understandable numerical methods. In many countries, students leave school without the mathematical capability to solve problems of the following kind:

> Suppose a family borrows $9000 to buy a car. The terms of the loan are that they must make a payment of $350 each month, and interest of 1.5% of the remaining balance of the loan is added each month. How long will it take to pay off the loan?

A graphics calculator provides ready access to numerical solutions of such a problem and, importantly, allows for variations such as the following to be explored:

> How much longer will it take to pay off the loan if the family can only afford to pay $300 every month?
>
> What will be the effect of an interest rate rise? For example, suppose the monthly rate is 1.8% instead of 1.5%; how much difference will this make to the time needed to repay the loan?
>
> How much more quickly will they pay off the car if they first pay a deposit of $2000 or trade in an old car for $2000, so that there is only $7000 to pay?

Later study of financial aspects of mathematics may be appropriate for some students, especially those who will make careers in the corporate world. However, all students ought to be entitled to leave school equipped to understand and to complete personal financial calculations of these kinds, which are not difficult if graphics calculators are available to them.

Conclusion

As noted above, the examples in the previous section are not exhaustive and no claim for completeness is made. Indeed, the value of a graphics calculator is related in no small way to the flexibility with which it can be employed for a variety of purposes, by both teachers and their students. So, it would not be sensible to attempt to produce the definitive set of ways in which a graphics calculator might be used in the lower secondary school, any more than it would be sensible to produce a myriad of 'calculator activities' in order to spice up the mathematical programme a bit.

A useful organising principle for student use of technology might be similar to that espoused by the Calculator Aware Number project, directed by the late Hilary Shuard in the

UK in the 1980's, and which involved allowing infants children unrestricted access to basic calculators:

> Children should be allowed to use calculators in the same way that adults use them: at their own choice, whenever they wish to do so. ... A significant finding from the project is that calculators should be viewed as an item of multi-purpose mathematical apparatus and that teachers do not need to design specific tasks to bring them into play. (Shuard et al 1991, p. 12)

It now seems time to make use of the personal technology of the graphics calculator in the junior secondary school in an analogous way, and to reconsider the experiences provided to students in various parts of the curriculum.

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