Technology for the 21st century: The case of the graphics calculator

Barry Kissane
Australian Institute of Education, Murdoch University, WA, Australia 6150
kissane@murdoch.edu.au

ABSTRACT:
The significance of technology for mathematics education into the 21st century is critically related to its educational aspects. The daily realities of societies, schools, teachers and curricula demand that technology is affordable, available and readily adaptable to accepted educational purposes if it is to be of importance to education. These constraints suggest that the graphics calculator merits special attention. Issues of economics, curriculum development, assessment and the professional development of teachers are addressed. The relationship of the graphics calculator to particular parts of the secondary school curriculum is highlighted, including algebra, calculus, probability and statistics; emphasis is placed on the potential of the calculator to permit student exploration in such areas. While secondary mathematics education in all societies in the early years of the 21st century can benefit by closer attention to the graphics calculator, particular attention is paid to the case of the developing world.

The closing years of the twentieth century have been a time of unprecedented change in the human condition. Among the changes have been mounting population pressures, urbanization, globalization, mechanization, greatly improved communication systems and topsy-turvy economic changes, to say nothing of the recent substantial political and military changes. Many of these changes are related to the development of new technologies such as telecommunications technology, computer technology, medical technology, agricultural and manufacturing technologies. Most of these changes have impacted sharply upon education in both developed and developing countries. Today we have increased expectations for education despite increased populations in school, with a declining quality of teaching force in many countries and a generally unsatisfactory funding base almost everywhere. This paper focuses attention on one small aspect of a large complex of issues: the place of technology in secondary school mathematics.

Even within education, the term 'technology' has many meanings, dating back to the time of Aristotle. For example, the multiplication algorithm for whole numbers can be regarded as a technology, as can the whiteboard and the compass. However, for the purposes of this paper, the term is restricted to refer to the electronic technologies of calculators and computers, as a careful consideration of these is arguably one of the most pressing problems for mathematics education at the turn of the millennium. Similarly, while the issues related to these technologies are important for mathematics education from kindergarten right through to graduate school, attention in the paper is focused on the secondary school.

Technology and mathematics education

It is possible to consider the relevance of technology to mathematics education from a number of different perspectives. These include the societal perspective, the mathematical perspective, the occupational perspective and the educational perspective.

Societal. It is clear from even a casual glance around the developed world and increasingly just as clear in the cities at least of the developing world that our societies have become steeped in modern technologies. In part because of the increasing evidence of globalism and in part because of rapidly declining costs, commerce and industry have accepted the microcomputer into their enterprises. Electronic funds transfer is increasingly commonplace, retail outlets routinely use sophisticated cash registers and internet addresses are becoming as prevalent as street addresses. Even in the apparently unsophisticated world of markets and bazaars, the four-function calculator is widely used in my experience throughout the developing countries of South East Asia. It would seem most unwise for education in general and mathematics education in particular to be unaffected by these significant societal changes. John Kenelly, a pioneer of the use of technology in mathematics education stated the case well from the perspective of the developed world:

Technology is changing the way we teach. Not because it's here, but because it's everywhere. Life itself revolves around electronics and we are in the information age. Today's automobiles have more computing devices than the Apollo capsule. Business is a vast network of word processors and spreadsheets. Engineering and Industry are a maze of workstations and automated controls. Our students will have vastly different careers and we, the earlier generation, must radically change the way that education prepares a significantly larger part of the population for information intensive professional lives. (Kenelly, 1996, p24)

Mathematical. There is of course a significant underlying mathematical presence in modern technologies. The scientists and technologists who develop computers and calculators rely heavily on mathematical expertise, concepts and thinking, as Devlin (1998) has made clear to the public. The mathematics associated with technology has given rise to increased interest in discrete mathematics, numerical analysis, computer modelling, Monte Carlo procedures, algorithmic thinking, computer programming and even the study of binary arithmetic. The incorporation of some elements of these into mathematics education at the secondary school level has reflected a growing awareness of their significance for today's students.

Occupational. While the market place, the factory, the shop and the bank have all been affected by technology, the impact upon mathematics, science, engineering, economics, finance and technology professionals has of course been very much greater. The computer is a necessary part of design and manufacturing, scientific work of many kinds, essentially all statistical analysis and increasingly, even mathematics itself in tertiary institutions. The penetration of computers into occupations requiring almost any level of undergraduate mathematics has been as deep as it has been rapid; the amount of change in a single generation has been breathtaking, and it is now clearly essential for undergraduates in the quantitative areas to develop some expertise with modern technology in order to be employable. It is inevitable that these changed expectations will have flow-on consequences for mathematics education in schools.

Educational. An educational perspective on technology focuses upon issues of teaching and learning. To an extent, computers have been adapted to educational uses in mathematics, notably through the production of educational software, but also thorough the use of software designed for other purposes. Computers have been used for demonstration purposes by mathematics teachers and for learning purposes by mathematics students although issues of access and software have remained problematic. In recent times, the Internet also has been suggested as an educational tool of significance. The most prevalent technology has been the calculator, however, and arguably the most important has been the graphics calculator, which (unlike its forebears) was developed for educational purposes and is still mainly found in educational settings.

It is suggested that the educational perspective is the most appropriate one to take when considering technology for secondary mathematics education into the 21^{st} century. The next section suggests that the graphics calculator is the most promising example of technology for secondary school mathematics into the near future.

The graphics calculator

Graphics calculators have now been available to the public since 1985 in the USA and a little more recently in other developed countries. It thus seems slightly strange to be describing it as an example of a 'new' technology, although there are still many settings internationally where this is an appropriate description, most notably in the developing world. Unlike other forms of technology, the graphics calculator is almost exclusively used within educational settings, for which it has been custom developed.

Since 1985, graphics calculators have undergone a number of changes. At present, there are four major international companies manufacturingcalculators at a range of levels. For the most part, the calculators available today represent the latest in a series of iterations of calculator features and design. Three levels of sophistication are identifiable. At the lowest level are calculators (such as Casio's fx-7400G and Texas Instruments' TI-80) designed with the needs of early to middle secondary students in mind, focussing on computation, function graphing and tabulation and data analysis. Mid-level calculators (such as Casio's cfx-9850G+, Hewlett Packard's HP-38G, Sharp's EL-9600 and Texas Instruments' TI-83) include more extensive graphing and numerical equation solving capabilities as well as providing access to matrices, complex numbers, recursively defined functions, numerical calculus capabilities and elementary statistical hypothesis testing. High-end calculators (such as Casio's Algebra fx 2.0, Hewlett Packard's HP-48G and Texas Instruments' TI-89) are distinguished by their symbolic manipulation capabilities in addition to the capabilities of less sophisticated machines, allowing for most of the standard symbolic work of elementary algebra and calculus to be completed. All graphics calculators are programmable with a fairly primitive, but accessible, programming language and have a reasonable amount of long-term memory storage. Most calculators above the low end models have communication capabilities for transfer of information between similar models or between a calculator and a computer.

There are three aspects of the graphics calculator which help to make it is a most satisfactory educational device. Firstly, its physical size makes it much easier for students and teachers to use than a computer. It is small enough to slip easily into a school bag, can be readily taken from class to class by a student, is portable enough to be taken to an examination or out of doors and can be readily transported in bulk (enough for a whole class) by a teacher. All of these attributes are of considerable educational and practical significance, when comparisons are drawn between calculators and computers.

In the second place, graphics calculators are an economically more affordable form of technology than other kinds of computers, such as laptops or desktop computers. In developed countries such as Australia, a class set of graphics calculators costs around the same amount of money for a school as does a modern computer, especially if the cost of the computer software is taken into account. Indeed, in many parts of the developed world, most students can afford to purchase their own graphics calculator or pay a hire charge for its exclusive use over the course of two or three years. While individual ownership is not a reasonable short-term expectation in most parts of the developing world, it is still much easier for a school or school system to provide a measure of effective access to graphics calculators than to computers.

The third aspect concerns the calculator capabilities. Since the software is built into the calculator, it does not have to be upgraded or maintained, certainly a major problem for much software for educational computing. Indeed, the calculators of today have been designed with educational help and advice to provide the main kinds of mathematical capabilities that students are likely to need at a personal level. As graphics calculators are relatively less sophisticated than other computers, it is relatively easier for students to learn to operate one, and international experience suggests that students can do this remarkably quickly, especially with the relatively user-friendly models of today. The use of batteries rather than electricity means that a graphics calculator is not dependent on a reliable power supply, which is a fragile expectation in several developing countries.

Unlike the commercial world, the educational world is not competitive, so that change cannot be expected to come about as a consequence of competition. In fact, change is only likely to come about when there is a reasonable likelihood that all components of an educational system can change at the same time. For this reason, a form of technology that is not fairly likely to be accessible to all members of a groups students, teachers and schools in an educational system or a country is unlikely to be seriously adopted. (Kissane, 1995) Thus, although graphics calculators are considerably less sophisticated than desktop, laptop or mainframe computers, they enjoy a number of educational advantages which collectively render them much more accessible for the great majority of secondary school students. Indeed, to paraphrase Schumacher, graphics calculators might well be regarded as "... *intermediate technology* to signify that it is vastly superior to the primitive technology of bygone ages but at the same time much simpler, cheaper and freer than the super-technology of the rich." (1974, p.128) To continue this analogy, the 'super-technology of the rich' in many countries is still represented by the computer, which has not yet been taken seriously by educational authorities in devising curricula or examination structures. While computers continue to be genuinely accessible only to the few, it will continue to be the case that graphics calculators represent a superior form of technology for mathematics education. While they may well represent a form of 'intermediate technology', it will be surprising if they are not of continuing importance at least into the first decade of the 21st century in many countries and particularly in developing countries.

Curricular relevance

As suggested above, graphics calculators have direct relevance to typical secondary school curricula. In this section, some examples of the ways in which this relationship can be exploited educationally are suggested. Of necessity, given the constraints of space, there is no intention here to describe all the possibilities. Rather, a few examples suggestive of the kinds of explorations made possible with a graphics calculator are given.

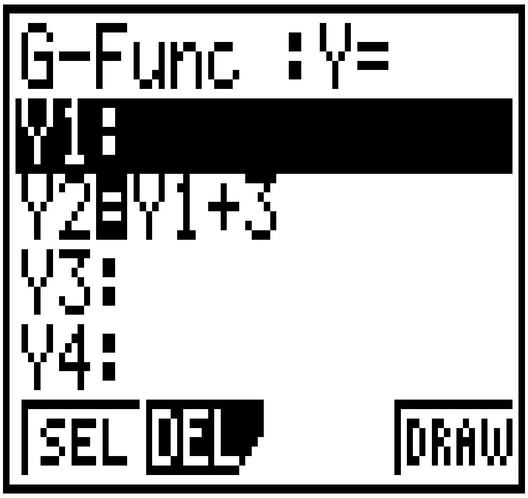
Algebra

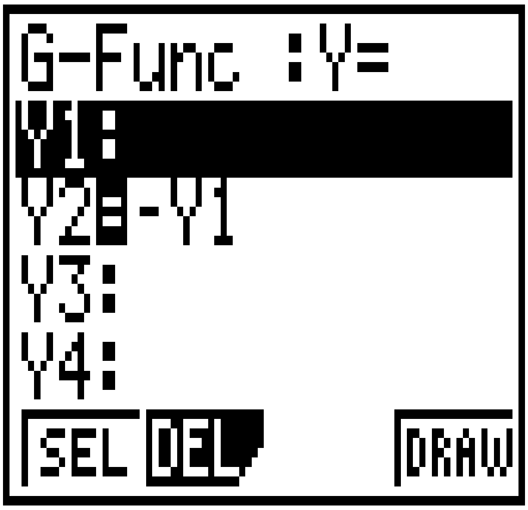
Graphics calculators have the potential to provide students with a variety of educational experiences related to algebra. The most obvious are those concerning the relationships between elementary functions and their graphs, giving rise to the (unfortunate) descriptive term, 'graphing calculator'.

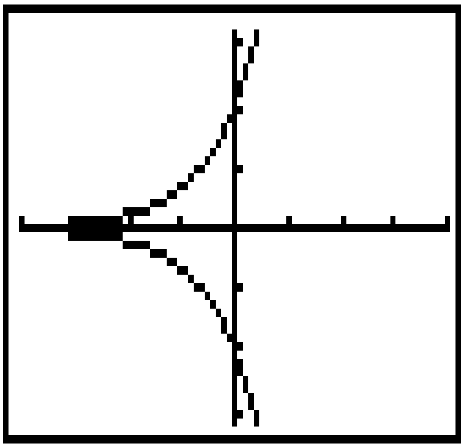
Kissane (1997) describes the idea of a 'function transformer', allowing students to see how the graphs of functions are affected by transformations. The following two screens show how an 'add 3' transformation and an 'opposite' transformation can be set up on the low-level Casio fx-7400G calculator.

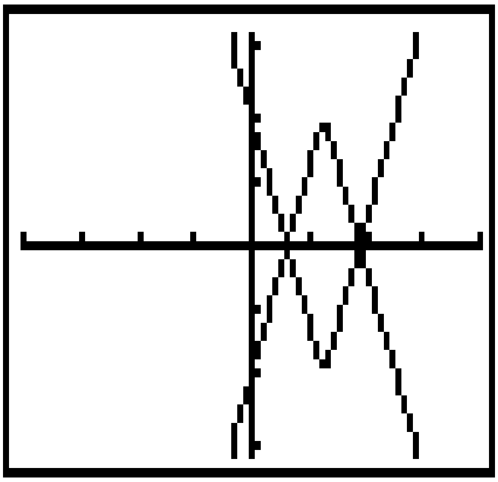
The screens below show some examples of the resulting graphs for the opposite transformer, after various functions have been defined in the Y1 position.

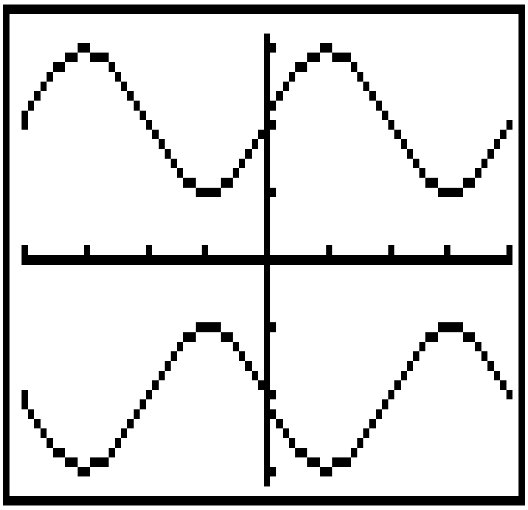
The ease of changing either the original function or the transformer allow students to explore relationships of these kinds efficiently and powerfully.


G-Func :Y=
Y1:
Y2=Y1+3
Y3:
Y4:
SEL DEL DRAW

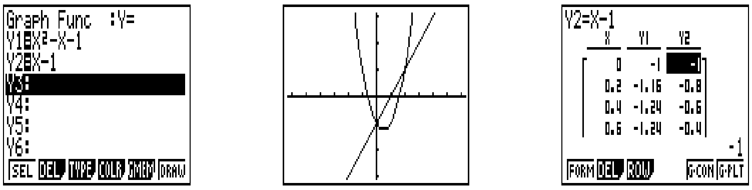

G-Func :Y=
Y1:
Y2=-Y1
Y3:
Y4:
SEL DEL DRAW



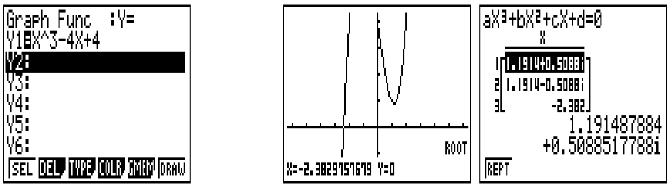




Modern graphics calculators allow for three representations of functions to be manipulated: symbolic, graphical and numeric (giving rise to what is often described as 'the rule of three'). The screens below show an example of this on the Casio cfx-9850G calculator:

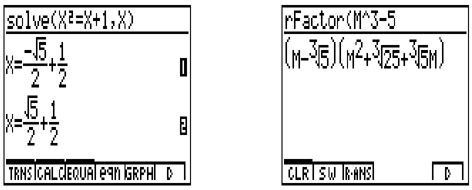


Mid-level graphics calculators like the Casio cfx-9850G allow students to explore relationships between functions, graphs and equations in ways that are not accessible without the aid of technology. The screens below show two (of several) ways of finding approximate solutions of the cubic equation $x^3 + 4 = 4x$. On a graphics calculator, functions and their graphs can be explored by students; prior to technological access, graphs were merely drawn by students.



In this case, the graph shows that there is only one real solution to the equation, while the equation solver shows that there are two additional (complex) solutions.

As their name suggests, algebraic calculators permit routine symbolic manipulation, increasing the responsibility of the student to represent situations algebraically, decide which manipulations are needed and interpret the results. The two examples shown below come from a Casio Algebra fx 2.0



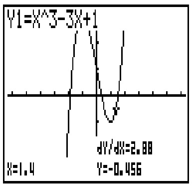
Together, these sorts of activities suggest both new opportunities for students to interact with and to learn about important algebraic ideas and also new demands to reconsider the content and balance of the curriculum, should most students be provided with access to experiences of these kinds.

Calculus

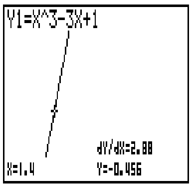
Early graphics calculators were restricted to numerical approximations to calculus concepts such as differentiation and integration. The recent arrival of algebraic calculators offer also symbolic calculus commands which may give rise to anxiety that calculators can be used to obviate important conceptual thinking. However, graphics calculators with calculus capabilities are best used to provide opportunities for students to personally explore and experience the concepts of calculus, as suggested by Roberts (1996):

"The case for the use of technology can be summarized this way. In a course where the goal is to teach the calculus, computers or calculators should only be used when there is a sound pedagogical reason for doing so, in which case they should certainly be used. But when used, thought should be given to whether or not they are being used most wisely for the problem at hand. Their proper role is as a tool for experimenting, for discovering, for illustrating, or for substantiating. That is, they are to be used for developing intuition and insight; they should not be used just so the user can crank out answers to ever larger and more complicated exercises". (pp2-3)

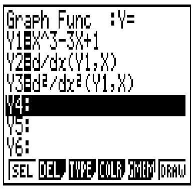
Some examples are given in Kissane (1998a). One of these involves using the derivative trace on the Casio cfx-9850G as shown below on the graph of $f(x) = x^3 - 3x + 1$. Moving the cursor to trace shows the student both the coordinates of each point and the derivative of the function at each point, helping to develop a tangible meaning for the derivative concept.

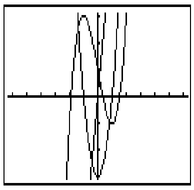


Similarly, access to a graphics calculator allows students to zoom in on the graph of a function several times in succession to see that elementary continuous functions are locally linear. This powerful and fundamental idea provides an opportunity to explore the concept of the gradient of a *curve* instead of the much more troublesome idea of the gradient of a *tangent to a curve*, an idea originally popularised by David Tall using computer software.



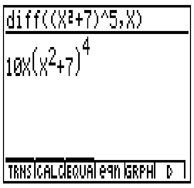
While a calculator can evaluate the derivative of a function at a point, a major consequence of this for developing insight is its capacity to do this for many points at once, leading to the idea of a derivative *function*. The screens below show an example of this.

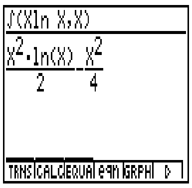


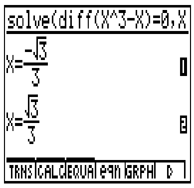


Students can learn much of importance about functions and their derivatives by actively exploring sets of graphs like these. For example, the relationships between the intercepts and the turning points of the graph of the derivative function and that of the original function are especially illuminative.

Generations of students have interpreted the calculus as a complicated system of symbolic manipulations. When students have access to calculators such as the Casio Algebra fx 2.0 which deal with routine manipulations such as those shown below, it is possible that more time and encouragement to concentrate attention on the underlying ideas and their significance will become available:



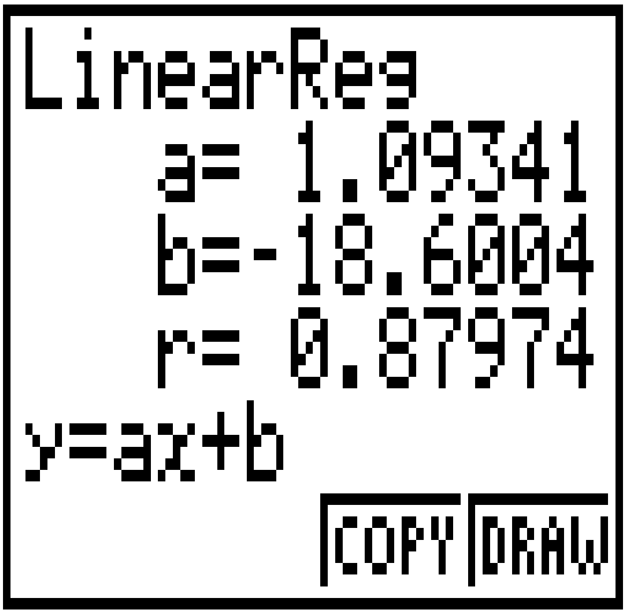


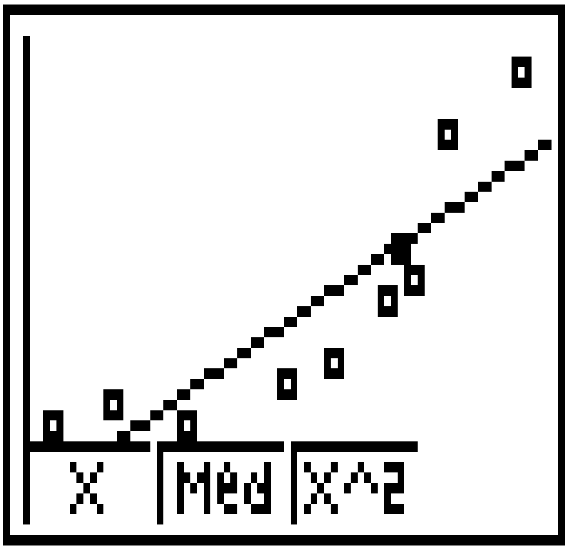


Probability and statistics

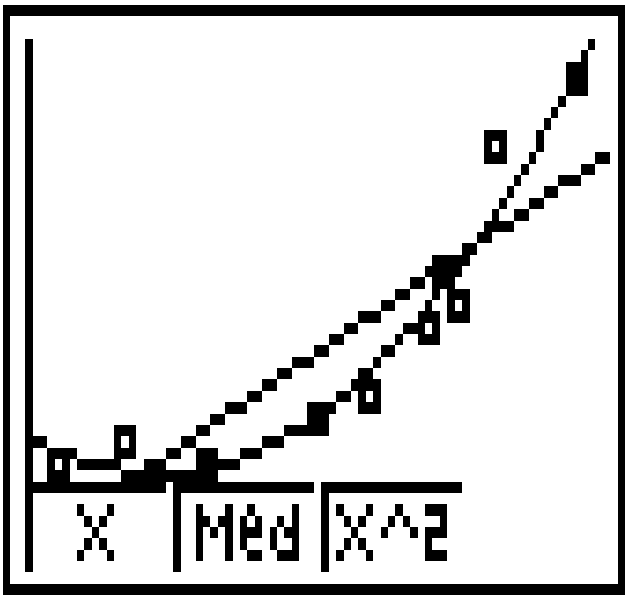
Apart from the obvious capability to represent data graphically, graphics calculators differ from scientific calculators as far as data analysis is concerned in a fundamental way: the actual data are stored in the calculator. A consequence of this is that the data can be checked, edited and transformed and can also be analyzed in a number of different ways. These capabilities together suggest that a graphics calculator permits students to engage in data analysis not unlike the opportunities provided by a computer statistics package, albeit in a scaled down way. For example, the

Casio fx-7400G screens below shows a small scatter plot with a linear regression line and some associated numerical data.



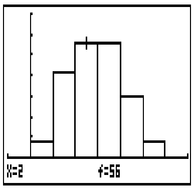


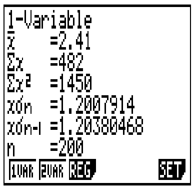
In this case, it seems that a linear model is not particularly appropriate for these data. The calculator allows a student to explore the appropriateness of a different model for the data, as shown on the screen below. In this case, a curvilinear model seems to fit the data quite well.



A graphics calculator allows a student to readily store and to analyse her own data, collected in order to solve her own problems. The task of the student with such a calculator is to make decisions about her data, making good use of the analytic tools provided by the calculator. This is of course the essence of data analysis.

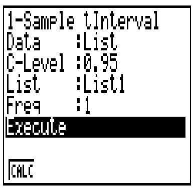
Graphics calculators also permit data to be generated at random, of value for some kinds of learning experiences. For example, the Casio cfx-9850G screens below show a set of 200 simulated sets of six coin tosses for a biased coin with a probability of 0.4 of landing heads. (That is, it is a simulation of a binomial random variable with n = 6 and p = 0.4.) It is relatively calculator for both numerical and graphical analysis.

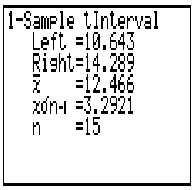




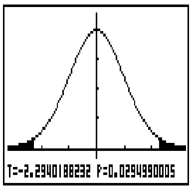
Again, it is difficult for students to get experiences of these kinds without access to suitable technology.

At a more sophisticated level, recent graphics calculators such as Casio's cfx-9850G+ allow for elementary hypothesis testing to be undertaken. The screens below show the construction of a confidence interval about a single mean, using Student's t-distribution.





Similarly, the next screen shows a graphical representation of a two-tailed *t*-test of the difference between two means.



For too many students, the study of inferential statistics has been made more difficult than necessary because of the computational demands of conducting the relevant tests. A graphics calculator allows for attention to be concentrated on the important inferential ideas and how to choose and interpret statistical tests. (Kissane, 1998b). While the same can be said for computer statistics packages, the graphics calculator has the potential to be much more accessible to many more students.

The change process
Adapting to changing circumstances is never easy; the adaptation of mathematics education to new technologies is no different from other ventures in this respect. A critical part of the change is to choose a technology that is relevant to the enterprise and reasonably likely to be available when needed. The earlier parts of this paper suggest that the graphics calculator is an optimal choice for education, even though it is clearly not the most powerful choice available. But there is much more to change than choosing the correct technology. At the very least, curriculum development, assessment traditions and the professional development of teachers need to be carefully considered if genuine progress is to be made.

Genuine individual student access to technology provides a realistic opportunity for associated curriculum development, unlike occasional access through a classroom demonstration computer or occasional access to a computer laboratory or the Internet. Ian Stewart (1995) has characterized some of the recent changes in mathematics itself as a move towards 'experimental mathematics'; in a limited way, graphics calculators provide students with both the opportunity and the machinery to engage in experimenting with mathematical ideas in new ways. Similarly, the graphics calculator may render some aspects of mathematics less important than previously (e.g. curve sketching), while other aspects may take on a new significance (such as iteration and recursion). Our expectations of students and our expectations for mathematics education may both change. For example, an important objective of the mathematics curriculum ought be for students to 'select and use appropriate technologies' for mathematics. (Curriculum Council of Western Australia, 1999). With a wider perspective, Gomez (1996) suggested that the integration of graphics calculators into the curriculum offered some prospect that an alternative vision of mathematics might be entertained and even brought about. The traditional vision has been "essentially procedural and symbolic" (p.61); graphics calculators might be consistent with a wider move towards a new and more personally powerful and engaging vision of mathematics as both an object and a practice. Such new visions of mathematics are already evident in developed countries such as the USA and Australia. As Gomez (1996 p.62) points out, however, graphics calculators may have a quite different relationship with curriculum development in developed countries (where they may be seen to be "a catalyst in a process that is already underway") than in developing countries, where traditional interpretations of mathematics are still very strong.

Issues of assessment are also important ones for many countries, notably those for which formal mathematics assessment has a significant role in student learning. In situations in which public examinations carry significant personal weight (e.g. determining graduation, tertiary entrance or employment prospects), curriculum development must be accompanied by a coherent assessment regime. In the case of graphics calculators, considerable difficulties arise if students are permitted or encouraged to use them during instruction but expected to not do so during examinations. Indeed, one of the advantages of graphics calculators over other forms of technology is that it is realistic for students to use them during examinations and to be expected to have learned to make discretionary and efficient use of them when appropriate. (Kissane, Kemp & Bradley 1996).

Finally, change will not be effective unless the professional development needs of teachers are taken carefully into account and accommodated. In many countries, the present generation of teachers (most of whom will also comprise the next generation of teachers) does not have a history of comfortable relationships with technology. Even within developed countries, many teachers are in workplaces in which computers are not yet everyday devices and inhabit homes in which there is not routine use of computers. Many of them learned mathematics without any real contact with computers or even calculators. Many of them have reservations about the use of technology and are anxious about seeming less capable than their own students (which is in fact generally the case). In developing countries, mathematics teachers are unaccustomed to change in mathematics education. Thus, Gomez observed that many Colombian mathematics teachers were "... ill-prepared both in their pedagogical and mathematical knowledge. They are not accustomed to innovating. Their work is restricted to following the guidelines imposed by an antiquated official curriculum." (1996, pp.64-65.) Similarly, Scott (1996) noted in the South American context that, while there was strong support from many teachers and others for the use of calculator technology, there was also some strong, and understandable, resistance to such changes. For example, he noted that, in Chile:

"There was also some fear expressed that they would not know how to use all the features of more complicated calculators and would lose authority in relation to their students. An important conclusion reached is that the introduction of calculators should be accompanied by a curriculum change strategy that could deal with any detected resistance." (p.172)

Of course, such problems are restricted to neither Colombia nor Chile; nor indeed are they restricted to the developing world. However, the successful integration of technology into mathematics education, including graphics calculator technology, must take them into account in some way. Well-crafted collaborations between mathematics educators, education systems, technology manufacturers and teachers offer the best prospect for successful change.

Conclusion

The graphics calculator represents the best chance for mathematics education to adapt to technological change in the early years of the 21st century, mainly because it is the most likely to be available in sufficient quantities. Taking advantage of this chance will require careful attention to curriculum development, assessment practices and the needs of teachers.

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