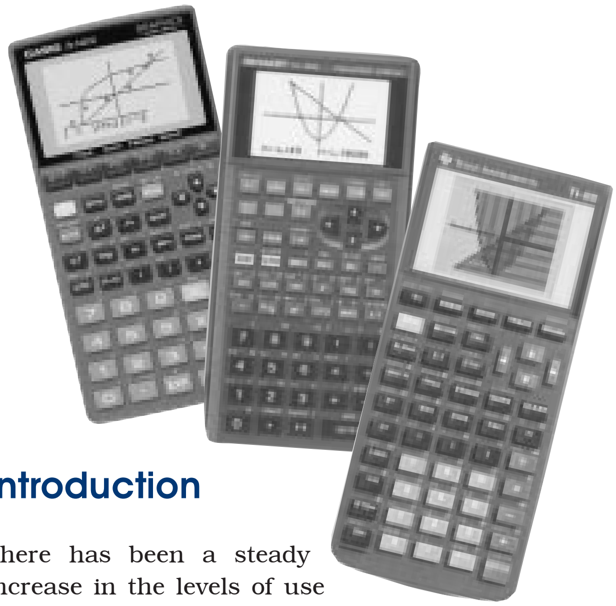
Graphics Calculators and lower secondary mathematics

Barry Kissane and Marian Kemp



Introduction

There has been a steady increase in the levels of use of calculators in both primary and secondary schools over the 1980s and 1990s, despite initial scepticism and caution. Today, the use of scientific calculators has become commonplace in lower secondary school mathematics classrooms in Australia. Although some schools began using scientific calculators through the purchase of class sets for upper secondary school students, these days most students are expected to own and learn to use their own personal calculator and begin to do so from the lower secondary level. This paper is concerned centrally with the appropriateness of lower secondary students now using graphics calculators instead of scientific calculators.

Although they are sometimes referred to as a new form of technology, graphics calculators are in fact an older form of technology than household items such as the compact disk player and have been available to schools now for around twelve years. Calculator capabilities vary from model to model. Less sophisticated versions have the standard calculating capabilities of typical scientific calculators and in addition allow users to draw graphs of functions, produce a table of values of a function, deal with statistical data both numerically and graphically, manipulate lists of numbers and generate random data. Each calculator is programmable and has sufficient memory to store data and programs when turned off. More sophisticated models include a variety of other features, including automatic numerical equation solving, matrix arithmetic, complex arithmetic, numerical calculus, a variety of function graphing and tabulation alternatives, elementary inferential statistics and financial mathematics capabilities.

Present day machines can be regarded as third generation calculators; they have been carefully designed and re-designed with user-friendliness and the particular needs of secondary school students in mind. Unlike computers and even the early versions of scientific calculators, graphics calculators have been designed for the principal purpose of providing secondary school students with a form of personal technology appropriate to their mathematical needs. Of particular relevance to this paper is the observation that there are now versions of graphics calculators specifically designed to meet the needs (and the budgets) of lower secondary school mathematics courses. Three companies (Casio, Sharp and Texas Instruments) market entry-level graphics calculators for this audience, essentially similar to their calculators for more senior students. The differences tend to be in screen resolution, memory size, the range of mathematical capabilities built in and, of course, price. These entry-level calculators already cost significantly less in real terms than did scientific calculators of the early 1980s.

In recent years, graphics calculators have become increasingly popular in upper secondary schools in some Australian states. In at least two states, they are now commonly used by almost all tertiary-bound students and either permitted or expected to be available to students in tertiary entrance level external examinations. Indeed, the acceptance or otherwise of graphics calculators by schools is related in no small way to their official acceptability by examination authorities.

It is against this background that we consider the relevance of graphics calculators for lower secondary level mathematics teaching and learning.

The case for graphics calculators in lower secondary school

The principal argument in favour of any change to conventional practices of mathematics education ought to be related to student learning and the graphics calculator is no exception to this. Graphics calculators provide opportunities to improve student learning that are not readily available otherwise for most students. In particular, they provide opportunities to investigate and to explore mathematical ideas, objects and data in new and productive ways, some of which are described briefly below.

Novice users of graphics calculators, both teachers and their students, do not necessarily notice the kinds of opportunities afforded by this kind of technology. Part of the reason may well be related to the language used to describe the device. For a generation now, we have learned to associate the term ‘calculator’ with ‘calculating’ and hence not surprisingly regard things called calculators as devices to find numerical answers to essentially arithmetical questions. Understandable as this perspective might be, it is not a helpful way of thinking about graphics calculators, mainly because it is too restrictive. In the same vein, a common misconception of new users of graphics calculators is that they are mainly useful for drawing graphs of functions, a perspective fuelled in part by the unfortunate descriptive term, ‘graphing calculator’. Graphics capabilities of calculators go beyond the graphing of functions, however, and include statistical graphs as well as other kinds of visual representations relevant to mathematics. The critical advantage is that graphics calculators allow students to analyse and to explore rather than merely to calculate. Analyses can be graphical or numerical or both. These characteristics together provide significant advances on the capabilities afforded by scientific calculators.

A second kind of argument involves economic issues. As noted above, most Australian students in the lower secondary school are expected to purchase a scientific calculator these days. A rapidly increasing number of students is expected to purchase or make use of a graphics calculator in the upper secondary school, so that they will probably get only two or three years’ use of their scientific calculator. However, for not much more money than the price of a scientific calculator, students can purchase a low-end graphics calculator model instead. While some students may need to purchase later a more sophisticated calculator to suit their mathematical needs, for the majority of students an entry-level calculator will suffice for all of the secondary years. Another way of looking at the economic questions of technological access is to note that a graphics calculator purchased (or hired or borrowed) for the upper secondary years is relatively expensive, since it provides technological support for only two years. If a graphics calculator is purchased in the lower secondary school years, the average annual cost of ownership (or the annual hire charge) is correspondingly more reasonable. Some of these arguments are revisited later in this paper.

Thirdly, choosing and using appropriate technology ought to be part of the mathematics curriculum these days. But lower secondary students cannot be expected to decide for themselves when and whether to use technology to help them to think about a particular mathematical situation if the technology is not available to them. Personal access to technology is critical here. At the moment, for essentially economic reasons, graphics calculators offer the only means for almost all students to have individual access to technology. Although microcomputers or laptop computers are clearly much more powerful then graphics calculators, even an affluent country like Australia is some years away from a situation in which each student in a class has unrestrained access to such devices, except in more privileged schools and communities. Students will not learn to make decisions about the use of technology in a computer laboratory, which is available only at school and only at some times, usually under the control of the teacher. Furthermore, when students are taken to a computer laboratory, it is clear to them that they are expected to use the computers, so that we cannot expect them to develop discrimination skills in such a setting. The situation is similar when a teacher brings a set of graphics calculators to the classroom: clearly they are expected to be used by the students. It is only when students have ongoing, personal access to technology that it is reasonable to expect them to develop the necessary expertise to decide when its use is appropriate or inappropriate and to be able to use it efficiently when the need arises.

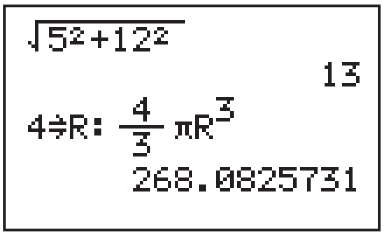
Finally, but importantly, mathematical practices and hence the mathematics curriculum are changing to accommodate technological changes. Some examples of this phenomenon include approximate (and probably iterative) solutions of equations, ‘data analysis’ rather than just ‘statistics’ and probability simulation as a tool for problem solving. As for other areas of the school curriculum, it is reasonable to expect mathematics to adjust to changing societal and technological circumstances. But the curriculum cannot and will not change until it is reasonable to assume that essentially all students have reasonable access to the appropriate technology. It is for this reason that mathematics curricula in schools have been almost entirely unaffected by the microcomputer revolution of the past generation. If it is not tenable to assume that essentially all students have reasonable access to the technology required, then curriculum agencies and authorities will consider it unwise and premature to change. Graphics calculators offer the possibility of a level of assurance that essentially all lower secondary students, rather than just the privileged few, can undertake mathematical work with appropriate technological support. In such a circumstance, curriculum change and improvement are possible.

Some specific advantages

In this section, we note some of the particular advantages for lower secondary school mathematics of using a graphics calculator rather than a scientific calculator. Space precludes a complete treatment of these or elaborated examples. Interested readers are referred to Kissane (1997) and Kissane, Bradley & Kemp (1997) for extensive elaborations.

A better scientific calculator

As noted earlier, a graphics calculator replaces rather than supplements a scientific calculator; consequently, it can be used to carry out calculations of the kinds for which scientific calculators have become convenient. Unlike early scientific calculators, however, graphics calculators use standard mathematical syntax. For example, to find the square root of a number, the radical sign is entered before the number is entered. In addition, the calculator screen shows what has been entered using conventional syntax to do so. What students see on their screens is consistent with what they see on whiteboards and in textbooks. Of major importance is that students can see both their input and their output on the screen at the same time. Calculator inputs can be edited, either to correct errors or to perform a succession of related calculations efficiently. As several lines of calculation are shown, students can trace their working to an extent.



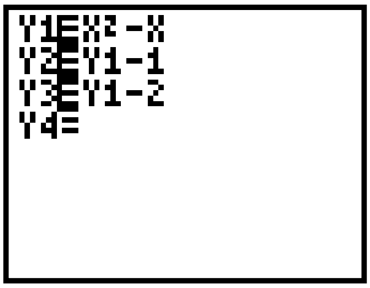
Apart from these kinds of screen capabilities, graphics calculators allow for the storage of data in alphabetically labelled memories. Thus a radius can be stored in a memory labelled R and a height in memory labelled H, so that students are unlikely to forget where values are located. Calculator syntax and normal algebraic syntax are consistent, so that a circumference can be found by evaluating $2\pi R$, with juxtaposition interpreted correctly as multiplication. For students encountering early ideas in algebra and the use of formulas, such consistencies are important.

Graphics calculator capabilities generally exceed those of scientific calculators, with many mathematical functions built in, even on the entry-level calculators. Indeed, as noted above, for the majority of students, an entry level calculator will serve their mathematical needs throughout the secondary years, not only for the lower secondary years.

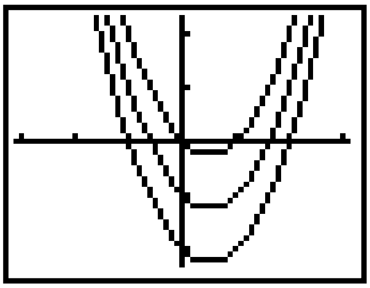
Generally speaking, schools in Australia have not used programmable scientific calculators, probably because they have not been permitted for use in end of school examinations. However, all graphics calculators are programmable, so that students can both explore the nature of algorithms and elementary programming as well as exploit this feature to upgrade the calculator capabilities where necessary.

Functions and equations

Some refer to graphics calculators as 'graphing' calculators, thereby highlighting the capability of the devices to draw graphs of functions. A graphics calculator provides access to lots of information about functions quickly, as the tedium of hand drawing of graphs (after laboriously calculating a collection of points) is eliminated. With a machine that provides graphs on demand, student attention can focus on interpreting, explaining understanding and using the graphs; prior to the availability of the graphics calculator, most of the potential thinking time was spent actually producing a graph. Calculators allow graphs to be explored, using tracing and zooming functions, rather than just being drawn.



Information about functions is not restricted to the graphical, moreover. Symbols, graphs and tables of values are mutually interactive on modern graphics calculators. Early versions of graphics calculators did not provide tables of values, but all recent models routinely do so. For lower secondary students, there are significant advantages for learning about functions in being able to jump from the symbolic form to the graphical to the tabular: the so-called 'rule of three'. The calculator allows for functions that are hard to graph by hand to be examined, with no more difficulty than for those that are relatively easy to graph by hand. Thus, lower secondary students can explore the nuances of exponential functions, crucial for understanding growth, just as readily as linear or quadratic functions. Transformations of functions, to aid understanding, are easily generated by drawing a succession of related graphs or editing the formula for a function and noting the change to its graph or the associated table of values. By such explorations, students can come to see the significance of both the form of a function and of its coefficients, in ways that were not readily accessible previously.



There are important connections between functions, their graphs and associated equations. Coming to understand these is an important outcome of elementary algebra in the lower secondary school as well as the upper secondary school. The calculator provides ready access to these connections through the medium of exploring graphs and tables.

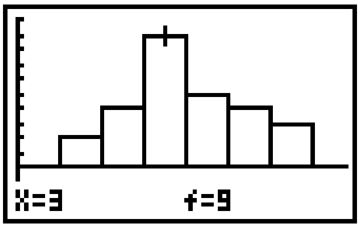
X	Y1	Y2
1	0	-1
2	2	1
3	6	5
4	12	11
5	20	19
6	30	29
-1		

To date, the study of equations in the lower secondary school algebra has been restricted almost entirely to those which could be solved with exact algorithms such as linear or quadratic equations. But equations that could not be solved analytically can be numerically solved, provided students have access to appropriate technology. On a graphics calculator, good approximations to solutions of other equations (such as exponential equations) are readily available by systematically refining solutions in either graphical or tabular domains. Thus, students gain access to powerful and understandable methods of general applicability, rather than being artificially restricted by a lack of suitable technology.

Data analysis

The most conspicuous advantage of a graphics calculator over its scientific predecessor is the availability of graphical representations of data as well as the standard numerical summaries. Graphical summaries are readily drawn and redrawn on a graphics calculator, in stark contrast to the case of doing such things by hand. But there are less obvious advantages as well.

Although scientific calculators have included some statistical capabilities for many years now, they are extremely restricted by the calculator memory size. In contrast, actual data (rather than just numerical summaries of them) are stored in a graphics calculator, so that data entry can be checked and data edited.



As well as providing a possibility for checking (the inevitable) key punching errors, graphics calculators allow for alternative analyses of the same data set to be undertaken. With graphics calculator access, the focus of student attention can be centred on the interpretation (of graphs or statistics) rather than on mere computation. Students can explore their own data in a number of different ways and learn both about their data and statistical ways of looking at them. For example, choosing different grouping intervals for the data produces different histograms. While very tedious to do this by hand, an experienced calculator user can do so quickly and learn both about histograms and the particular situation for which they have collected data. They can also consider the comparative advantages of using a histogram and a box plot to represent the same data visually. As a more sophisticated example, a bivariate data set can be examined for linearity and, if clearly nonlinear, alternative models can be compared. When outliers are detected (and they are more easily detected with a graphics calculator than with a scientific calculator) the offending data points can be removed and statistical computations repeated.

Data transformations can be of practical or conceptual use; on a graphics calculator, an entire data set can be transformed at once. For example, data can be transformed from one measurement unit to another or residuals from a model can be constructed and analysed.

All graphics calculators include a capability to generate random numbers. This can be used to generate sets of random data. Efficient random simulation of data provides for rich chance explorations by students. Once simulated, large sets of data can be analysed efficiently and students can compare their results and their conclusions with each other to learn about the nature of randomness or to solve otherwise intractable mathematical problems.

Personal finance

It is perhaps surprising that students in lower secondary school do not learn enough about loans to understand what for many of them will become critical matters later in their adult lives. The reason for this is that the mathematics curriculum has been restrained by the reliance on hand-held computation to unrealistic situations such as simple interest (rather than compound interest) and has avoided reducible interest for a similar reason. A graphics calculator, however, allows for the efficient generation by students of successive terms, perhaps representing the monthly balance on a car or home loan.

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1000
            1000.00
Ans×1.01-90
             920.00
             839.20
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Students can solve practical problems using relatively direct numerical methods on a graphics calculator. For example, they can work out how many monthly payments they will need to repay a loan at a certain interest rate, or what the effects of varying the interest rate will be on the duration of a loan.

Some unresolved tensions

When considering the relevance of graphics calculators to lower secondary students, schools and teachers are likely to encounter some important issues. In this section, some of these are identified and commented upon, although it is recognised that this practice does not resolve the associated tensions.

In many schools, and even in school systems, economic issues are likely to be significant for some time to come. Even in an affluent country like Australia, there are many families for whom the costs of sending children to school are very difficult to bear, and for whom the present requirement that students purchase a scientific calculator is beyond their means. Prevailing political views of user pays, economic rationalism and declining school budgets together have exaggerated the economic issues. This is not a new problem, of course. Schools have long worried about the abilities of some families to meet the costs of schooling, leading to book hire schemes, for example. On a larger scale, programs to deal with the needs of schools in disadvantaged communities have been around for some time. It may be that the costs of calculator purchase need to be thought about in the same sorts of ways; graphics calculator hire schemes and buy-back schemes have already proved to be useful in schools around Australia, although even these will not entirely resolve the problems. Although the purchase of class sets of calculators for students to share may seem an attractive option, it is only a temporary solution, as was the case for class sets of scientific calculators in the late 1970s and early 1980s. Despite the difficulties, it needs to be recognised that the cost of a suitable graphics calculator for lower secondary school use is now around the cost of two CDs, and thus well within the means of the overwhelming majority of Australian families.

The balance of traditional mathematical activities and those related to technology is also likely to give rise to some spirited debate in schools. As for the use of hand calculators in the primary school, some will argue that students need to have a sound grasp of the 'basics' before they are allowed to use a graphics calculator. However, the tension here is in precisely what is regarded as basic. For example, some have argued that they want students to be able to draw graphs of functions by hand before they are permitted to use a calculator to obtain a graph (in the analogous way to those who have argued that students should not be permitted to use a four function calculator until they have memorised the multiplication tables). But it may be that students will learn more about graphs of functions by exploring them on a calculator than by drawing them by hand; at the very least, they are likely to experience many more examples of functions and their graphs with a calculator then when relying on a hand sketch (and, to continue the analogy, students might well learn their multiplication tables in part by using a calculator to see the results of particular products).

Nobody wants to breed a nation of button pushers and mathematics teachers widely agree that mathematics is about thinking, not just getting answers electronically. But we may deprive our students of important opportunities to learn and to think for themselves by not allowing them access to graphics calculators. As an example of this kind of thinking, *Access to Algebra* (Lowe et al., 1993) uses function graphing technology to help students understand the nature and significance of graphs of functions, rather than waiting until all the necessary learning has taken place to introduce technological help. It is also worth noting that the less powerful graphics calculator models actually require more thinking by students, and are favoured by some teachers for lower secondary school use as a consequence. An example of the difference is that more sophisticated models often provide automatic means of interrogating graphs, such as finding roots or points of intersection, while less sophisticated models demand that students trace their graphs or read approximate results off the screen.

Some have suggested that mathematically weaker students do not need graphics calculators, which are mainly relevant to stronger students, such as those likely to proceed to significant mathematics study beyond lower secondary school level. Such arguments are often problematic, in part because they assume that 'weaker' and 'stronger' designations are fixed for all time. In addition, perhaps weaker students are precisely those who most need help to think about, connect and interact with mathematical ideas and hence may be especially likely to benefit from a benign technology. It is also difficult, of course, to differentiate students too strongly too soon in lower secondary school on the basis of their mathematics performance to date, if we are to ensure that all students have access to a suitable mathematics curriculum.

Tensions will arise, too, in situations where school courses have not yet changed to accommodate technologies such as graphics calculators. Thus, an algebra curriculum with a large focus on traditional symbolic manipulation, such as factorisation, may not immediately welcome the use of a graphics calculator which may render the technique less important. (The major real use of factorisation in school mathematics appears to be the solution of quadratic equations, for which more efficient technologies are available, not the least of which are the quadratic formula, which can be easily programmed into a calculator, and graphical approximation, readily carried out on a graphics calculator). There is always going to be an element of the chicken and the egg in such debates, of course; if we always waited until the curriculum changed to change the technology, nothing would ever change in school mathematics.

There are related issues of technology use in assessment at the lower secondary level. For example, there are potential inequities if only some students have access to graphics calculators, or students have access to calculators with a range of capabilities. Ideally, there will be some congruence between the curriculum, student assessment and graphics calculator availability. One solution to this sort of concern is to not allow any students to use graphics calculators in examinations until we are pretty sure that all students have access to them, but this can be all too easy converted into a recipe for curriculum stagnation. A more sensible solution may be to expect and assume that all students have access to graphics calculators and then design curriculum and student assessment accordingly, while attending to the particular needs of students who have trouble accessing calculators. The solution of 'calculator-neutral' assessment is not, however, a sensible solution; not the least of the reasons is that it sends the contradictory messages that calculators are useful but unnecessary and not likely to give students any advantages.

As suggested above, ideally technology will be integrated into curricula, rather than being tacked on in the form of one-off tasks, using 'calculator activities' to spice up the classroom experience a bit. The major advantage of graphics calculators over alternative technologies is their potential availability to students in a range of circumstances, such as home, school, assessment, projects and assignments. When students have only periodic access to a calculator, through the use of a class set shared among teachers, they may need to relearn how to use the calculator each time they access it, and are unlikely to gain full advantage of the opportunities provided. While shared sets of calculators and one-off activity sheets may be a useful way to get started with a new technology at the lower secondary school level, we should be aware of the limitations of this and try to move past them.

Professional development needs of teachers are always important with any innovation. In the case of graphics calculators, the most urgent need is probably to provide ready access to the technology for individual teachers as well as some local help. Not surprisingly, diffident teachers are unlikely to use graphics calculators in their classroom, where the risk of being shown up by students is considerable, until they feel comfortable with the devices themselves. Fortunately, the cost of providing each teacher with a graphics calculator for personal use is relatively small, and will go some way to getting many teachers started. The growing number of graphics calculator publications designed for school settings also offers relatively easy entry points for teachers, as do websites provided and maintained by calculator companies and others.

Choice of calculator will be a point of debate in many schools. Of course, there are significant advantages of using the same calculators throughout a particular school, not the least of which are the potential benefits for teachers to learn from each other about useful classroom practices and specific calculator techniques. However, student movement and individual choices will probably mean that in time teachers will need to be comfortable with a range of calculators. While there are good arguments to be made for the use of the less expensive entry level calculators, some students will need to have access to a more powerful calculator as they progress into the senior years. It will often to be difficult to predict which lower secondary students will later need more powerful models, and hence have to find a way to upgrade their calculator. Schools may offer to buy back entry-level calculators from upgrading or departing students and sell them second hand to younger students. Rather than upgrading as mathematical needs get more sophisticated, some students may prefer to get a more powerful calculator from the start of lower secondary school, but this may prove too costly for many students, and may increase inequities. The issue is compounded by rapid obsolescence, which may mean that a calculator purchased for lower secondary level use will seem less useful four years later when students are in the upper secondary school, primarily for reasons of calculator developments. An example of this is the development of tables of values for a function, which were not provided on early model calculators, but which are now routinely provided on all models.

Conclusion

Graphics calculators represent the first opportunity for schools to provide all students with a mathematics curriculum informed by appropriate and available technology. In the lower secondary school, entry level graphics calculators open many new opportunities for learning by students, and are well within the reasonable economic circumstances of typical Australian families and schools. While the use of graphics calculators by lower secondary students raises a number of issues for which resolution needs to be sought, these ought best be used to stimulate healthy debate and experimentation in schools, rather than act as justifications for inaction. In fact, experience in a number of schools suggests that by taking the first steps into graphics calculator use in the lower secondary school, we are likely to find that some of the reservations prove unfounded and that the resolution of some of the tensions is clearer.

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Barry Kissane The Australian Institute of Education
Murdoch University, WA
email kissane@murdoch.edu.au

Marian Kemp Teaching and Learning Centre
Murdoch University, WA
email kemp@murdoch.edu.au