

Mathematics Education: A Lifetime Juggling Act

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What is juggling?

- juggle:** *n.* –*v.t.* 1. to keep (several objects, as balls, plates, knives) in continuous motion in the air at the same time by tossing and catching.
2. to manipulate or alter by artifice or trickery: *to juggle accounts.*
- v.i.* 3. to perform feats of manual or bodily dexterity, such as tossing up and keeping in continuous motion a number of balls, plates, knives, etc.

The Macquarie Dictionary, 1981

Juggling two things



- John Mason has described some tensions of mathematics teaching
- Handling these tensions – finding a balance – seems very much like juggling:
- Eg, Keeping control vs allowing pupils to explore
- Eg, Time tensions: ‘cover’ the material vs understanding it well
- Eg, Challenge and support: too much of either or too little of either creates problems
- Eg, Intervening in student work: earlier or later?

The didactic contract



- Pupils learn and teachers teach
- Brousseau's observation of an implicit contract between the two
- Pupils want to know what to do and how to do it: to what extent should we just tell them?

“The more the teacher is explicit about what behaviour is wanted, the less opportunity the pupils have to come to it for themselves and make the underlying knowledge and understanding their own”.

Mason, J. 'Tensions' in Pimm, D. (ed) *Mathematics, Teachers and Children*.
Oxon: Hodder & Stoughton, pp 164-169.

Juggling the syllabus



- By nature, a syllabus is constraining, sometimes *too* constraining
- “Will this be on the test?”
- How do we provide for a wider view of mathematics in such a circumstance?
- Where is mathematics in the wider world?
 - Bookshops
 - Libraries
 - Media

Mathematics in the wider world



- Bookshops rarely have a category called ‘mathematics’
- We are *not* a branch of science, but in the public domain we seem to be regarded as such
- School libraries often contain almost nothing to read (or watch) in mathematics
 - Despite a wide expansion of popular mathematics books in recent years
 - Textbooks are rarely suitable for reading!
- Mathematics is almost unknown on television

Some examples of recent popular mathematics books



Ian Stewart, 1998. *Life's Other Secret*, Penguin.

Paul Hoffman 1998. *The Man who Loved Only Numbers*, Hyperion.

Malba Tahan 1993. *The Man Who Counted*, Canongate.

Denis Guedj 1998. *Numbers: The Universal Language*, Thames and Hudson.

Thomas Banchoff 1996. *Beyond the Third Dimension*, Scientific American.

Richard Phillips 2004. *Numbers*.

Ian Stewart 2001. *Flatterland*, Pan.

Margaret Wertheim 1997. *Pythagoras' Trousers*, Fourth Estate.

Richard Mankiewicz 2000. *The Story of Mathematics*, Cassell.

A few more examples

Clio Cresswell 2003. *Mathematics and Sex*, Allen & Unwin.

Hans Magnus Enzensberger 1997. *The Number Devil*, Granta.

Ivars Peterson 1998. *The Jungles of Randomness*, Penguin.

Ian Stewart 2001. *What Shape is a Snowflake?* Weidenfeld & Nicolson

Rob Easterway & Jeremy Wyndham 1999. *Why Do Buses Come in Threes?* Robson

Mario Livio 2002. *The Golden Ratio*, Review.

Keith Devlin 1998. *Life by the Numbers*, Wiley.

A. K. Dewdney 1993. *200% of Nothing*, Wiley.

[Clifford Pickover](#) books

Reading mathematics

- We almost seem as a profession to assume that no-one would want to spend *any* of their 'leisure' time concerned with mathematics, such as
 - Reading
 - Television
 - Excursions
- Almost as if maths is *useful*, but not really *interesting* ...
- Our colleagues teaching English, Science, Drama, Media, Social Studies, Dance, Art, Physical Education, Home Economics, Photography and many other things at school seem to make the opposite assumption

Why?



- Do we consciously project mathematics as ‘work’, and not something that might be enjoyed in one’s spare time? Why?
- We are surprised and disappointed that students follow other interests, and do maths only because they ‘have to’? Why?
- We are desperately short of mathematics teachers in Australia; *anything* that might help this is worth trying.
- See the [AAMT catalogue](#) and AAMT journal book reviews for many examples of materials for libraries.

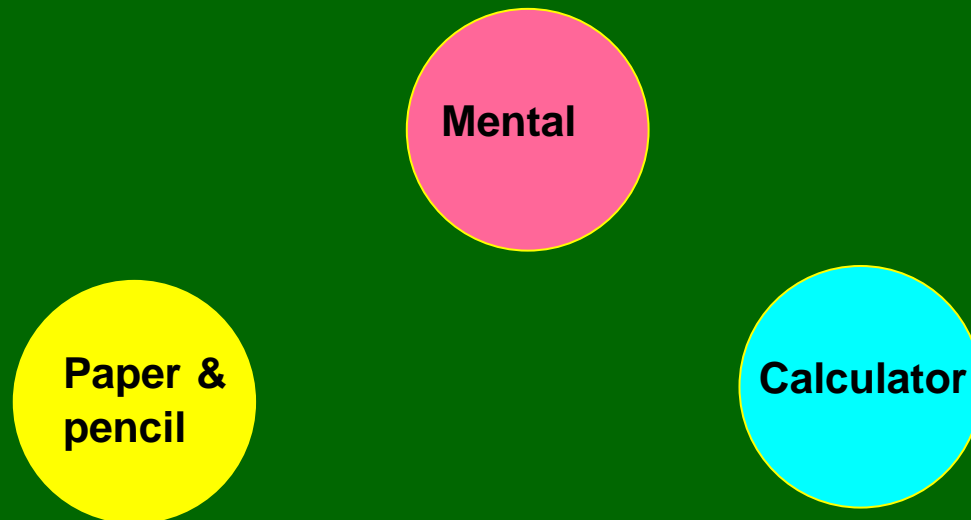
Web-based mathematics

The Internet these days provides some wonderful opportunities to see a wider view. Here are a few examples:

- [Plus](#) magazine
- Keith Devlin MAA column, [*Devlin's Angle*](#)
- Ivars Peterson's [Math Trek](#) column
- MAA has home page links to [mathematics columns](#)
- [Centre for Popularisation of Mathematics](#)
- [Mathematics Museum](#)
- [Fibonacci Numbers](#)
- [Cut The Knot](#)

Juggling three things

- I've been told we're not really juggling until there are *three* things involved ... which I don't accept.
- Consider the example of technology: All Australian curricula these days attempt to juggle three means of computation.



This is an old issue

- This famous 16th century engraving shows Boetius, calculating by writing, competing against Pythagoras, using an abacus
- Written calculation is clearly favoured by Lady Arithmetic, who is looking on
- Mental calculation is not made explicit in this case



A lesson of history ...

“From the 12th century on, the abacus was gradually replaced by the dust board as a tool of calculation. ... This development did not come about without a struggle between those who, evoking the ancient Greek mathematician Pythagoras, championed the abacus and those who became masters of algorism, the new Arabic number system. In this competition between the Ancients and the Moderns, the former often saw themselves as the keepers of the secrets of the art of computation and the defenders of the guild of professional calculators, with interests paralleling those of the Christian church.”

... still relevant today?



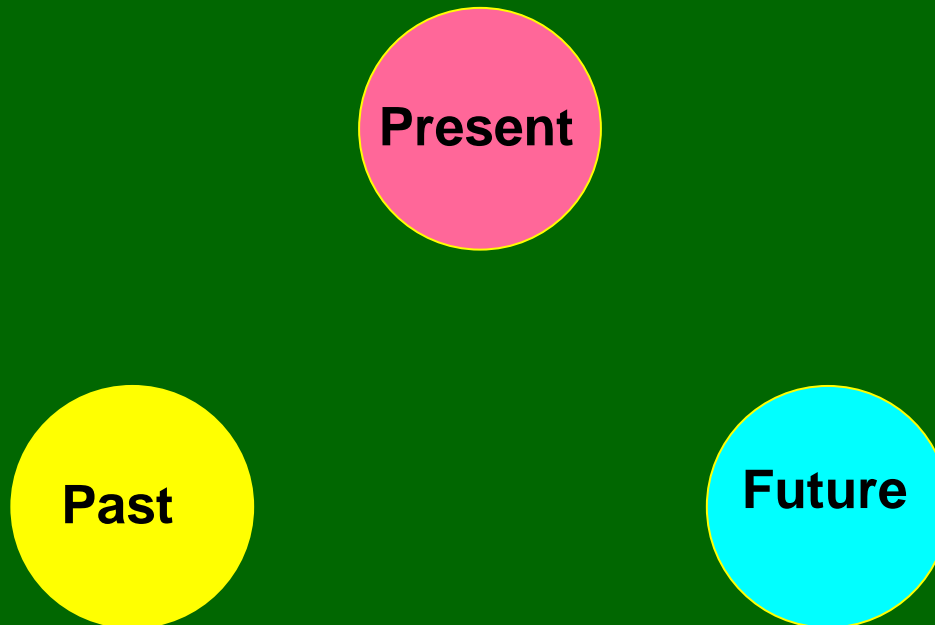
“The introduction of the new system indisputably marked the democratisation of computation: its simplicity and lack of mystery made its widespread use possible. Computation was no longer an esoteric art practiced within the limited circles of specialists.”

Guedj, Denis (1996) *Numbers: The Universal Language*,
London: Thames & Hudson (p 52)

Relevant today? Consider the analagous case of CAS
(computer algebra systems) ...

Juggling in time

A common juggling act seems to involve three *temporal* things



Mathematics education



- Mathematics education has a past, present and future
- Although mathematics teaching preceded Plato, the profession of mathematics education is fairly recent
 - Eg AAMT was founded in 1966
 - Eg first ICME was held in 1969
 - Eg MASA was founded in 1959
 - Eg, JRME was founded in 1970
- It is even more recent at the tertiary level

Juggling mathematics education in time



- We seem often to be engaged in juggling in mathematics education
 - Preserving the past
 - Working in the present
 - Looking to the future
- Or is it:
 - Mindful of the past?
 - Coping with the present?
 - Anticipating the future?
- Consider the case of technology

Solving an equation

- A major component of elementary algebra involves solving equations and inequalities
- Eg, Find the numbers whose cubes are five more than twice the number itself.
- I.e., solve for x : $x^3 = 2x + 5$
- In how many ways can you do this?
- What are the solutions?

The past



- When I left school, I could not answer this question
- The only cubic equations I could solve were those that could be factorised
- Iterative procedures such as Newton-Raphson were known about, but impractical at school and not part of the syllabus

The present

- These days, students have access to technologies that make such a task accessible
- The most likely technology to be generally available is a graphics calculator
- Here are a few possibilities now on the Casio cfx-9850GB PLUS
 - Guess, check and improve
 - Graphical
 - Solver
 - Cubic solver

The future



- More sophisticated technologies may become available for equation solving
- The [ClassPad 300](#) is a good example
- Although it still does not provide an exact solution in this case
- We may also wish to reconsider the balance of the curriculum

Analytical

- Provide *exact* answers
- Are available for only some equations (most notably linear, quadratic and (some) exponential)

Numerical



- Provide numerical, approximate answers (to any desired accuracy, usually)
- Available for all equations. Can deal with practically any 'realistic application' of mathematics

Analytical

- Guarantee the solutions if procedures correctly followed

[Maybe that's why students often seem disinclined to check the solutions?

Maybe also why 'show your working' is emphasized?]

Numerical

- Checking the solution is often part of the solution process



Analytical

- Long history in school mathematics, which has traditionally been restricted to algebra of equations with exact solutions

Numerical



- Have not featured in curricula or textbooks, presumably because solutions were unavailable without technological help?

Analytical

- Two excellent general strategies:
- (i) DSBS, balance idea, which is very useful for linear equations
- (ii) Multiplication Property of Zero: $pq = 0$ iff $p = 0$ or $q = 0$, which is very useful for quadratic, factored polynomial ... one of the few reasons for factoring?

Numerical



- Often several methods accessible via a graphics calculator.
- Part of the task is to decide on what kind of method is best for a particular equation.

Analytical

- Unavoidable if exact or general solution is needed

Numerical



- Unavoidable when no exact solution method known. Require a conscious choice of a suitable level of accuracy (presumably to fit the data and the context).

Analytical

- A major reason for learning about symbolic manipulation is to be able to transform expressions in equations into a tractable form for analytic solution (especially linear and quadratic)

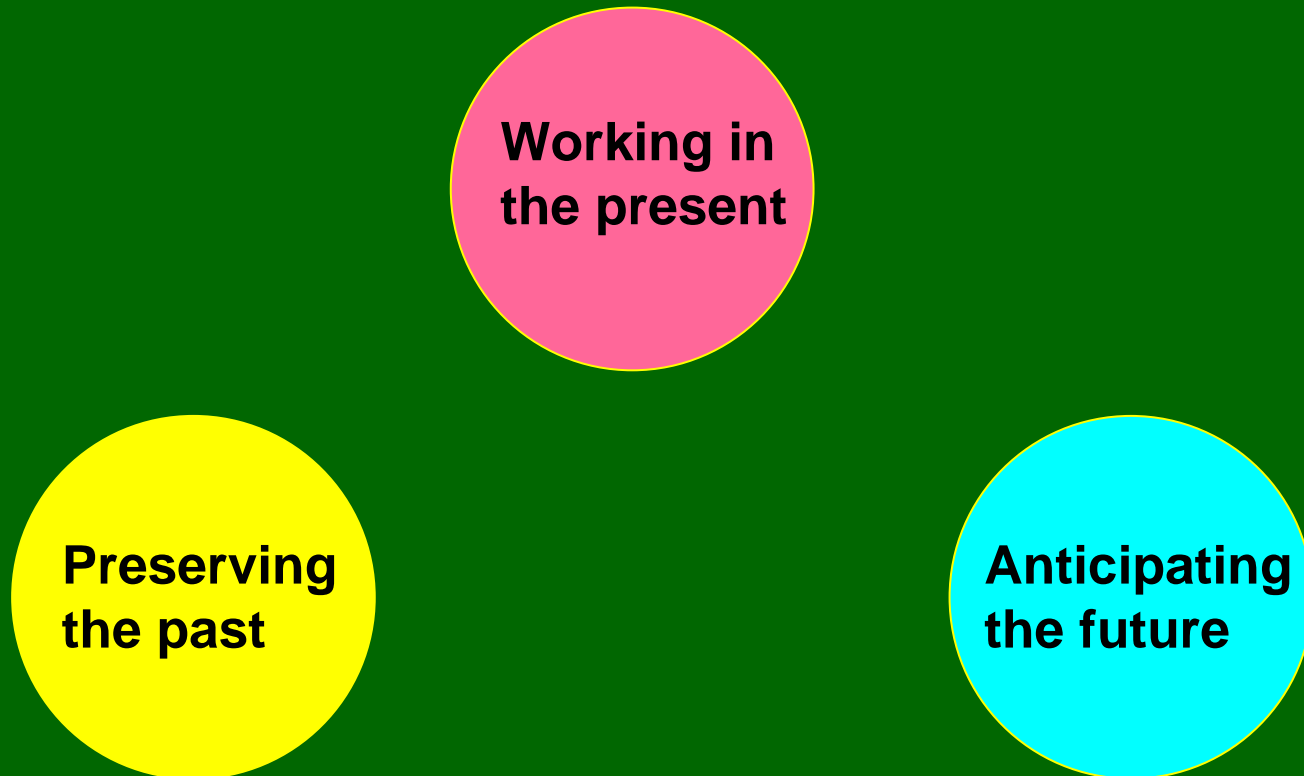
Numerical

- Less dependent on symbolic manipulation



Juggling equations

Deciding how to proceed feels very much like a juggling act:



A change of emphasis?

- Helping pupils to ***express*** relationships algebraically
- ... rather than on manipulating the expressions themselves
- Helping pupils to ***formulate*** equations and ***interpret*** solutions
- ... rather than on the algebraic manipulations required to solve equations

Juggling symbolic manipulation

- A great deal of formal school mathematics of the past involves symbolic manipulation
- ... (too) much of which seems to be learned as a ritual activity, rather than a meaningful activity
- For many pupils of the past, algebra actually meant these tasks, such as:
 - Factorising
 - Expanding
 - Solving
- Indeed, even calculus meant symbolic tasks such as:
 - Differentiating
 - Integrating

Symbolic manipulation today

- For many of us, not much has changed (yet)
- Many algebra texts of today do not differ markedly from those of yesterday
- ... except for niceties of colour, images and other cosmetic changes
- In some places, computer algebra systems (CAS) are beginning to be used
- ... in other places, they are actively banned
- Reminiscent of banning (English) books or of Christian Church's heresy-prevention mechanisms?

Symbolic manipulation tomorrow?

- For how much longer can we pretend that symbolic manipulation by machine is not available?
- *The Emperor's New Clothes?*
- Will we continue to have a choice?
- Consider, for example, the ClassPad 300 as an integrated device for school mathematics
- Why would anyone want to produce technology to support mathematical work that did not support symbolic work?

What is involved in symbolic manipulation?



- Representing a situation in symbolic terms
- Deciding what manipulations are needed to solve a problem
- Performing the manipulations
- Deciding when to stop
- Interpreting the result

How well do typical pupils do?

- Representing a situation in symbolic terms
- Deciding what manipulations are needed to solve a problem
- Performing the manipulations
- Deciding when to stop
- Interpreting the result
- Poorly, often, according to many teachers
- We usually do this for them (eg “solve”, “factorise”, “evaluate \int ”)
- Done to exhaustion, and takes most of the time
- We often do this for them (simplify, answer to 2 dp)
- Often handled poorly, especially when non-routine

Arcavi's *Symbol Sense*



- An understanding of and aesthetic feel for the power of symbols
- A feeling for when to abandon symbols in favor of other approaches
- An ability to manipulate and to “read” symbolic expressions as two complimentary aspects of solving algebraic problems
- The awareness that one can engineer symbolic relationships and the ability to do so

... Arcavi continued



- The ability to select a possible symbolic representation of a problem, to acknowledge dissatisfaction with a choice and to be resourceful in finding a better replacement
- The realization of the constant need to monitor and compare the meanings of symbols with one's intuitions when solving a problem
- Sensing the different roles symbols can play in different contexts

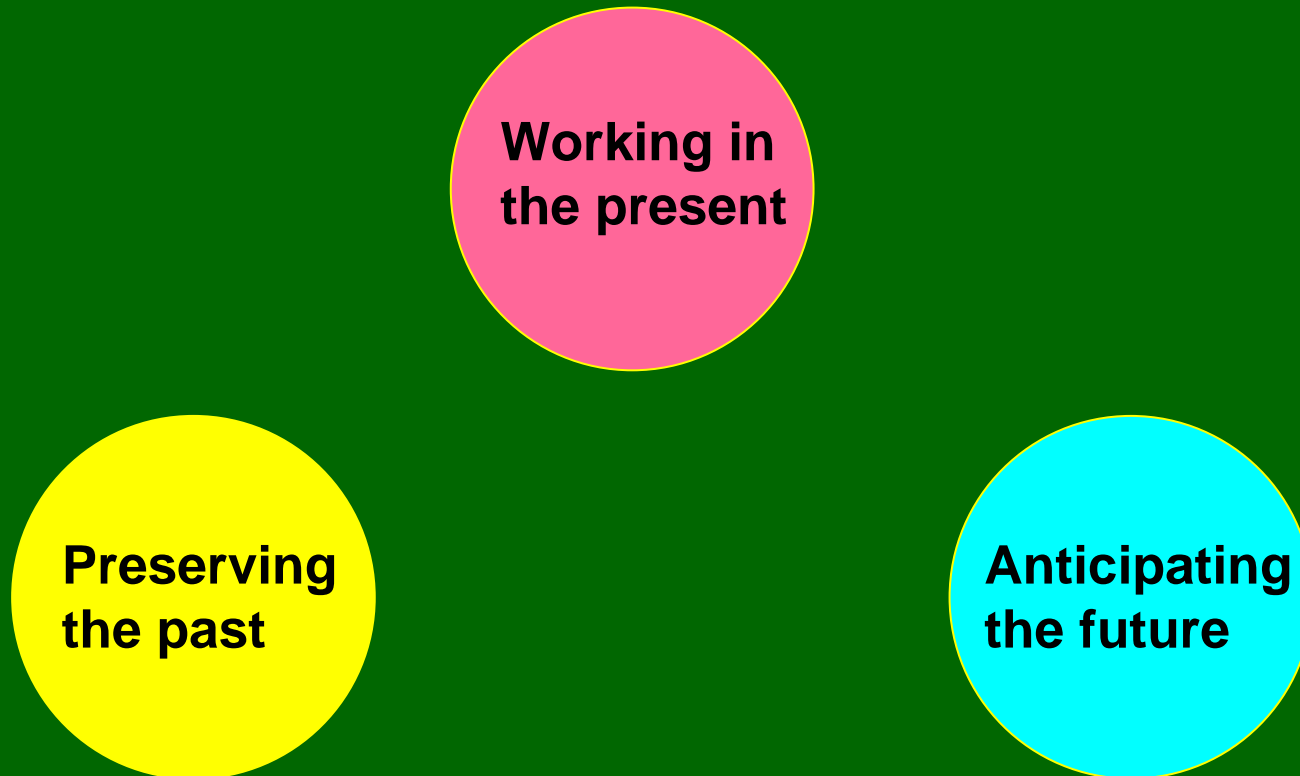
Arcavi, A. 1994. Symbol sense: Informal sense-making in formal mathematics, *For the Learning of Mathematics*, 14, 3, 24-35.

A role for CAS?

- Most of the thinking associated with symbolic manipulation is not completed by a CAS
- Perhaps there is some prospect of more time being devoted to the hard things
- ... only if the routine things are handled to an extent by a machine
- Consider for example the way the ClassPad 300 naturally uses symbolic representations

Juggling symbol manipulation

Deciding how to proceed with CAS feels very much like a juggling act:



Professional juggling



- The most significant recent achievement of the AAMT involves the *Standards for Excellence in Teaching Mathematics in Australian Schools*
- The *AAMT Standards* development is ongoing, with details accessible on the [web](#). [A copy can also be downloaded from the web.]
- The *Standards* are organised into three domains

Domain 1: Professional Knowledge

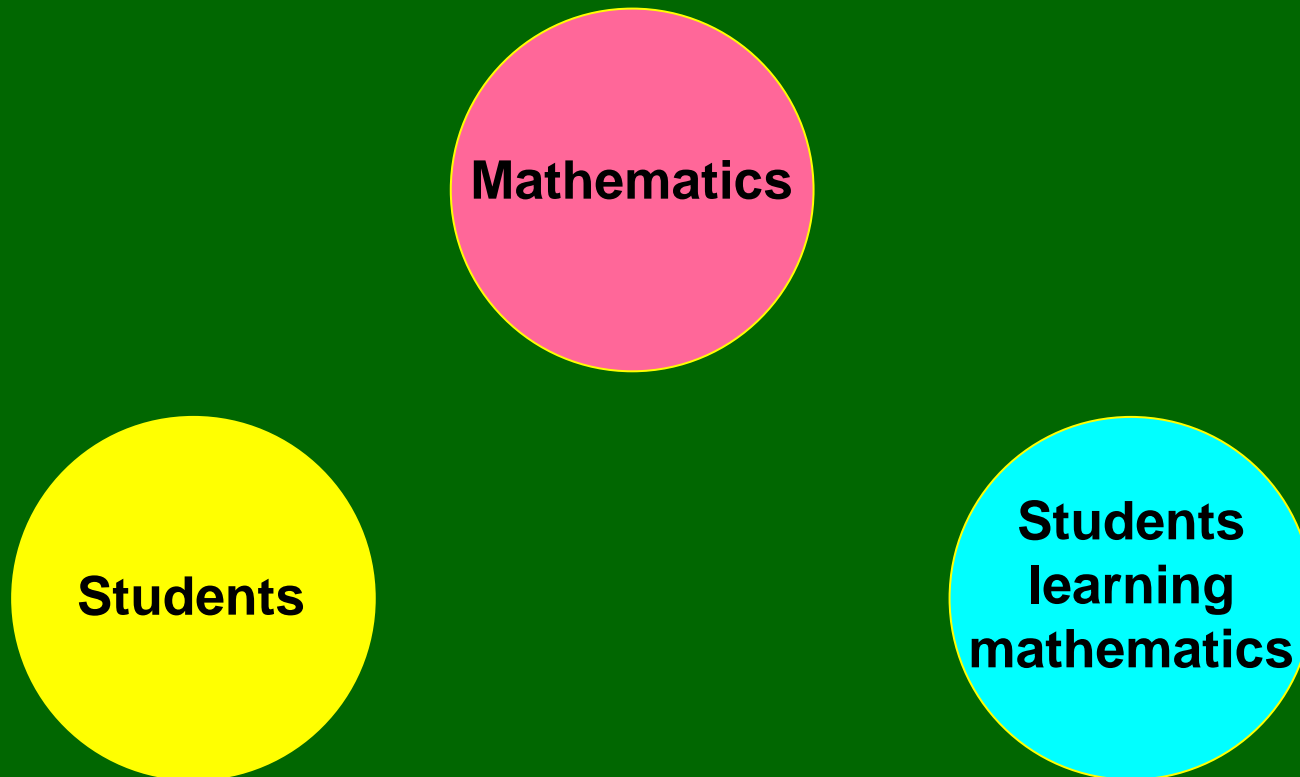


Excellent teachers of mathematics have a strong knowledge base to draw on in all aspects of their professional work, including their decision making, planning and interactions. Their knowledge base includes knowledge of students, how mathematics is learned, what affects students' opportunities to learn mathematics and how the learning of mathematics can be enhanced. It also includes sound knowledge and appreciation of mathematics appropriate to the grade level and/or mathematics subjects they teach.

Juggling Professional Knowledge



All three balls need to be kept in the air at once



Domain 2

Professional Attributes



Excellent teachers of mathematics are committed and enthusiastic professionals who continue to extend their knowledge of both mathematics and student learning. They work creatively and constructively within a range of ‘communities’ inside and beyond the school and set high, achievable goals for themselves and their students. These teachers exhibit personal approaches characterised by caring and respect for others

Juggling Professional Attributes



**Personal
professional
development**

**Personal
attributes**

**Community
responsibilities**

Domain 3: Professional Practice



- Excellent teachers of mathematics are purposeful in making a positive difference to the learning outcomes, both cognitive and affective, of the students they teach. They are sensitive and responsive to all aspects of the context in which they teach. This is reflected in the learning environments they establish, the lessons they plan, their uses of technologies and other resources, their teaching practices, and the ways in which they assess and report on student learning.

Juggling Professional Practice

All four balls must be kept in the air at once

Planning for learning

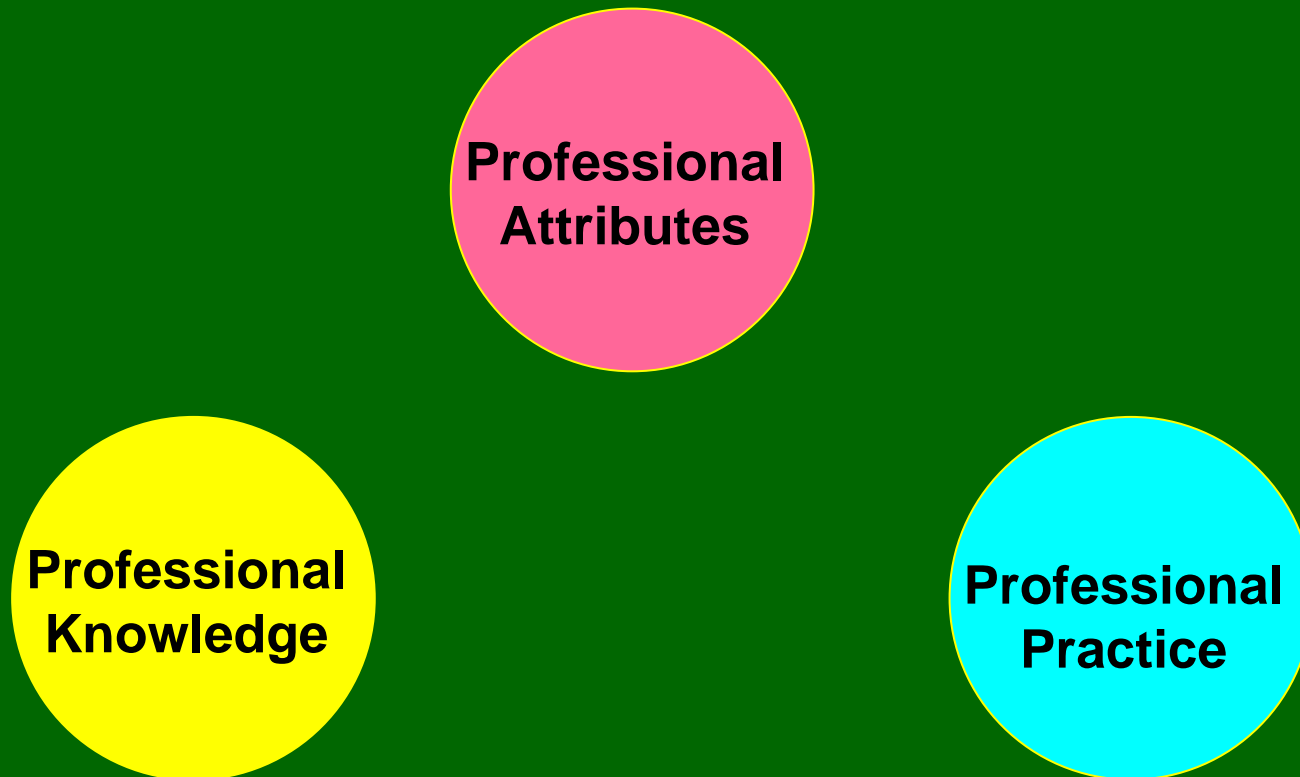
Teaching in action

The learning environment

Assessment

Juggling *AAMT Standards*

Attending to all the Standards involves complex multi-faceted professional behaviour:



AAMT Standards



- Just as juggling can be learned, so too can the AAMT Standards be attained
- Indeed, attending conferences of this kind is one of the good ways of continuing the learning process
- National conferences provide unique opportunities to see the world through the eyes of others
- Think about the [AAMT Biennial Conference](#) now and plan around it!

Juggling time on the web

- The World Wide Web has become enormously significant in recent years
 - How quickly we tend to take new developments for granted!
- How can we juggle the time so that we become aware of what is available
 - For ourselves?
 - For our students?
- What is worth the expense?
 - Student learning time
 - School capital resource investment?
- What can our students do at home now?

What should we learn?

Email?

XML?

All of these?

JavaScript?

Browser?

Shockwave?

Macromedia
Flash?

Java?

HTML?

None of these?

Other things?

Some good examples

- Encyclopaedic information on a mathematical topic
- More engaging material than a text can provide
- **Reference materials such as dictionaries**
- New ways of looking at things
- Fresh opportunities for learning important concepts
- Innovative collections of information
- Interactive information about all kinds of mathematics
- Professional affiliation and community support

How to find out more

- MASA [links](#) list
- Other annotated lists (eg, [New Scientist](#))
- Regular reviews
 - ENC [Digital Dozen](#)
 - [Math Forum Internet Newsletter](#)
- Periodicals
 - [Journal of Online Mathematics and its Applications](#)
 - [Spreadsheets in Education](#) journal
- Professional reading
 - such as Esther Loong's nice 2003 paper about the use of the web, in *AMT*, 59(1), pp 23-29.

Juggling for mathematics teachers



- A modern professional mathematics teacher needs to learn to juggle their time well between
- Working life
- Professional life
- Personal life

Working life



- What we are paid for
- ... with industrial overtones, since we are mostly employees
- With many external constraints
- With many connections to our professional lives as well, of course
- The secret to a happy life is to find out what you like to do, then find someone who will pay you to do it!

(Autonomous) Professional Life



- Reading
 - a website or a software manual
 - a journal article or a new mathematics book
- Writing
 - a paper for *AMT* or *ASMJ*
 - a letter to the editor of *The Advertiser*
- Speaking
 - presenting a conference workshop, a talk to parents
 - conducting within-school PD, talking to colleagues
- Listening
 - attending a MASA committee meeting
 - going to the AAMT Biennial Conference

Personal Life

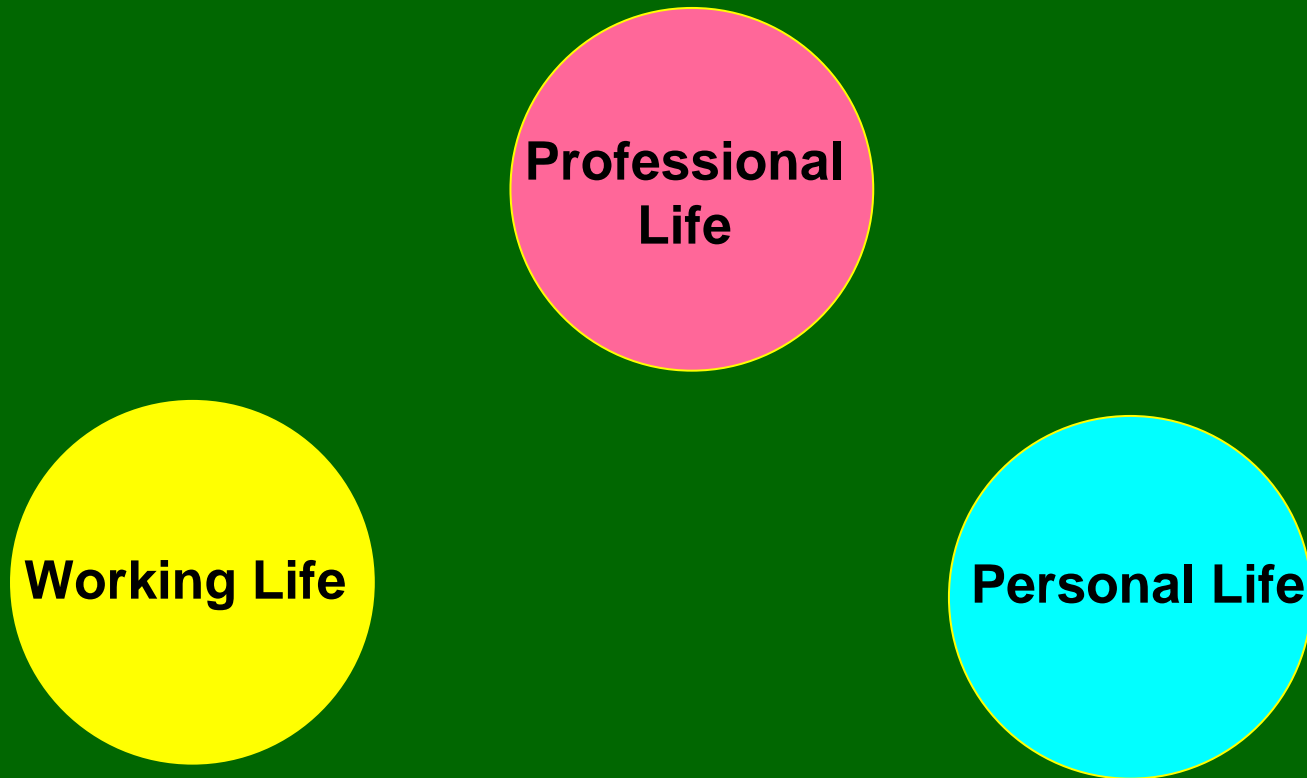
- Personal relationships need attention in the same way as professional and occupational relationships
- Families are very important
- So are friends
- So is leisure

Why juggling is hard in mathematics education



- Everything is changing
 - Occupational life
 - Professional life
 - Personal life
- And this will not stop
- Lienwand on change:
 - It is unreasonable to expect a professional to change more than 10% per year
 - And it is unprofessional to change less than 10% per year
- We need to acknowledge the changes and work with them, to avoid the need for catastrophic change
 - Eg, if we do nothing about technology for years, it won't go away!

Happy Lifetime Juggling!





Thanks!

David Martin



MASA Conference Committee



St Peter's for facilities

MASA for AAMT AGM this afternoon

You ... for being here!