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Teaching and Learning Probability in an Age of Technology

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Abstract: *Modern classrooms have access to a range of potential technologies, ranging from calculators to computers to the Internet. This paper explores some of the potential for such technologies to affect the curriculum and teaching of probability in the secondary school and early undergraduate years, rather than relying on the classical and formal approaches focusing on set theory and counting techniques. Different approaches to probability, including the study of risk, are identified. We describe some of the ways in which the teaching of probability might be supported by the availability of various forms of technology, including calculators, computer software and the Internet. We consider especially the role of simulation as a tool for both teachers and students, focusing on activities that are not possible without the use of technology. Modern technology provides an excellent means of exploring many of the concepts associated with probability. Many of these opportunities for learning were not available before the technology development and access within schools we enjoy today. This paper suggests some of the avenues for exploration, research and development and provides advice on where to get further information and resources.*

1. Introduction

Probability is a recent addition to school curricula, perhaps in part because it is a recent addition to mathematics itself [13]. To date, introductions to the study of probability have often been very formal, with students being provided with probability ‘rules’ and definitions, based on sample spaces and equally likely outcomes. This paper explores the nature of the probability curriculum and outlines the promising influences of technology for teaching and learning in this area [3][4].

The main motivations and arguments for inclusion of the study of probability in school curricula include developing an understanding of everyday chance phenomena, and thus the improvement of people’s everyday decision-making. Since most events in practice have a chance component, there would seem to be a good case for helping students and future citizens understand how mathematics can be used to guide action in the presence of uncertainty. Contexts of interest include the interpretation of medical test results or jury verdicts (recognized as not being deterministic in nature), as well as a thorough understanding of key ideas in statistical inference for more advanced students. It has been also suggested [29] that a good understanding of gambling is important for all citizens, thus providing a significant incentive for work on probability to be included in the curriculum for all, not only for those who are more likely to use statistical methods.

The relationships between technology of various kinds and probability education have been surprisingly neglected in the research literature. In particular, a search of recent ATCM conferences revealed little work at all in this area, so that at this stage it is premature to summarise empirical evaluations of the effects of using technology for probability instruction. Indeed the evaluation of the advantages and disadvantages of using technology in educational settings are more complex than commonly recognized, for a range of practical reasons. Instead, this paper offers some analysis and overview of the possibilities offered by technology, considers their implications for the probability curriculum, and incidentally offers some advice on suitable teaching resources.

2. Definitions of probability

Of critical importance to this paper, and indeed to probability education, is the nature of probability itself. Three different conceptions have emerged from recent work.

The first of these involves a ‘classical’ approach, building on the work of Kolmogorov in the early twentieth century. The key feature of this approach is the formal definition of probabilities in terms of sample spaces of equally likely events, with an associated algebra, based on a strict set of axioms. Students encountering this approach to probability typically learn a set of rules for the computation of probabilities, often with a set theoretic platform, and do not necessarily associate this work with practical everyday matters, even though the work is frequently introduced in a pseudo-practical way, such as with ‘balls’ being selected from ‘urns’. The work is frequently connected with aspects of combinatorics, necessary for counting sample spaces in more sophisticated situations. It seems unlikely that technology is particularly helpful to support the development of expertise in such a context, relying mostly on strict application of the various rules.

A second approach to probability is the so-called ‘frequentist’ approach, which defines the probability of an event as the long-run relative frequency of the event. In practice, this definition of probability lends itself to realistic applications, which can rarely be described in terms of sets of equally likely outcomes. This approach relies on the assumption that the random events of the past are a good indication of what is likely to happen in the future. While there is no alternative in practice to making an assumption of this kind, it is not without its problems as noted in [13]:

The frequentist approach is not devoid of problems, as is apparent in the way individuals interpret probabilistic assumptions, as became apparent in the recent financial meltdown of 2008. The common issue, rarely discussed in depth, is why one might think that a pattern from the past may be replicated in the future, particularly when variation is a key feature (p. 4).

There is not space in this paper to explore this issue, which has significant philosophical overtones, but nonetheless it is important to recognize that there is an issue worthy of exploration.

A third approach assigns probabilities on the basis of subjective assessments, perhaps informed by frequencies. An example might be the claim that there is a 30% chance of rain tomorrow. Having made this claim, probabilities can be used to model situations (such as, if the chances of rain are 30% each day of the week, how likely is it to rain on only two of the weekdays?)

Recently, and particularly in the light of events related to the global financial crises, the notion of ‘risk’ as distinct from probability has become more prominent, and arguably represents a fourth way of thinking about probability. The work of Gerd Gigerenzer [8][24] has been seminal in this respect, suggesting that more attention be paid to ‘natural frequencies’ rather than ‘probabilities’ in order to understand situations involving risk. While this approach to everyday phenomena (such as understanding results of medical screening tests) may be more interpretable for many people, and has been celebrated internationally, it has not yet featured strongly in mathematics curricula [15].

3. Experiencing chance with technology

An important contribution of technology concerns the production of data that allows students to both experience chance events and also to use the data for practical purposes, especially simulation. Prior to the age of technology, the main means of achieving these ends were either physical (such as rolling dice) or through the use of random number tables. For neither of these was it easy to generate a lot of data efficiently. Technology allows for random data to be generated and also to be used much more effectively than these previous methods.

Most random data generated by technologies such as computers and calculators are in fact not random, but pseudo-random, through the use of internal machine algorithms. Truly random data are

rather more difficult to access. A good source is the web site at [10] which relies on atmospheric noise. While truly random data are important for some applications (e.g., those concerned with gambling, for which it is critical that data not be periodic), for the purposes of simulation and modeling, as well as teaching, pseudo-random data are adequate and can be efficiently generated.

Today's scientific and graphics calculators, as well as spreadsheets such as Microsoft's *Excel*, routinely have a pseudo-random number generator, typically generating a uniform distribution on (0,1). It is important that students learn how to use this facility, and in particular, how to transform the data generated into a more useful form for a particular purpose. As an elementary example, Figure 1 shows a transformation of the random number, *Ran#*, to give a random integer in the closed interval [1,10], simulating rolling a 10-sided die. While recent models of calculators offer some inbuilt transformations (e.g., to produce integers on an interval), it is unwise for students to rely on these, and preferable for them to learn how to make elementary transformations of random numbers for themselves to suit a particular purpose. Indeed, it has been argued that such transformations ought to be a routine part of a modern school mathematics curriculum [17].

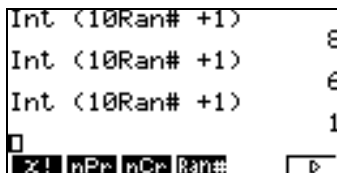


Figure 1: Rolling a 10-sided die on a Casio fx-9860G calculator

Figure 2 shows a more sophisticated calculator-based example, generating a table of successive sums of 100 rolls of a pair of standard dice, and representing these as a histogram.

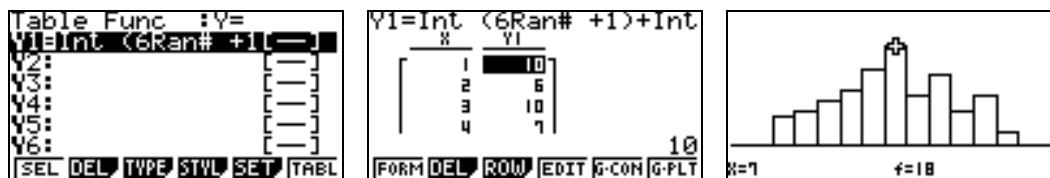


Figure 2: Generating a 100 rolls of a pair of dice on a Casio fx-9860G calculator

When students are able to generate chance data like this, they can see for themselves that random processes produce different results each time and also that there are (relatively) predictable patterns in these. Thus, if each student in a class generates a histogram such as that in Figure 2, and comparisons of these are made, they can see that the general shape is similar each time, but never quite identical. Experience of this kind is important for understanding the nature of chance events.

4. Simulation

The major contributions of technology to the teaching and learning of probability concern simulation, drawing on the capacity of technology environments to simulate random events in various ways.

Simulation allows students to experience random events at first hand, and to use these to help understand key probability concepts. The previous section showed an example of this, using a graphics calculator to simulate rolling a pair of dice. The whole of Chapter 6 of [18] describes in some detail how a graphics calculator can be used to simulate events to understand the nature of

chance phenomena, even before a formal analysis is accessible to students. These all depend on the use of the pseudo-random number generation capabilities of the calculator.

Experience with simulation can be used to address well-known misconceptions associated with probability, such as that it is more difficult to toss a six rather than other numbers on a standard die, most likely arising from young children’s experiences of playing games for which it is necessary to throw a six before starting. As a second and more sophisticated example, Figure 3 shows the calculator being used to simulate events that are very likely to happen, to address student misconceptions that ‘very likely’ means ‘almost always’ (and that ‘very unlikely’ means ‘almost never’). Figure 3 shows a calculator command to simulate the very likely event of surviving an operation that is 95% successful. It is clear that the first three simulated ‘operations’ were successful, represented by the results of 1. (A zero would indicate an unsuccessful operation).

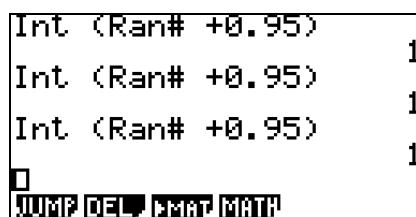


Figure 3: Using a Casio fx-9860G calculator to simulate a very likely event three times

In five classrooms of thirty students, however, when each of the students simulates an operation in this way on their own calculator, the probability that each student’s first operation will be successful drops dramatically. In about four out of these five classrooms, at least one of the students will have an unsuccessful operation at their first attempt, which most students find surprising. If each student regards the calculator as simulating his or her own personal operation, this quickly leads to a different interpretation of the meaning of this ‘very likely’ event in everyday terms. In addition, students can use the same idea to simulate a succession of operations (assuming they are the surgeon and not the patient) through forming a table of results. In the example in Figure 4, the third operation is in fact a failure, which may help to address the common (optimistic?) misconception that the rare failures are only likely to occur after a sequence of many successes.

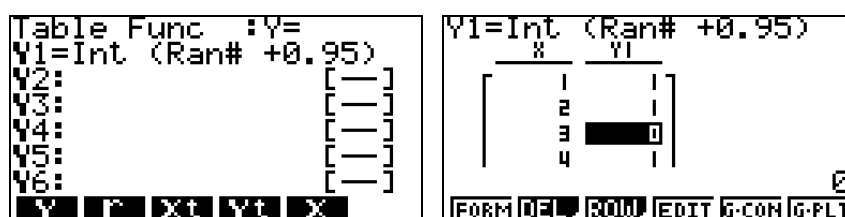


Figure 4: Using a fx-9860G calculator to simulate a succession of very likely events

These simulations of course rely on an assumption that the events are independent. The experience with simulation can help to motivate a study of this assumption and its consequences, or might take place well before the sophisticated idea of independence of events is introduced. In this context, discussions about the effects of particular hospitals, surgeons or facilities may occur.

Some software has been designed especially to support student simulations, in order to offer organized and systematic experiences with random events. Both the statistics packages of *Fathom* [15] and *TinkerPlots* [20] include good capabilities for this purpose [26]. *Probability Explorer* [22] has been designed entirely for this experience, allowing research into student understanding of

probability [6][23][33]. As Figure 5 illustrates, students can use this software to generate random data and to represent the results in various ways—graphical, numerical and tabular—in order to experience the nature of random events. In this case, the software shows that the number of times a head and a tail both appear is similar to the number of times both coins are the same.

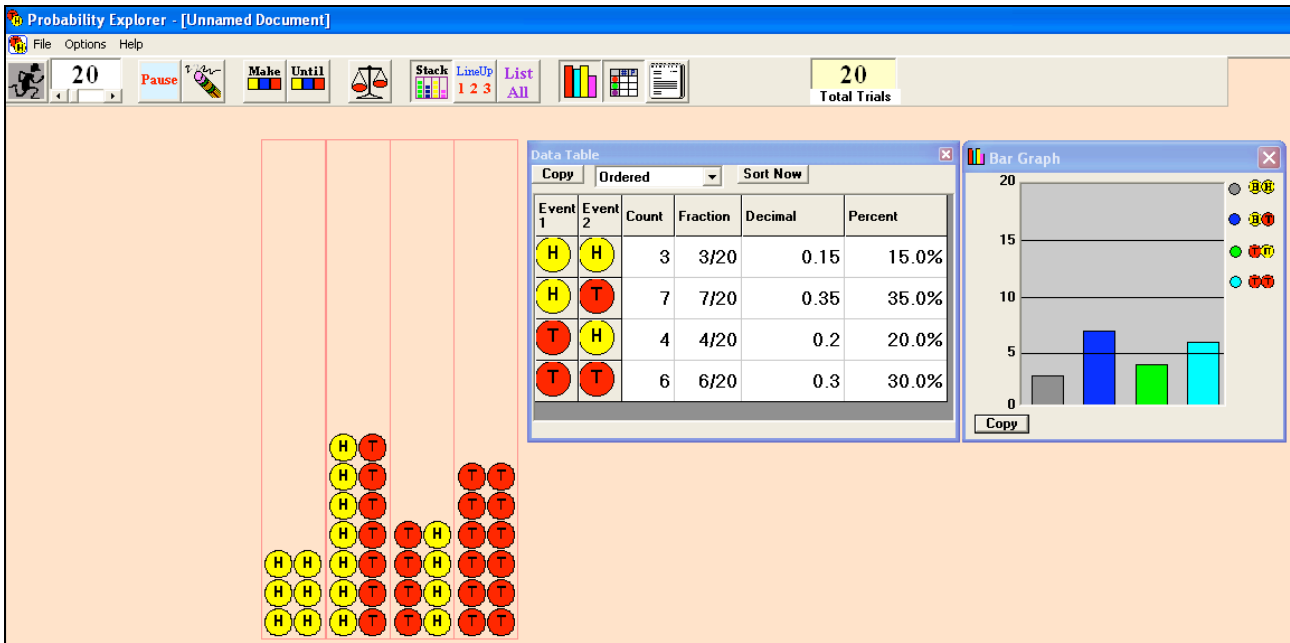


Figure 5: Use of *Probability Explorer* [22] to explore tossing a pair of coins

Of key significance for such work is an appreciation that short-term behaviour is not a good guide to eventual, long-term behaviour. As noted earlier, the frequentist view of probability is premised on the position that in the long run, random events are (surprisingly) predictable. The classical approach to probability does not offer much insight to students regarding such matters, and it seems likely that experience with software of this kind might be useful both as a supplement, and as part of the foundational experience upon which the formal work can be undertaken.

In a similar way, a graphics calculator can be used to see what happens in the long run, with some stability emerging for random events. Figure 6 shows an example of this, using a short program *Longrun* [11] on a Casio fx-9860G graphics calculator to simulate a succession of events with the same probability (such as tossing a fair coin in this case) and displays the proportion of the time that a head was shown. The results of three separate uses of the program shown in Figure 6 suggest that, despite initial wavering, in the longer term, the results are quite stable.

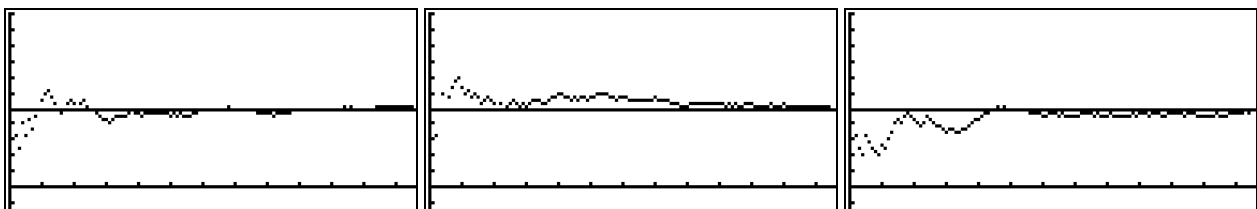


Figure 6: Use of calculator program *Longrun* [11] to explore tossing a fair coin

In a similar vein, a package for teachers from the *Nrich* website [27] offers an opportunity to simulate a succession of events, using a Flash applet suitable for classroom projection, including the

possibility of using an interactive white board for this purpose. Figure 7 shows the applet being used with a typical result showing the remarkable convergence, using the powerful graphical representation first developed by the DIME project in Scotland.

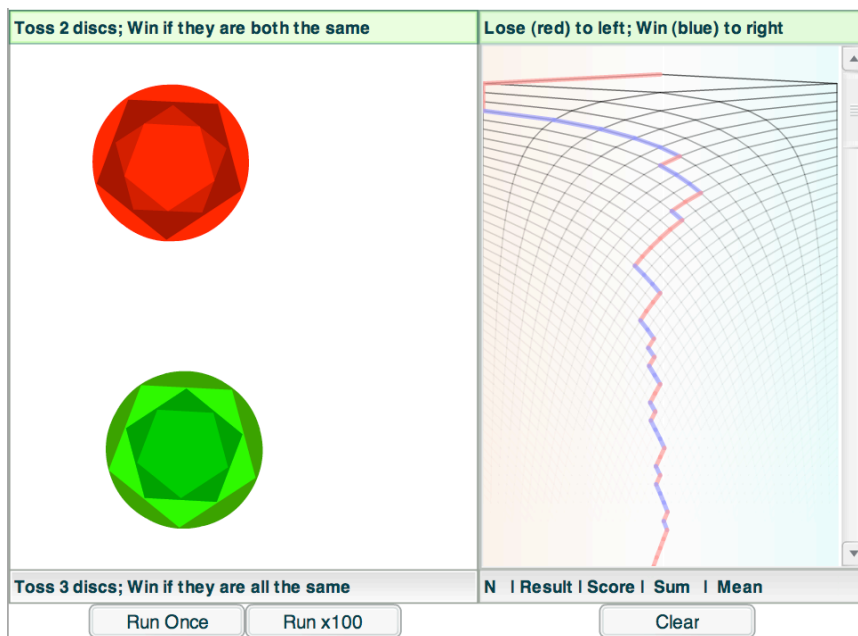


Figure 7: Use of a Flash applet from the *Nrich* website [27] to simulate an experiment with coins

A second role of simulation is to allow students to address practical everyday problems through random simulation (so-called Monte Carlo methods) rather than through theoretical analysis. In some cases, there is no alternative to the use of simulation, while in other cases, a simulation may be followed by a formal analysis.

Good examples of such problems accessible to younger students are the Cereal Box problem [1], the Monty Hall problem [34] and the well-known birthday problem. It seems important for students to have experiences of the solution of problems using simulation as an alternative strategy to mathematical analysis, to understand the power of such a method. Both the Monty Hall and Birthday problems can be addressed by secondary school students using formal methods as well, and undergraduate students can be helped to analyse the Cereal Box problem analytically. As a further example, a ‘replacement unit’ for early secondary students, *Assessing Chance* [11], was constructed and used with secondary students as an alternative to traditional approaches to probability. The unit explores several problems through the use of simulation, using graphics calculators and spreadsheets, illustrating the role of simulation very well. An example of stochastic modelling using graphics calculators is also described in [17].

Computer software, such as *Fathom* [15] and *TinkerPlots* [20], designed for use by secondary and middle schools students respectively, include simulation capabilities of various kinds, allowing for suitable problems to be addressed in this way as a new problem solving strategy. Recent work with middle-school students using simulations with *TinkerPlots* has been successful [14] while a report on work with *Fathom* highlights the apparently slow growth of student understanding [26].

5. Probability distributions

Prior to the advent of technology, probability distributions such as the normal and binomial have been printed in tables. Of necessity, such tables have been limited and have also required students to

develop table interpretation skills to suit the format. Thus, only some probability values have been tabulated for the binomial distribution, and tables of the standard normal distribution have had to be interpreted appropriately to suit other normal distributions (or even for negative values for z). These limitations have been removed by modern technologies, such as graphics calculators with inbuilt probability distributions. For example, Figure 8 shows the probabilities of obtaining various numbers of boys in a family of five children, when the probability of a baby being male is assumed to be 0.52 (a figure that tables would not normally accommodate).

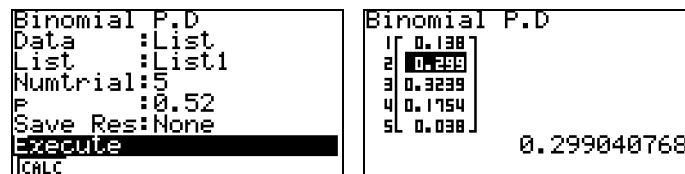


Figure 8: Probability distributions on a Casio fx-9860G calculator

As well as providing numerical values, technology can be used to provide visually informative probability distributions as well. Figure 9 shows an example of the use of *GeoGebra* software to evaluate normal probabilities [17].

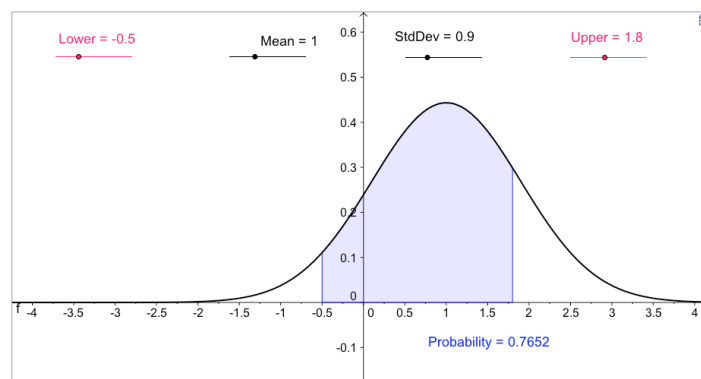


Figure 9: Evaluating normal probabilities using *GeoGebra*

These two examples illustrate that some of the limitations of tables for probability distributions can be overcome with appropriate use of technologies.

6. The Internet

The Internet offers many opportunities for both teachers and students of probability. Many of these involve using Java or Flash applets of some kind, and offer opportunities to develop key concepts associated with probability. The extent to which such resources are useful depends on the nature and quality of the access to the Internet in classrooms. Helpfully, some materials are available for downloading and use off-line, thus alleviating the problems of slow, unreliable or non-existent classroom Internet access; in some cases, CD-ROM versions are available for purchase. Some Internet materials are intended for use by the teacher with a class, in demonstration mode, to provide explanations for key phenomena or to generate a class discussion. Other materials are intended for individual use, which for some students can be done at home as well as at school, but these too can typically be used by teachers with a whole class when facilities are limited.

In this section, we review briefly the major kinds of Internet opportunities presently available, recognizing that some of these inevitably will change with the passage of time.

Dedicated probability websites. Some websites have been constructed to offer a wide variety of resources associated with teaching and learning probability. Good examples are *The Probability Web* [5] and *The Probabilistic Learning Activities Network* [12], constructed mostly to support teachers of the International Baccalaureate, but of wider use as well. The *Chance* website [31] offers a range of teaching resources, including video and audio materials, from Dartmouth College in the USA, together with many examples of undergraduate college courses elsewhere. The Mathematical Association of America's Special Interest Group on Statistics Education [25] also offers resources, as probability is frequently taught in conjunction with early courses in statistics.

Virtual laboratories. Some websites include a comprehensive collection of materials related to probability (and statistics), allowing the user to experiment with the concepts in various ways. One example is the Rice Virtual Lab in Statistics [21], which includes an online hypertext book, in which there are many aspects of probability included. The focus is on undergraduate statistics, although some of the less sophisticated material is accessible to secondary school students. A second example is the Virtual Laboratories in Probability and Statistics at the University of Alabama in Huntsville, USA [30]. Again, the focus is on the early undergraduate years, with the website providing a rich collection of materials for both students and teachers, with many opportunities to experience chance phenomena and to experiment with the key concepts via Java applets. As for the earlier example, the connections between probability and statistics are highlighted. In both of these cases, the authors have generously allowed users to download interactive materials without charge for use in educational settings.

Virtual manipulatives. The concept of virtual manipulatives recognizes the value of students manipulating objects for learning purposes, as has been common especially in elementary school. A distinguishing feature of virtual manipulatives is that they have been designed for educational use by school students, and advice of various kinds is provided for teachers (or even for parents). The best example is perhaps the National Library of Virtual Manipulatives [34], a comprehensive freely available collection of Java applets intended for K-12 education and based at Utah State University in the USA. To ease access problems, the entire library is available for purchase on a CD-ROM. A discussion of the successful use of several of these manipulatives for probability for younger students is given by [2]. Another good collection of virtual manipulatives is the *Illuminations* project of the National Council of Teachers of Mathematics [28], several of which are concerned with aspects of probability. Although designed for school students, virtual manipulatives of these kinds are also potentially helpful to older students for introductory, less formal study of probability.

Java applets. While some of the preceding classifications have included Java applets, it is worthwhile to highlight this kind of Internet resource separately as well. Java applets can be used both by teachers and by students, and provide interactive and exploratory opportunities not previously available. Some have been specifically designed for instruction at the undergraduate level. A good collection of applets is described in [32]. Some of these offer probability examples, with some advice for lecturers, while many are concerned with aspects of statistics. Similarly the collection in [7] has many examples, mostly concerned with the early undergraduate years.

Risk. The concept of 'risk' has attracted a good deal of attention recently in the wider world, as societies have attempted to come to terms with the many risky aspects of the modern world. While risk is necessarily connected with ideas of probability and chance, the focus is a little different and some websites have recently appeared to support thinking differently about everyday events from this perspective. A good example is *Understanding Uncertainty*, a UK site [35] which contains materials of various kinds, including videos and animations, with many topical examples. The author of the definitive public domain text [8] is a key person in the *Harding Center for Risk Literacy* [24] in Germany, offering a range of useful materials including an interesting Risk Quiz.

7. Conclusion

This paper has sampled from the available technologies that learners or teachers may have access to for teaching and learning probability in the early years of the twenty-first century. Calculators, computers and the Internet can be used to illustrate the claim that there are many ways in which teaching and learning might change, when compared with earlier approaches. A major change is that students can interact directly with probabilistic ideas through the medium of technology in more active ways than have previously been available with paper and pencil alone. While the possibilities in any classroom will of course be dependent on which particular technologies are available, there seems much to be gained from exploring these new opportunities.

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