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Integrating technology into learning mathematics: the special place of the scientific calculator

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Integrating technology into learning mathematics: the special place of the scientific calculator

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Abstract. Technology for learning mathematics and for STEM more generally can take many forms, but this paper argues that the most likely technology to have an impact for all students in many ASEAN countries is the scientific calculator. Integration of technology into mathematics education in the twenty-first century requires good technology, an appropriate curriculum, well-educated teachers and an assessment regime that recognizes how important technology is for mathematical activity. While popular misconceptions that a calculator is only helpful for arithmetic persist, a four-part model for understanding the educational potential of scientific calculators is described and exemplified, recognising the significance of representation, computation, exploration and affirmation. Alternatives to scientific calculators include online calculators with visual and computer algebra capabilities, which might be appropriate provided the educational environment supports their use in all aspects of education, including formal assessment. The paramount significance of the teacher for successful integration is highlighted.

1. Introduction

In recent years, the importance of STEM (Science, Technology, Engineering and Mathematics) has become clear in all communities, and many efforts have already been expended to connect these key quantitative areas together, both in schools and beyond. The importance of teaching and learning mathematics well has never before been more evident, as mathematics is the glue that binds together the various STEM components.

In this paper, the significance of technology for learning mathematics in the twenty-first century will be discussed and illustrated. While technologies in general have increased in availability and penetration in all countries, in recent years their use in mathematics education in some ASEAN countries is still lagging somewhat in many communities. At the same time, recent research summaries [1] have made clear both that calculators can be effectively used to support mathematics education and also that they do not bring about the educational harm to students that has been expected and feared by some. The paper elaborates the case for calculators offering the best prospect for successful integration into the mathematics curriculum, and describes some of the key features of doing so.

2. Technology for STEM education

There are many kinds of technologies available to STEM educators these days, and many manufacturers of those technologies are keen to both support and benefit from the large marketplaces of schools and universities. Technologies differ in terms of their capabilities, their properties, their availability, their ease of use and, of course, the expense associated with integrating them into the world of teaching and learning. Affluent communities may well be able to take advantage of all the available technologies, but most communities in the ASEAN region – as well as most communities elsewhere – need to consider the various options carefully and to make a sound selection from them. Among the technologies available are computers, tablets, smartphones, the Internet and calculators of various kinds. To enable a comparison, in this section, some criteria for selection are proposed and briefly described.

2.1. Students as users
Technology for education should mostly be used by the student, not only the teacher, although the teacher might use technologies to support instruction. Modern technologies have become simultaneously more powerful and easier for students themselves to use, so that there is little to separate the various choices on this dimension. Indeed, many have observed that students quickly develop proficiencies with technologies before their parents and teachers do, as further testimony to the modern designs. Nonetheless, it is important to remember that the key significance of any technology for learning will be addressed when it is the students themselves that are using it. A consequence of the student being regarded as the user is that many more pieces of hardware are needed than is the case if the teacher is the user. Like smartphones, the small size and weight of calculators render them especially convenient, more so than the heavier and larger laptop computers, and they can be easily taken from place to place, such as from home to school or from classroom to classroom.

2.2. Ease of use for mathematics
Ideally, a technology would provide easy and quick access to mathematics and mathematical operations. In this respect, calculators and tablets (with installed apps) are superior to computers, smartphones and the Internet. In the case of the computer, suitable software needs to be sourced, installed, updated and accessed and the various nuances of computer operations need to be navigated. Although there are some mathematical opportunities provided by the Internet, it can take some time to find these and of course they are not accessible unless a device is online, which is frequently problematic. While smartphones offer access to some apps of mathematical value, the small screen size is a significant problem for many of these, and some apps that are adequate on a tablet are not available or functional on a smartphone. Mathematical operations are immediately available to calculator users, as soon as they are turned on, as the calculators do not have other uses.

2.3. Designed for mathematics education
Many of the most powerful technologies were designed for use in a range of settings and for a range of purposes. Although they can be adapted for use in education, they were not designed for education and require modifications to do so – such as the purchase, installation and maintenance of special software or the inclusion of safeguards, such as Internet filters. While some software, such as spreadsheets, is useful in education, others have described these as ‘solutions in search of a problem’ as they are of more general use. Smartphones were clearly designed for interpersonal communication purposes in the first instance, while the Internet is powerful because of its wide applicability to many fields – many of them commercial in nature. The calculator continues to be the only technology expressly designed for use in education, so that it is unsurprising that it has great appeal for education, and for mathematics education in particular.

2.4. Not too distracting
One of the disadvantages of flexible and powerful technology is that it might serve as a significant distraction as well as having desirable properties. While being distracted is problematic for adults, it seems to be especially so for adolescents, who are easily lured by social media into various kinds of conversations with friends, or other sorts of engaging activities, rather than the intended educational use of the technology. While this is a problem for computers, tablets and the Internet, it is widely recognized as a major problem for smartphones. Indeed, in some Australian locations recently, smartphones have been officially banned from school use, because of the problems of ensuring that their use is benign. A

recent OECD report [2] similarly observed that the mere presence of ICT in schools did not routinely improve learning, most likely for similar reasons. In sharp contrast, while calculators can also be distracting, the distraction is from one aspect of mathematics to another, which is a much less serious educational problem.

2.5. Self-contained

Ideally, a technology for STEM would not require extra resources in order to be effective. Yet, ICT investments in schools require significant extra expense beyond the cost of the hardware concerned. Thus schools require specialized IT staff, extra cabling and Internet structures, software purchases and licenses and even specialized rooms as well as reliable electricity in order to install, maintain and update computers and tablets and provide Internet access on a reasonable scale free from viruses. Similarly, tablets and smartphones do not routinely come with the necessary apps and other software, which must be sourced, maintained and at times paid for. In contrast, calculators typically come as a package with both hardware and software provided from the outset and only very rarely need to be maintained with software upgrades; the only regular need for maintenance is for battery replacement, although increasingly many calculators even have a solar battery, so that even replacement of batteries is unnecessary.

2.6. A reasonable lifespan

In the world of business and commerce, a frequent need to replace IT infrastructure is recognized as an ongoing expense. However, it is rare for education systems to be funded in a similar way, so that the lifespan of technologies needs to be considered. With rapid developments in operating systems and computer requirements, computers and tablets are frequently regarded as outdated and in need of replacement after as little as three years, while smartphones are often replaced after an even shorter period. Even modern batteries in tablets and laptops become problematic after two or three years, requiring significant extra expense. In contrast, students can often use the same calculator effectively for several years, and amortize the expense of purchase over a period approximating their term in secondary school.

2.7. Acceptable for assessment

Perhaps the most critical criterion to consider when technologies are considered is the extent to which – if at all – their use is officially sanctioned by curriculum authorities and especially by examination authorities. There is a clear and widely-recognized problem with students using technologies in school, but being prevented from using them in examinations, rendering the examination process as a different one from the normal educational process. To date, it is very rare around the world for students to be permitted to use any technology in examinations except for calculators. Examination authorities have (understandable) concerns about inequities associated with students in examinations accessing extraneous information on a hard drive, or via a piece of software, or by personal communication of some kind, and have generally not permitted computers, tablets or smartphones to be used during examinations or connected to the Internet. In stark contrast, calculators have been routinely used in official and high stakes STEM examinations in most countries in the western world for around forty years now. In the last thirty years, this policy has been extended to graphics calculators for many STEM subjects in many countries, and in some cases even CAS (computer algebra system) calculators are assumed to be widely available to support the mathematics curriculum.

3. The place of the calculator

The previous section suggests that there are significant reasons for a scientific calculator to be a better choice of technology for mathematics education than other forms of technology. In the first place, calculators offer special advantages for mathematics education over other forms of technology for schools in the ASEAN region, especially if the needs of all students are to be accommodated. In the second place, calculators offer a range of opportunities for educational use beyond the mere 'calculation' suggested by the term 'calculator'.

3.1. Technology for all in developing countries

During the latter years of the twentieth century, it may have been appropriate to think of technology in schools for only some pupils, as it was too expensive to consider doing so for all pupils. However, it is now clear that little change in mathematics education can be effected if only some students have access to better learning conditions and curricula. It is critical that the technology used for learning in classrooms also be visible in other parts of the curriculum, such as in textbooks, official curriculum descriptions and in formal high-stakes examinations. [3]

Indeed, if the technology used is available to all students, it becomes possible for it to be included in some sense in the curriculum itself (e.g. specifying that some mathematical operations can be completed using technology, rather than relying on paper and pencil alone). Furthermore, when there is an artificial divide between the use of technology for learning and the use of technology for assessment, it is natural for teachers, students and others to privilege the conditions of assessment over other conditions. In short, if students are not permitted – even expected – to make appropriate use of technology in examinations, there is little incentive for textbook writers or for teachers to take the technology seriously and devote sufficient attention to using it well.

While the term 'developing country' is inherently problematic, nonetheless there is a genuine problem associated with either limited family resources or limited societal resources in many countries in Southeast Asia, certainly including Indonesia. If a technology is to become a realistic and credible component of the mathematics curriculum, it is essential that it be relatively inexpensive, so that it is a reasonable expectation that all students can be provided with access to it and the curriculum can be designed accordingly. At present, the only technology that meets these sorts of requirements for STEM subjects in general and for mathematics in particular is a scientific calculator.

3.2. A model for learning

Like computers, it is insufficient for calculators to be merely available to students in order for their use to have a positive impact. Of critical importance is how they are actually used and how they are regarded by teachers, students and others. To understand this matter better, a model for calculator use in education was constructed and proposed [4], in part to address the common misconception that the only purpose for a calculator is to undertake calculations, as its name suggests. As well as computation, they suggested that calculators are important because of their potential to offer representation of mathematical ideas, opportunities for exploration and a context for engaging in affirmation of thinking.

There are certainly some situations in mathematics where machine calculation, rather than calculation by hand, is important. For example, statistical analysis of data can be intensively arithmetic in nature if it is conducted entirely by hand, with most of students' time spent in calculations, rather than in interpreting the results of the calculations. Whether data are obtained by students directly, such as measurements in a science laboratory, or acquired from another source, such as official statistical records, it is inevitable that they will be inconvenient for hand calculation. One 'solution' to this problem is to expose students only to artificial data, constructed deliberately to ease computations. Another 'solution' is to focus on methods of computation designed to facilitate arithmetic (such as the use of raw-score formulas for finding regression coefficients). But neither of these solutions is a satisfactory alternative to analyzing the data with a calculator quickly and efficiently, and devoting correspondingly more attention to interpreting the results; and neither will persuade students of the importance of the mathematics involved.

Discussions about scientific calculators in school are frequently hampered by limited views of the nature of modern calculators, the extent of their capabilities and the range of ways in which they might support mathematics education. So the following section offers a little insight on these matters.

4. Examples of calculator use

While computation is clearly an important and widely recognized part of a model for learning with a calculator, the other three aspects of suggested calculator use: representation, exploration and affirmation are less widely recognized as important. This section of the paper offers selected examples of each of these aspects. For convenience, a particular modern calculator, the CASIO *ClassWiz* (fx-991 EX) is used for most of these, although other calculator models will be helpful for some of them.

4.1. Representation

Unlike early models of thirty and forty years ago, modern calculators like the *ClassWiz* have been designed to use conventional mathematical notation and syntax, consistent with that routinely used in textbooks and on school whiteboards. This kind of change has been deliberate, helping to remove unnecessary differences in representation between calculators and other methods of communication of mathematics. Figure 1 shows some examples of this, showing how fractions and radicals are represented on the calculator screen in conventional ways.

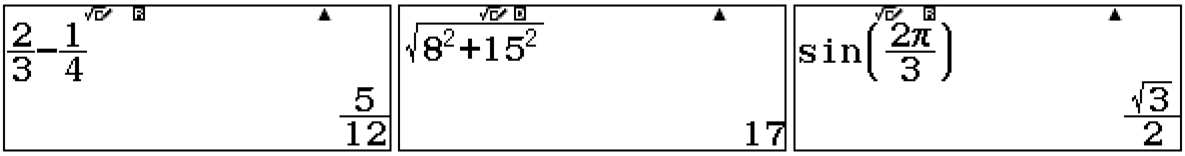

Figure 1: Representing and evaluating mathematical expressions in conventional syntax

Recent calculator developments have adapted the suite of calculator functions to those that are frequently used in senior school and early undergraduate mathematics, so that the devices are of value to students in different mathematics courses. Some more advanced examples are shown in figure 2, including some related to discrete mathematics and calculus.

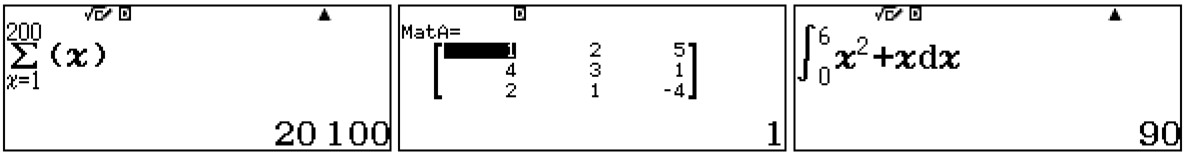

Figure 2: Representing and evaluating some more advanced mathematical functions

Many calculator operations involve entering an expression into the calculator, resulting in a re-presentation (that is, a fresh presentation of the mathematical object). Frequently, this routine process offers students some insight into the mathematical objects involved. For example, figure 3 suggests that percentages might be represented by either fractions or decimals.

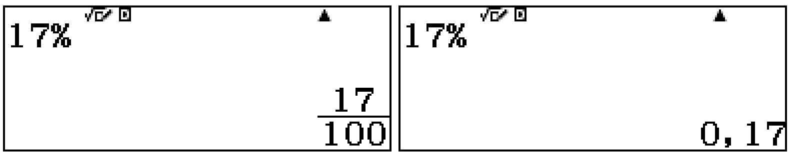

Figure 3: Representing percentages as decimals and as fractions

Similarly, in figure 4, an integer division has been routinely represented as a fraction, a surd has been represented in its simplest form, and a division of complex numbers has been represented as a complex number with a real and imaginary component. Transformations of these kinds offer students opportunity for insight regarding the mathematical objects involved. When students use calculators themselves, re-presentations of these and other kinds happen frequently, so that thoughtful use of the devices provides a means for students to understand better the meanings of the mathematical objects involved.

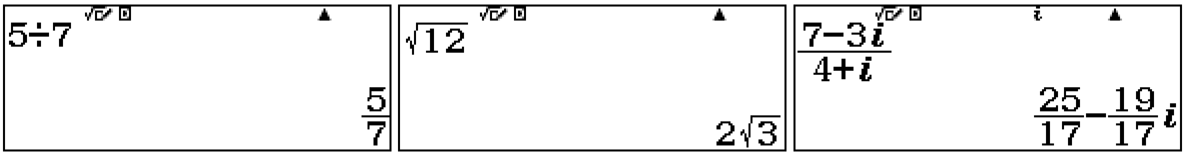

Figure 4: Various 're-presentations' of expressions

Representation of mathematical objects on calculators can offer insight into their meanings in other ways, not just those associated with evaluation of an expression. For example, the first two calculator screens in figure 5 show how a table facility on the *ClassWiz* allows students to see that a function can be represented both symbolically and numerically. The third screen shows how a sequence of powers of 2 can be represented on a spreadsheet, via a suitable recursive definition. The calculator allows students to represent these mathematical objects in a helpful way. The function representation is helpful for students to see the inherent symmetry of a quadratic expression, as well as the two roots of the function.

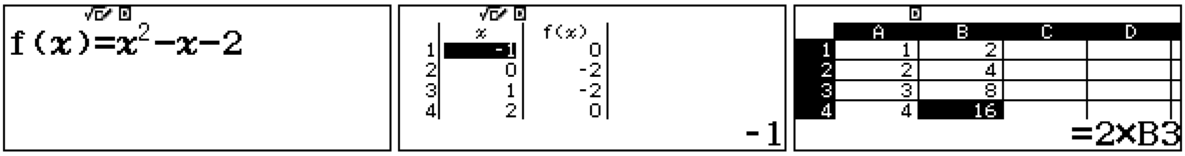

Figure 5: Representing a function in a table and a sequence in a spreadsheet

The spreadsheet representation in figure 5 of the sequence of powers of 2 given by $T_n = 2^n$ allows students to see that the same sequence can be represented recursively as well as explicitly, via the relationship that $T_n = 2 \times T_{n-1}$, $T_1 = 2$.

Together, these selected examples suggest that a calculator can represent, and can re-present, various mathematical objects of central importance to secondary school mathematics using conventional mathematical syntax and in productive ways. The calculator has been designed to support students learning about and using various mathematical objects, and their associated concepts, and offers many opportunities for students to represent these on a calculator screen to develop insight into their meanings.

4.2. Computation

As noted above, numerical computation is advantageous when real data are being used in school, both in statistics and in other contexts in the everyday world that involve measurement of some kind. Many of these arise in STEM contexts, such as in physics, chemistry and engineering laboratories and experiment. However, the benefits of computation are more substantial than merely completing efficiently and accurately calculations of these kinds.

In the first place, a modern calculator provides students with opportunities to undertake numerical methods, previously too cumbersome for everyday purposes. For example, Indonesian children studying population growth can use information provided by [5] to estimate future population growth, without yet having access to all the necessary mathematics. The 2019 population of Indonesia is approximately 271 million people and the annual growth rate is about 1.1%. Figure 6 shows that students can use these data to make successive improved estimates of the likely time when the population will reach 300 million people, under the (increasingly dubious) assumption that annual growth rate remains constant.

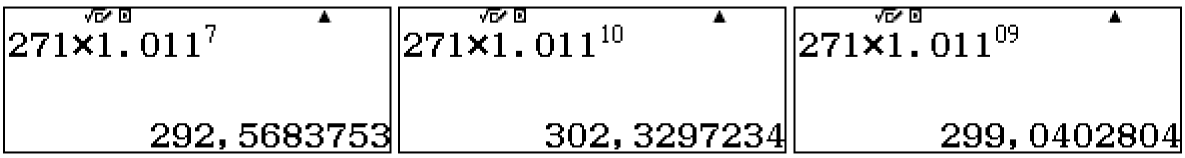

Figure 6: Estimating the likely population growth of Indonesia from 2019

Numerical methods like this are sometimes called ‘trial and adjustment’ or ‘guess, check and improve’ and represent a fresh way for students to engage in everyday mathematical modelling without a requirement for sophisticated mathematics. In this case, the first estimate of seven years (after 2019) results in a projected population that is too low; the second, improved, estimate is a little too high, while the third estimate of about nine years is the closest of these three. While the calculator is undertaking the computation, students are undertaking thinking and evaluating the next step. Work of this kind seems to be valuable, before later more formal methods (such as those using logarithms) become accessible.

In a similar way, students might address the same question using a table of values, such as that shown in figure 7. Again, such a process might serve the useful purpose of engaging in informal mathematical modelling, and with adequate recognition of the assumptions and approximations involved, suggesting that at best approximate solutions to such questions can be determined. For work in STEM generally, as well as everyday uses of mathematics, understanding work of this kind seems to be important and worthy of some curriculum emphasis.

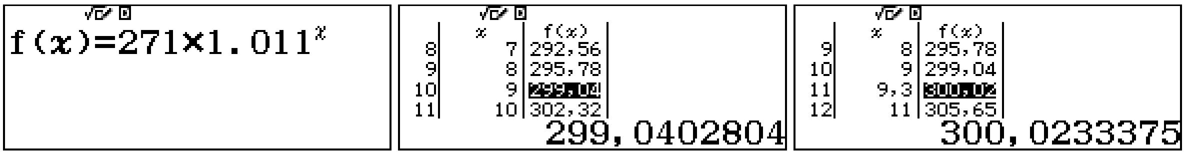

Figure 7: When will the population of Indonesia reach 300 million?

Figure 7 also shows that students might seek a more precise estimate of the time needed for the population to reach 300 million, by adjusting manually the number of years of growth. The third screen shows an ‘improved’ value of 9.3 years, or about nine years and four months, but should also prove to be a fertile discussion topic regarding what levels of accuracy are justifiable, given the assumptions involved and essential problems in the data. Such discussions are an important aspect of learning about practical applications of mathematics, but are frequently neglected when numerical methods are not used and published information is (optimistically) regarded as exact.

In the second place, modern calculators also offer tools for standard tasks like equation solving, which previously required an excessive amount of routine and tedious symbolic manipulation and elementary arithmetic. In addition, a calculator often provides more than one way of completing a task. For example, consider again the problem described in figures 6 and 7. This might be addressed on a calculator as solving an equation, such as that shown in figure 8.

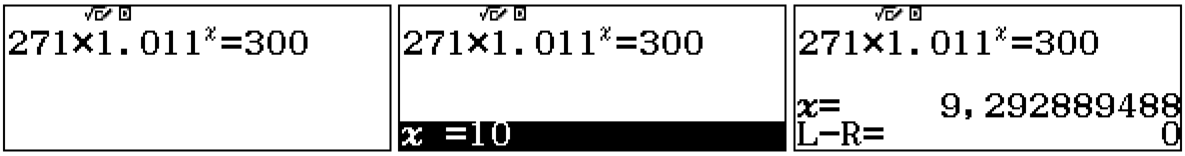

Figure 8: Solving an equation to find the time needed for a population of 300 million

The first screen shows the problem represented as an exponential equation, while the second screen initiates a numerical solution method with an initial guess of ten years. The third screen gives a value of about 9.293 years for the time required for the population to reach 300 million, close to the previous approximate figure but with more precision (of contestable merit, however). A different numerical procedure, using different mathematical ideas, is shown in figure 9, producing the same result as the equation solving procedure, with perhaps a little more insight provided.

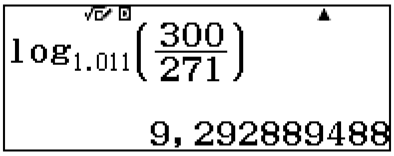

Figure 9: Using logarithms to solve the exponential equation for population growth

A great deal of time is expended with equation solving in typical secondary school curricula; while the effort is strictly necessary in order to get the solutions, the processes involved provide only modest insight, and are frequently error-prone and uninteresting. Yet good, reliable numerical methods are routinely available on calculators, dealing with most equations of the kinds studied in secondary schools, including systems of linear equations and polynomial equations of order 2 or 3, which will answer very quickly most practical questions likely to be encountered by students to a high degree of precision. Space prevents extensive illustration of these, but a single illustrative example is shown in figure 10.

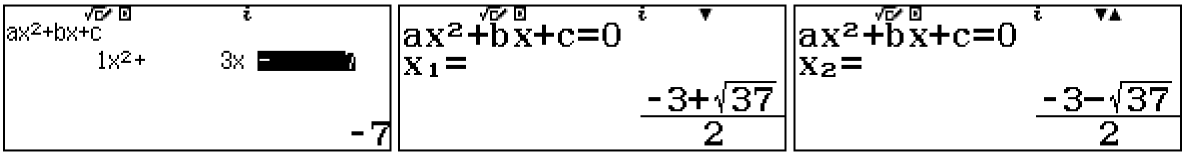

Figure 10: Solving the quadratic equation $x^2 + 3x = 7$

The calculator procedures require that equations be represented in a particular way, which requires some re-ordering in this case, but relatively little effort is needed to obtain both of the solutions in exact form. In addition, the process also provides helpful information about the associated function in order to sketch a graph that will help students appreciate some properties of the parabola involved, including its symmetry around the line x = -3/2, as suggested by Figure 11.

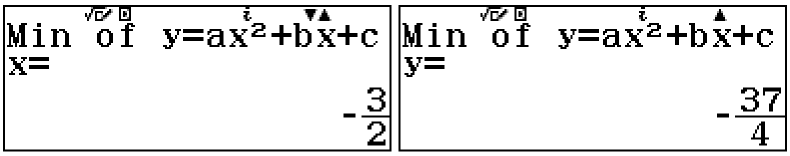

Figure 11: Further information about the associated graph of $y = x^2 + 3x - 7$

A modern calculator provides many other instances of facilitating computation, such as those associated with numerical calculus. The *ClassWiz* provides commands for both numerical integration and differentiation, so that effectively any integral or derivative likely to be encountered by school students can be evaluated to adequate precision by numerical means. For STEM students in science, engineering and mathematics, such capabilities allow practical problems of many kinds to be numerically resolved. Figure 12 shows some illustrative examples.

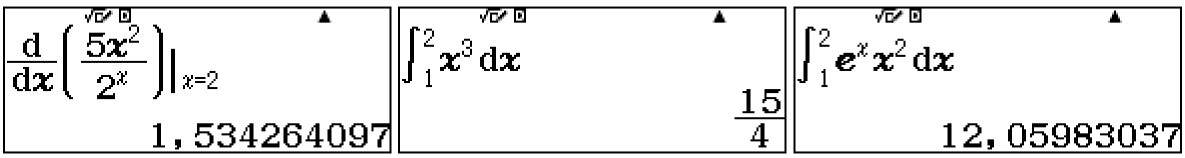

Figure 12: Examples of numerical calculus

The second example in Figure 12 gives an impression that some numerical calculus computations provided by *ClassWiz* are exact – when in fact they are not – but they are simply good numerical approximations. Although there are calculators (such as CASIO's *ClassPad* series) that provide exact derivatives and integrals, through use of a computer algebra system, such capabilities are not contained in scientific calculators at present.

When the computational capabilities of devices like *ClassWiz* are examined, it is clear that a fresh consideration of what is important in school mathematics is needed. On the one hand, it continues to be important that students develop an understanding of various mathematical ideas and processes. Yet on the other hand, care is needed to not spend excessive time learning to do relatively poorly what a small device can do much more efficiently and quickly. A new curriculum and teaching balance is needed, assuming that a major purpose of STEM education is not to merely produce human calculators

4.3. Exploration

An import educational opportunity afforded by calculators like *ClassWiz* involves that for students to explore mathematical ideas for themselves, and to discuss their observations with their peers. There are many examples of such opportunities throughout Kissane (2015), with instances for almost all of the topics studied in secondary school. Some of these are relevant to younger students, such as the learning opportunities provided by the fraction and decimal capabilities routinely provided. Figure 13 shows that a fraction-decimal conversion key allows students to see that fractions and decimals are merely different versions of the same number: that a fraction is a number (not merely a *pair* of numbers) and has a unique place on the number line. Numbers can be represented in one way with a decimal (ignoring trailing zeros) but in many different ways with a fraction.

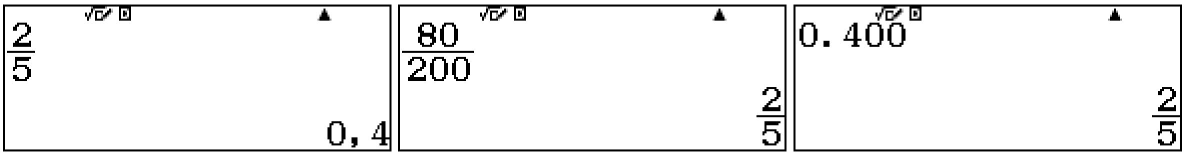

Figure 13: Different representations of the same number

On some calculators (such as the CASIO fx-991 ID PLUS, designed for Indonesian schools), even further exploratory opportunities are provided for understanding fractions and decimals. As Figure 14 shows, some fractions can be represented as either decimal approximations or exact decimals in the form of a recurring decimal, which facilitates an exploration into the nature of both fractions and decimals.

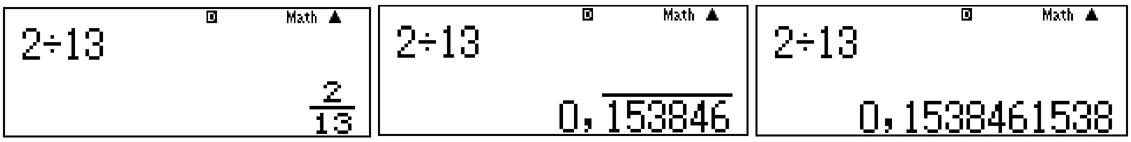

Figure 14: Representing fractions as decimals on a CASIO fx-991 ID PLUS

Many aspects of school mathematics can be readily explored on a calculator, sometimes in surprising ways. Thus, a *ClassWiz* command to find prime factors of integers (and to represent them using indices in a natural way) will allow students to see for themselves how the laws of multiplication and division of indices work. The basis for this kind of exploration is suggested by figure 15.

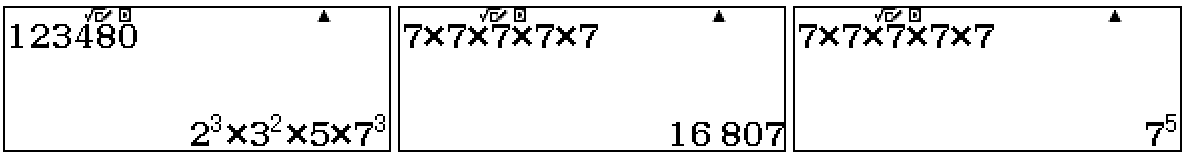

Figure 15: Using a prime factor command to explore indices

The first screen in figure 15 shows how the prime factor command replaces a number with its prime factors. The second two screens show how the result of an integer combination of numbers is shown firstly as a large number and then as its prime factors, as in the final screen. Students can use this capability to see how index notation works. Then the capabilities can be used for explorations, as shown in figure 16.

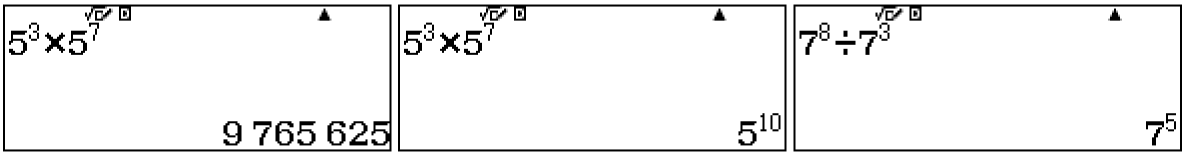

Figure 16: Exploring the index laws

At a more sophisticated level, calculators permit students to explore difficult mathematical ideas encountered in the calculus such as those of continuity, limits and convergence. The informal idea of getting 'closer and closer' to a limit can be explored via numerical means on *ClassWiz* as shown in Figure 17.

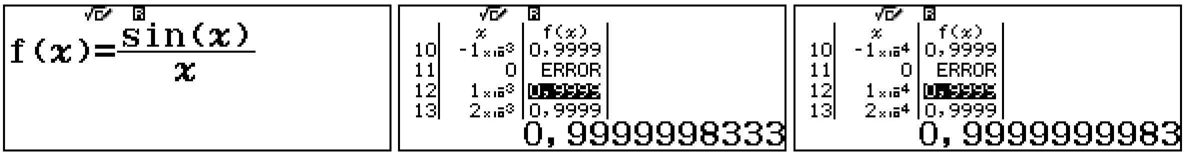

Figure 17: Exploring a trigonometric limit as x tends to 0

This particular limit is fundamental to learning about derivatives of circular functions and the idea of a limit is itself fundamental to the concept of a derivative at a point. The concept of the ratio getting closer and closer to one as the value of x gets closer and closer to (but not actually reaching) zero from either side is readily explored by students in this environment, building the important intuitions about the mathematical ideas involved.

In a similar way, students can explore the nature and properties of a derivative of a function at a point by evaluating successive values, as shown in figure 18.

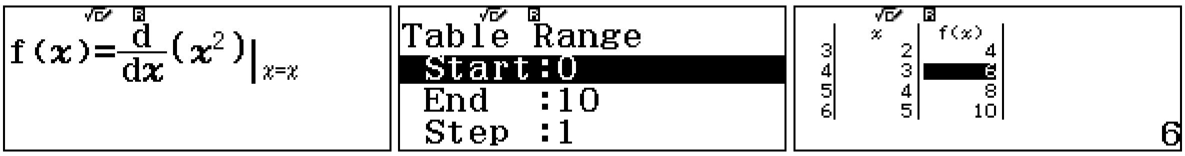

Figure 18: Exploring the derivative function of $f(x) = x^2$

In this case, the idea of a derivative at a point needs to be first understood as the gradient of a curve at a particular point, after zooming in sufficiently, not unlike the process illustrated in figure 17. The *ClassWiz* has a command to evaluate numerical derivatives at a point, but not to evaluate them in general (which requires a computer algebra system).

Figure 18 illustrates that students can see for themselves the remarkable pattern that the value of the derivative in this particular case is twice the associated x-value, providing a rich experience of the concept of a derivative function, and the particular case here that $f'(x) = 2x$. In this case, and in many other cases, the calculator provides students with access to many different kinds of experiences to support their thinking and learning through personal exploration. Many of these kinds of experiences were not available to students before technology came to classrooms.

4.4. Affirmation

The fourth and final aspect of the model proposed by [4] involves students using a calculator thoughtfully, rather than casually, and considering the information provided by the device. From this perspective, the calculator can be regarded as a personal device for testing hypotheses, checking assumptions and in general seeking reassurance that the ideas involved have been mastered. Students should always be encouraged to think of their expectations when using a calculator, and asking themselves what will happen if a particular process is undertaken (*before* they complete it).

To illustrate, figure 19 shows three examples of this. In the first screen, the values of the sine function are being tabulated with $f(x) = \sin x$, with angles measured in degrees. Before using the cursor to examine values for x > 35, a student ought *expect* that they will increase so that sin 40^o is expected to be greater than 0,5735. A more sophisticated (and more observant) student might expect that the increase is consistently decreasing so that sin 40^o is expected to be greater than 0,5735 but less than 0.6470. Such thinking helps to build an intuition about the shape of the graph of the sine function and also (in the more sophisticated case) develops a sense of anticipation that the derivative of the sine function is always less than one in the first quadrant.

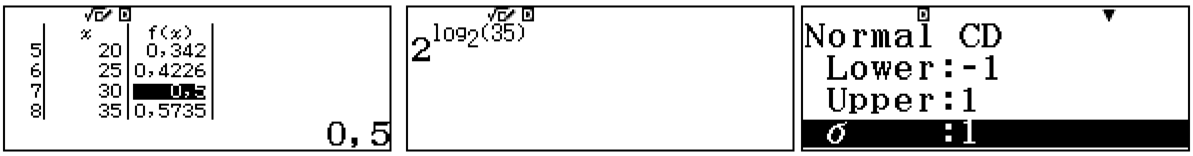

Figure 19: Illustrating thinking *before* the calculator keys are pressed: what will happen next?

The second screen in figure 19 gives another example. Students who are learning about logarithms ought to anticipate that raising the logarithm base to the power of a particular logarithm will by definition produce the number involved - in this case 35. It is reassuring to see one's thinking affirmed by the calculator in this sort of way. Similarly, the third screen permits students already familiar with the result that about 34% of the area under a standard normal curve is obtained between $z = 0$ and $z = 1$ to make a good prediction. Given the symmetry properties of the normal curve, they might anticipate a result of about 68% for the area shown; again, there is a satisfaction in making such predictions correctly.

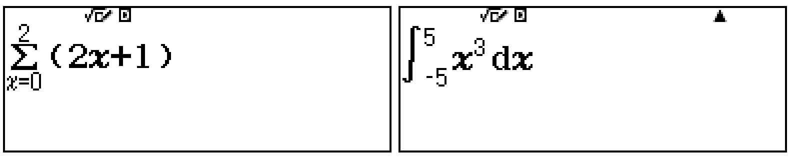

Figure 20: What will happen next? What further examples might be examined?

Thoughtful use of the calculator in these kinds of ways might be a good stimulus for students to explore the mathematical ideas for themselves, in productive ways. For example, the first screen in figure 20 offers students a chance to check that their understanding of the summation notation is correct, anticipating the result to be 1 + 3 + 5 = 9, before tapping the equals key. But then editing the command to evaluate sums of further terms, such as 1 + 3 + 5 + 7 = 16 might encourage them to look for and to see the pattern that the results of adding successive odd numbers always results in a perfect square and to start to look for reasons for such a surprising result. The second screen will allow students to visualize the integral and to anticipate (correctly) that the result will be zero, because of symmetry properties of the graph of the function. This might encourage them to edit the expression to construct other odd functions for which a similar result holds, such as $f(x) = x^3 + x$ or $f(x) = x^3 + x^5$.

While reassurance of these kinds is helpful, perhaps ironically the calculator might be of even more value to students when it does *not* affirm their thinking, generating a form of cognitive conflict and forcing some reconsideration. While students regularly develop misconceptions, and dealing with them productively is always a valuable exercise, the capacity of the calculator to be always correct is a helpful property that can lead to important learning if well-harnessed.

Some elementary examples for young students are shown in figure 21. In the first screen, Young students not yet confident with decimal place value, and reading 0.11 as 'zero point eleven', might expect a result of 0.18, since 11 + 7 = 18. The surprising (for them) result of 0.81 will force them to reconsider their interpretations of decimal numbers. Similarly, accustomed to the calculator expressing decimal numbers as fractions, a young student might reasonably expect that 0.4 will result in a fraction of four tenths, not the figure of two fifths shown in the second screen. In this case, their thinking is not incorrect, but the calculator's routine use of fractions in simplest form might encourage them to encounter the concept of equivalent fractions and at least appreciate that the same number can be represented as a fraction in more than one way.

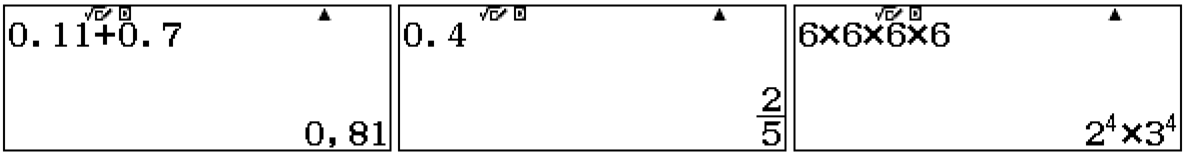

Figure 21: Some examples for which student expectations might *not* be affirmed.

The third screen in figure 21 might be a surprise to a student coming to terms with index notation, who expected a result of 6^4. The result shown is a consequence of the calculator finding *prime* factors, but might encourage students to explore other relationships between indices.

These examples highlight the possibility of a calculator being helpful for student learning, provided that students are encouraged to be thoughtful users, and don't merely regard the calculator as a device to obtain numerical answers to questions.

5. An online version of a calculator

In some international contexts, in which students already have good access to technologies like computers, tablets and the Internet and there are processes in place that permit free use of these regularly in all settings, such as classrooms, home and external examinations. In such a situation, an online version of a scientific calculator might be advantageous (despite the fact that the contexts have some undesirable consequences, most notably the opportunity for distraction, as noted earlier.). For this kind of context, CASIO have developed an online version [7] of their popular *ClassPad* calculator, which is a graphics calculator with a suite of computer algebra system (CAS) capabilities included. The *ClassPad.net* software is intended for both classroom and examination use, and so is mostly important in contexts in which formal examinations permit computers to be used. (In Australia at present, although the *ClassPad* calculator is accepted for use in examinations in some states – and hence is widely used in classrooms – the online version is not permitted in formal examinations, which still rely on written responses by students restricted to using hand-held versions of the *ClassPad* and similar devices.).

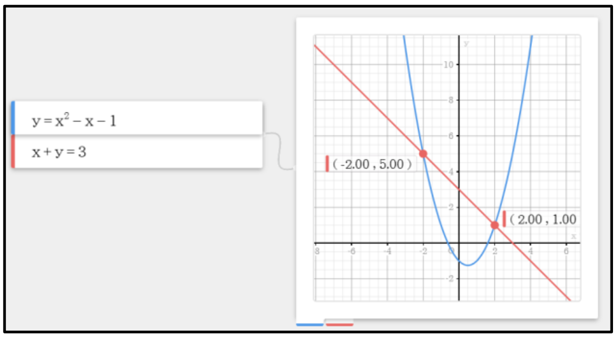

Figure 22: Use of *ClassPad.net* to examine graphs of functions

Figure 22 shows an example of the use of *ClassPad.net* to undertake a graphing task. Each task is in the form of a separate sticky note (similar to the small sticky paper notes sometimes used by people). The graphics environment shown is not available on calculators like *ClassWiz* (and graphics capabilities are not permitted in many ASEAN locations at the time of writing), but is nonetheless very helpful for students, illustrating in this case the shape of a quadratic function and its intercepts with a line.

Use of a graphics calculator, including an online version such as *ClassPad.net*, allows for geometric objects to be constructed and explored for various purposes, as shown in figure 23. Activity of that kind is also not normally available on a scientific calculator.

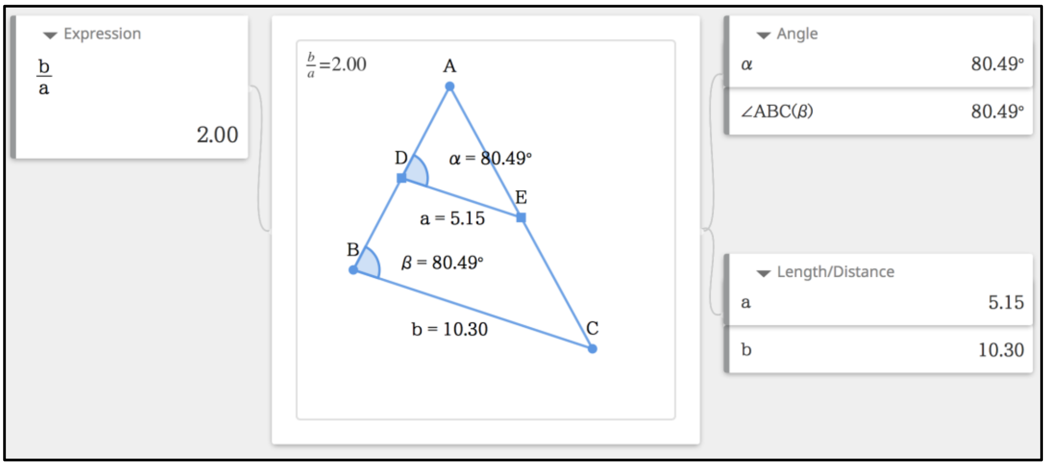

Figure 23: Using *ClassPad.net* to explore triangle properties

While guest access to *ClassPad.net* is freely available through a standard browser, more sophisticated capabilities (such as those shown in figure 24, which are reliant on CAS capabilities and others usually require a paid subscription.

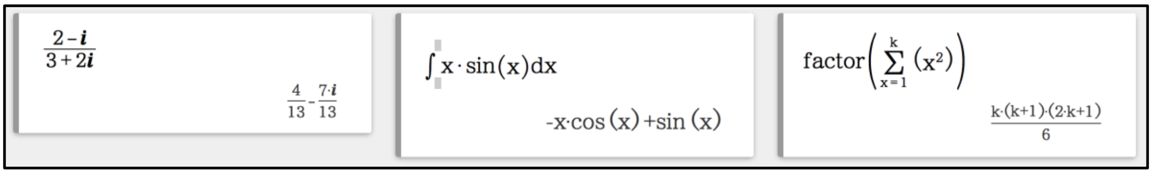

Figure 24: Some more sophisticated operations with *ClassPad.net*, require a subscription.

These few examples illustrate that an online calculator can offer further capabilities for students than those that are restricted to a scientific calculator. Of course, the available financial resources and the conditions for both classroom teaching and formal examinations are key features of an environment that is suitable for this kind of technology. The key features of a model for learning from technology that have been proposed in [4] and are illustrated in this paper are also relevant to this environment as well.

6. The place of the teacher

Integration of technology into the mathematics classroom requires careful attention to the choice of technology and also requires an environment conducive to making good use of it (such as encouragement in the official curriculum, support in the form of written materials such as textbooks, and an assessment environment that recognizes that technology should be part of the mathematical experience in school, at home and in external assessment. But these conditions are not themselves sufficient for successful integration; the teacher is the most important ingredient.

A modern scientific calculator can provide significant support for mathematics education and support for other aspects of STEM education, but it seems unlikely that it will be used to best advantage if students do not get sound advice and direction from their teacher. There is a persistent view in the community, including parts of the mathematics education community, that the main purpose of scientific calculators is to undertake arithmetic computation. Without suitable activity by the teacher, students seem likely to hold similar views and the potential advantages of the calculator may get overlooked.

Using a calculator effectively may require a pedagogical shift by teachers, away from a view that students learn by direct instruction, imitation and practice and towards a view that students need to be active participants in their own learning. [8] Such a shift is unlikely to happen without some support, so that published advice and materials such as [6] may be important and teachers may well develop the necessary expertise along with colleagues in schools, departments or professional workshops. Of course, a thorough knowledge of calculator operations and capabilities is necessary, some of which can be acquired from calculator manuals. However a deep knowledge of the educational properties of calculators is unlikely to be obtained from calculator instruction manuals. Many teachers find the use of computer emulators of scientific calculators to be helpful teaching aids, when their classroom permits computer projection, but these do not substitute for students using the calculators themselves, recognizing that the technology is intended to be personal in nature.

7. Conclusion

The scientific calculator continues to be the sole technology that has been developed specifically to support mathematics education in schools and the early undergraduate years, and modern versions have much to offer both students and teachers. While other technologies may be more powerful technically, the educational power of calculators arises from their unique mix of capabilities and properties that render them accessible to essentially all students while proving acceptable to curriculum and assessment authorities in many places around the world. In some contexts, online versions of calculators are appropriate and acceptable. Together with well-educated teachers, and a curriculum that recognizes the importance of modern technologies, the scientific calculator can have considerable impact upon the mathematics classroom, offering new opportunities for learning for all students, not only those in affluent communities. In turn, successful integration of scientific calculators into the mathematics curriculum can reasonably be expected to improve STEM education more generally.

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