Fuzzy Rule Interpolation for Multidimensional Input Spaces in Determining d50c of Hydrocyclones

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***Abstract*—Fuzzy rule-based systems have been very popular in many engineering applications. In mineral engineering, fuzzy rules are normally constructed using some fuzzy rule extraction techniques to establish the determination model in predicting the d50c of hydrocyclones. However, when generating fuzzy rules from the available information, it may result in a sparse fuzzy rule base. The use of more than one input variable is also common in hydrocyclone data analysis. This paper examines the application of fuzzy interpolation to resolve the problems using sparse fuzzy rule bases, and to perform analysis of fuzzy interpolation in multidimensional input spaces.**

***Index Terms*—d50c, fuzzy interpolation, hydrocyclone, mineral processing, sparse fuzzy rules.**

I. Introduction

HYDROCYCLONES [1] find extensive application in the mineral process industry where they are used for the classification and separation of solids suspended in fluids. Hydrocyclones normally have no moving parts. The feed slurry containing all sizes of particles enters the hydrocyclone. Inside, due to centrifugal force experienced by the slurry, the heavier particles separate from the lighter ones. After the particles suspended in the fluid are classified, they are discharged either from the vortex finder as overflow or from the spigot opening as underflow. The mathematical analysis of the separation is not always possible due to the complexity of the separation mechanism in the hydrocyclone, the interpretation difficulties of the physical behavior, the complexity of forces acting on the particles, and the variations in the operation conditions. However, much work has been concentrated on describing the hydrocyclone performance by means of mathematical models based on the empirical analysis [2]–[5].

The performance of a hydrocyclone is normally described by a parameter known as d50, which determines the classification efficiency. It represents a particular size particle with a 50% chance of reporting either to the overflow or to the underflow streams. The separation efficiency depends on the dimensions of the hydrocyclone and the operational parameters. Examples of the operational parameters are the flowrates and the densities of slurries. The d50 is not a monitored parameter, but can be determined from separation curves known as the tromp curves. They are used to provide the relationship between the weight fraction of each particle size in the overflow and underflow streams.

In practical applications, the d50 curve is corrected by assuming that a fraction of the heavier particles can enter the overflow stream. This is equivalent to the fraction of water in the underflow. This correction of d50 is designated as d50c. The correct estimation of d50c is important since it is directly related to the efficiency of the operations. Under normal industrial applications of hydrocyclones, any deviation from a desired d50c value cannot be restored without changing the operation conditions or/and the geometry of the hydrocyclone. Also, sensing changes in d50c is a difficult task, requiring external interference by taking samples from the overflow and underflow streams and conducting time consuming size distribution analyzes of these samples.

Gupta and Eren have discussed the automatic control of hydrocyclones [5]. The output signal d50c cannot be sensed or conditioned directly, thus d50c needs to be calculated from the operational parameters. Then the automatic control of hydrocyclones can be achieved by manipulation of the operational parameters such as diameters of the spigot opening (Du), the vortex finder height (H), the inlet flowrate (Qi), the density (Pi), and the temperature of slurries (T) for a desired value of d50c. The correct prediction of d50c is essential to generate the control signals for the actuators.

Eren *et al.* have proposed an artificial neural network (ANN) model which can incorporate many nonconventional operation variables easily [6]. This is normally very time consuming and difficult to be implemented using conventional empirical and statistical methods. Eren *et al.* [6] have also showed that the accuracy has improved when using the ANN technique. Although ANN techniques have proven to be useful for the prediction of d50c, the main disadvantage has been their inability to convey the acquired knowledge to the user. That is, the trained network is represented by a collection of inaccessible weights thus introducing difficulties for the user to understand and to modify the model.

At this end, fuzzy system which makes use of human understandable rules seems to be a better alternatives to the ANN. Besides, fuzzy set theory that is capable of handling vagueness and uncertainty in most engineering applications will be more appropriate [7]. A fuzzy set allows for the degree of membership of an item in a set to be any real number between 0 and 1. This allows human observations, expressions and expertise to be modeled more closely. Once the fuzzy sets have been defined, it is possible to use them in constructing rules for fuzzy expert systems and in performing fuzzy inference. This approach seems to be suitable to d50c determination as it allows the incorporation of intelligent and human knowledge to deal with each individual case. However, the extraction of fuzzy rules from the data can

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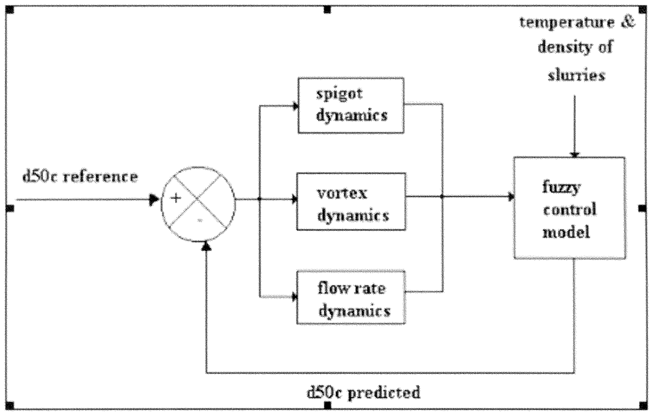

Fig. 1. Online fuzzy hydrocyclone control system.

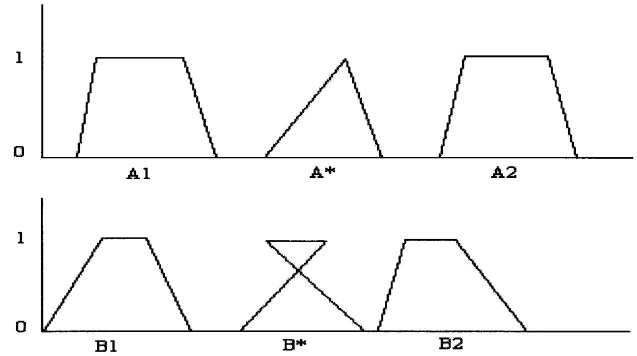

Fig. 2. Fuzzy rule interpolation.

be difficult for analysts with little experience. This could be a major drawback for use in determining d50c. If a fuzzy rule extraction technique is made available, then fuzzy systems can still be used for hydrocyclone control [8], [9].

Normally, the information embedded in the available training data is not enough to cover the whole population. With the use of any fuzzy extraction techniques, the fuzzy rules generated from these data form a sparse fuzzy rule base [10]. Classical fuzzy reasoning methods cannot be used to handle a sparse fuzzy rule base. This is due to the lack of an inference mechanism in the case when observations find no fuzzy rule to fire [11]. This is undesirable when using a fuzzy model for determining d50c of hydrocyclone. If more than half the input instances in the prediction cannot find any rule to fire, this determination model is considered useless. As there are a number of good fuzzy rule extraction techniques [8], [9] that can be used in extracting the sparse rule base used for determining d50c of hydrocyclone, they are not discussed here in this paper. A block diagram of the proposed online hydrocyclone control systems using fuzzy control scheme is shown in Fig. 1. The main purpose of this paper is to introduce a fuzzy rule interpretation technique that could solve the problem in a sparse fuzzy rule base for used in determining d50c of hydrocyclone.

This paper examines the practical use of fuzzy rule interpolation for multidimensional input spaces in determining d50c. Thus suggesting that fuzzy control system can be used for this application if fuzzy rule interpolation is incorporated. Next section will discuss on the theory of fuzzy rule interpolation. Section three will look at the improved multidimensional alpha cut based (IMUL) fuzzy rule interpolation technique, which is used to solve the problem in sparse fuzzy rule base. A case study and results will also be presented.

II. Preliminaries

The classical inference methods in fuzzy control by Zadeh, Mamdani and Sugeno deal with dense rule bases, in which cases are when the input spaces are completely covered by the rule premises. If the universe of discourse is not completely covered by the rule antecedents it may happen that for an observation finds no rule to fire. Fuzzy rule interpolation, proposed first by Kóczy and Hirota [12], is an inference technique for fuzzy rule bases, whenever the antecedents do not cover the whole input universe, i.e., for so-called sparse rule bases. There are a few reasons that this will happen. First, reducing the number of rules in a rule base and, subsequently, the complexity of the resulting fuzzy system by omitting redundant rules with proper technique can result in incomplete rule base. The use of sparse rule base allows removal of redundant rules with proper techniques even if the resulting set of rules contains "gaps." Second, the incomplete knowledge about the modeled system, regardless of the construction of the rule base can result in sparse rule bases. Originally, on the basis of Zadeh's concepts, fuzzy systems were constructed from linguistic IF-THEN-RULES provided by a human expert. More recently, learning techniques have increasingly been developed and applied to the construction of fuzzy IF-THEN-RULES from numerical sample data. Both ways of construction can result in sparse rule basis. In case of using learning techniques it may happen that the sample data do not represent sufficiently to map some regions of the input domain. In the case of rules obtained from human expertise, an incomplete rule base can be the consequence of missing knowledge for certain system configuration.

Fuzzy rule interpolation techniques provide a tool for specifying an output fuzzy set whenever at least one of the input spaces is sparse. Kóczy and Hirota [12] introduced the first interpolation approach known as KH interpolation. This is based on the Fundamental Equation of Rule Interpolation [refer to (1)]. This method determines the conclusion by its α-cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with that among observation and the antecedents for all-important α-cuts (breakpoint levels). This is shown in the equation as follows (refer to Fig. 2 for notations)

$$d(A^*, A_1) : d(A^*, A_2) = d(B^*, B_1) : d(B^*, B_2). \qquad (1)$$

Two conditions apply for the usage of the linear interpolation [12]. First, there should exist an ordering on the input and output universes. This allows us to introduce a notion of distance between the fuzzy sets. Second, the input sets (antecedents, consequents, and the observation) should be convex and normal fuzzy sets.

The KH interpolation possesses several advantageous properties [11]. First, it behaves approximately linearly between the breakpoint levels. Second, its computational complexity is low, as it is sufficient to calculate the conclusion for the breakpoint level set. However, for some input situations it fails to result in a directly interpretable fuzzy set, because the slopes of the conclusion can collapse as shown in Fig. 2. A modification of the

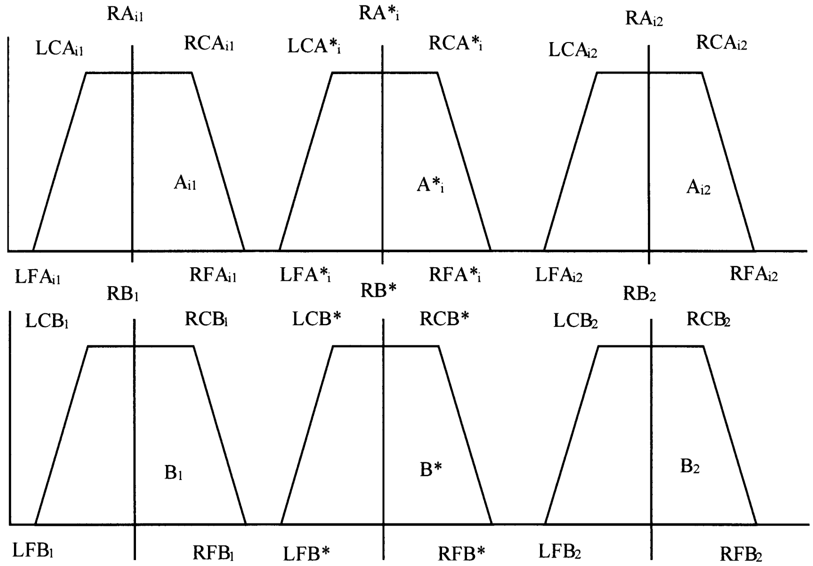

Fig. 3. Notations Used for Fuzzy Interpolation.

original method has been proposed in [13] technique, which can solve the problem of abnormal conclusions while maintaining its advantageous properties.

For ease of computation and ease of interpretability, normally triangular or trapezoidal membership functions are used in most engineering applications and thus will be used in determining d50c of hydrocyclone.

III. Calculations

In this paper, the improved multidimensional (IMUL) fuzzy interpolation technique [13] is used to generate d50c of hydrocyclone using the sparse fuzzy rule base. This method incorporates features of the modified alpha-cut fuzzy rules interpolation (MACI) [11] and the conservation of fuzziness technique [14]. It makes use of the vector description of the fuzzy sets by representing them in characteristic points, and the coordinate transformation features of the MACI. At the same time, it can take the fuzziness of the fuzzy sets in the input spaces at the conclusion as those presented in the conservation of fuzziness technique. The advantage of this improved fuzzy interpolation technique not only takes the fuzziness of the sets at the input spaces, but also make use of the information of the core at the consequents.

Fig. 3 shows the notations used for the following calculations. For k input dimensions, the reference characteristic point of the interpolated conclusion can be calculated:

$$\mathrm{RB}{*} = (1-\lambda_{\mathrm{core}})\mathrm{RB}_1 + \lambda_{\mathrm{core}}\mathrm{RB}_2 \tag{2}$$

where

$$\lambda_{\mathrm{core}} = \frac{\sqrt{\sum_{i=1}^{k}(\mathrm{RA}_i^* - \mathrm{RA}_{i1})^2}}{\sqrt{\sum_{i=1}^{k}(\mathrm{RA}_{i2} - \mathrm{RA}_{i1})^2}}$$

By using the above reference point, the left and right cores of the conclusion can then be calculated

For right core

$$\mathrm{RCB}^* = (1-\lambda_{\mathrm{right}})\mathrm{RCB}_1 + \lambda_{\mathrm{right}}\mathrm{RCB}_2 + (\lambda_{\mathrm{core}} - \lambda_{\mathrm{right}})(\mathrm{RB}_2 - \mathrm{RB}_1). \tag{3}$$

where

$$\lambda_{\mathrm{right}} = \frac{\sqrt{\sum_{i=1}^{k}(\mathrm{RCA}_i^* - \mathrm{RCA}_{i1})^2}}{\sqrt{\sum_{i=1}^{k}(\mathrm{RCA}_{i2} - \mathrm{RCA}_{i1})^2}}.$$

After calculating the cores of the two sides, the two flanks can then be found. When calculating the left and right flanks of the conclusion, the relative fuzziness of the fuzzy sets in all the input spaces are taken into consideration as follows, based on A_{i1} and B_1:

$$s_i = \mathrm{RFA}_{i1} - \mathrm{RCA}_{i1} \tag{4}$$

$$s' = \mathrm{RFB}_1 - \mathrm{RCB}_1 \tag{5}$$

$$r_i = \mathrm{LCA}_i^* - \mathrm{LFA}_i^* \tag{6}$$

$$r' = \mathrm{LCB}^* - \mathrm{LFB}^* \tag{7}$$

$$u_i = \mathrm{RA}_i^* - \mathrm{RA}_{i1} \tag{8}$$

$$u' = \mathrm{RB}^* - \mathrm{RB}_1. \tag{9}$$

In multidimensional input spaces

$$s = \sqrt{\sum_{i=1}^{k}(s_i)^2} \tag{10}$$

$$r = \sqrt{\sum_{i=1}^{k}(r_i)^2} \tag{11}$$

$$u = \sqrt{\sum_{i=1}^{k}(u_i)^2}. \tag{12}$$

To calculate the left flank

$$\mathrm{LFB}^* = \mathrm{LCB}^* - r_k\left(1 + \left|\frac{s'}{u'} - \frac{s}{u}\right|\right) \tag{13}$$

IV. Results and Novelties

Data collected from a Krebs hydrocyclone model D6B-12o-839 has been used. There are a total of 70 training data and 69 testing data used in this study. The training and

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Warning: no rule is fired for input [493.0 26.00 85.20 3.750 26.00 ]! 0 is used as default output.
Warning: no rule is fired for input [388.0 24.20 69.50 2.650 33.00 ]! 0 is used as default output.
Warning: no rule is fired for input [462.0 10.20 85.20 3.750 40.00 ]! 0 used as default output.
Warning: no rule is fired for input [267.00 24.50 85.20 3.750 34.00 ]! 0 is used as default output.
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Fig. 4. Warning message for input without firing rules.

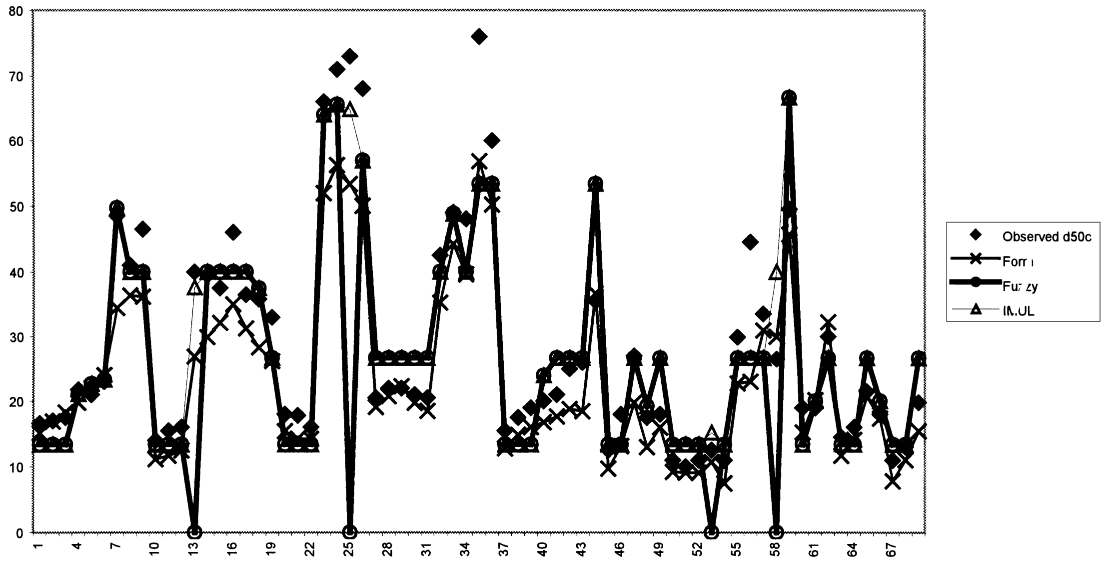

Fig. 5. Graphical plots of all predicted d50c and observed d50c.

testing sets are selected based on one in training and next in testing scheme. This is the normal way of splitting the training and testing data in establishing the d50c determination model [6]. The input parameters are diameters of the spigot opening (Du), the vortex finder height (H), the inlet flowrate (Qi), the density (Pi) and the temperature of slurries (T), and the output is d50c. The self-generating fuzzy rules technique [8] is used to extract fuzzy rules from the 70 training data. This fuzzy rule extraction technique is used as it is available at hand, however, any other fuzzy rule extraction techniques can be used to generate the sparse fuzzy rule base.

In order to show the applicability of this proposed method, the results are also used to compare with results generated from the on-line control model obtained from conventional statistical analysis discussed in [5].

The on-line control formula used in [5] is

$$d50c = 23.36\left[\frac{\mathrm{EXP}(-0.0125Du + 0.1031)Pi}{\mathrm{EXP}(0.2721Du)}\right] * \left[(-0.0229Du - 0.0211)\left(\frac{Qi}{Q\min}\right) + 0.0739Du + 0.9138\right] * \left[-0.426\left(\frac{H}{H_D}\right) + 1.42\right] * \left[0.2\left(\frac{T_n}{T}\right) + 0.8\right] \tag{14}$$

where

$$T_n = 25, \quad Q\min = 120, \quad \text{and} \quad H_D = 85.2$$

When the sparse fuzzy rule base are used to perform control on the testing data, 4 sets of input instances cannot find any fuzzy rules to fire, and the fuzzy inference system default the output to zero as shown in Fig. 4. In this case study, the number of input sets that cannot find any rule to fire is considered minimal. However, in some cases, this may not always be true. If more than half the input instances cannot find any rule to fire, this control system may be considered useless.

From observation and Euclidean distance measure on each input variable, the nearest fuzzy rules of the four input instances are determined to be used for the IMUL fuzzy interpolation. After performing the IMUL fuzzy interpolation, d50c for the four input instances can be interpolated straight away.

Percentage similarity coefficient (PSC) is used to perform the measurement of difference between the predicted d50c (T) and the observed d50c (O). The calculation is perform using the following expression:

$$\mathrm{PSC} = 200\frac{\sum_{i=1}^{P}\min(T_i, O_i)}{\sum_{i=1}^{P}(T_i + O_i)}. \tag{15}$$

The PSC value between the observed d50c and the d50c calculated using [5] is about 90%. The d50c generated from the fuzzy inference system without the IMUL fuzzy rule interpolation as compared to the observed d50c is about 88%. However, when the IMUL fuzzy rule interpolation is used, the PSC value increased to around 92%.

Fig. 5 shows the plot of all the predicted d50c as compared to the observed d50c. The sharp fall of value in the fuzzy curve is mainly due the default value of zero when no fuzzy rule is fired.

From these results, it has been shown that the fuzzy system with the IMUL fuzzy rule interpolation technique has better prediction results compared to those of the formula. This case study has also shown the importance of the IMUL fuzzy rule interpolation technique to be used in the practical situation where the observed d50c can only generate a sparse fuzzy rule base. However, in this case study, the number of input instances that cannot find any fuzzy rule to fire is considered minimum. If more than 20% of the input instances have the similar problem, then the fuzzy inference system will have no use in determining the d50c. Besides, with the use of the IMUL fuzzy interpolation technique, the number of fuzzy rules in the rule base is not increased. This is a desirable characteristic for on-line hydrocyclone control, as a minimum of fuzzy rules used will allow the analyst to better examine the determination model.

V. Conclusion

In the prediction of hydrocyclone parameter d 50c, the results of the best known conventional model have been compared with those results obtained from the fuzzy inference system with the IMUL fuzzy interpolation technique. The case study has shown that with the assistance of the IMUL fuzzy interpolation technique, the sparse fuzzy rule base extracted from the observed d50c can provide a good improvement in on-line hydrocyclone control. This is useful in the field of on-line hydrocyclone modeling as it allows the incorporation of fuzzy sets to allow better human understanding into the determination model.

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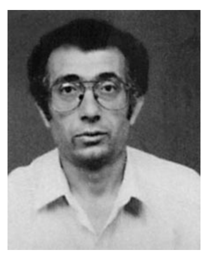

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