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# An Efficient Forward-backward Algorithm to MSDEPP Including Batteries and Voltage Control Devices

Anula Abeygunawardana, Ali Arefi, *Member, IEEE*, Gerard Ledwich, *Senior Member, IEEE*  
Electrical Engineering and Computer Science School  
Queensland University of Technology  
Brisbane, Australia

**Abstract**— Electric distribution networks are now in the era of transition from passive to active distribution networks with the integration of energy storage devices. Optimal usage of batteries and voltage control devices along with other upgrades in network needs a distribution expansion planning (DEP) considering inter-temporal dependencies of stages. This paper presents an efficient approach for solving multi-stage distribution expansion planning problems (MSDEPP) based on a forward-backward approach considering energy storage devices such as batteries and voltage control devices such as voltage regulators and capacitors. The proposed algorithm is compared with three other techniques including full dynamic, forward fill-in, backward pull-out from the point of view of their precision and their computational efficiency. The simulation results for the IEEE 13 bus network show the proposed pseudo-dynamic forward-backward approach presents good efficiency in precision and time of optimization.

**Index Terms**— Energy storage, Capacitor, Distribution expansion planning, Multi-stage planning, Pseudo-dynamic approach.

## INTRODUCTION

Long-term distribution expansion planning problems (DEPPs) are complex combinatorial problems due to the large number of variables involved. Further, as DEPPs are non-linear “NP-hard” problems, computational time generally increases exponentially with the number of variables. Therefore, exact mathematical methods can only be applied to solve DEPPs for small scale systems. For this reason, many authors have applied heuristic methods to solve large scale DEPPs. Even with heuristics, computing good solutions to large scale DEPPs remains a time consuming operation. One of the main factors that increase computational time of DEPPs, beside the large dimension of the networks is time dynamic nature of the problem. Due to this reason, researchers have used diverse approaches to simplify the dynamic nature of the DEPPs.

Most of early studies on DEP considered the study period as a single stage by ignoring the inter-temporal dependences and the dynamic nature of DEPPs [1, 2]. This approach is known as horizon planning and it finds the optimal state of the

network for a future fixed year. Since timing of decisions is not a decision variable in horizon planning, it may not be appropriate for long term planning considering economic aspect. Because it does not take into account the time value of the money and the optimal sizes of network expansion equipment for different load levels. In order to achieve a more precise and optimal modeling of DEPP, a dynamic planning approach that minimizes the overall present value of several planning periods should be used. However, exact dynamic planning is very complex, involves a very large number of variables and requires enormous computational effort to obtain the optimal solution. Therefore, exact dynamic planning works only for small DEPPs. A number of studies propose multi-stage distribution expansion planning (MSDEP) models based on exact dynamic planning approach for small networks [1, 3]. Due to the curse of dimensionality in applying exact dynamic approach for multistage planning of real-size networks, a number of algorithms based on pseudo-dynamic theory have been proposed in the literature. Pseudo-dynamic algorithms decompose multi-stage planning problems into a sequence of single-stage problems and each sub problem is solved independently and coordinated through different strategies.

### A. Forward fill-in planning

The most commonly used pseudo- dynamic approaches for DEP are the forward fill-in and backward pull-out approaches. In the forward fill-in approach, the static expansion planning problems are solved sequentially for all time stages starting from the first one, considering in the next time stage the equipment installed in the past. Forward fill-in is simple but it works best for very short-range planning and in cases where there are only one or a few load growth locations [4]. A few studies have used the forward fill-in approach for DEP mainly due to its simplicity [5, 6].

### B. Backward pull-out planning

In the backward pull-out approach, first the optimal set of investments are determined for the last stage (say stage T) of the planning period by considering the demand in the last stage and the network configuration at the beginning of planning period. Then the planning exercise proceeds

backwards in time analyzing stage T-1, then stage T-2, and so forth, to stage 1, using only the elements determined in the first step. Backward pull-out works best for multi-year planning situations [4] mainly because it takes into account demand growth for the whole planning period. This approach has been used by a number of studies [3, 7-10].

### C. Forward-backward approach

Although the backward pull-out approach gives reasonable solutions for long-term DEPPs, this approach decides the optimal set of investments for the horizon year demand level and finds the timing of these investments backwards. Since the backward pull-out approach optimizes the network for last time stage, it effectively does no splitting of the time stages. In the other hand, the forward fill-in approach does full-splitting of the problem into time stages and it does not take future load growth into account. Since most distribution equipment is available only in discrete sizes and there is usually a large economy of scale, investments decisions for each time stage should be made by considering future load growth. For the same reason, the optimal size of network expansion equipment for one particular demand level may not be optimal for another demand level. Therefore, both forward and backward planning can be combined to obtain the better results for DEPPs as a combined approach can take into account, future load growth, the discrete nature and economy of scale of distribution equipment, and the time value of money.

Few studies have used forward-backward approach [4, 5, 11]. Nara et al used a recursive forward-backward approach for multi-stage distribution expansion planning (MSDEP) using the branch and bound method [4]. In this approach, forward fill-in approach is initiated from the first time stage and proceeded to the second stage and then backward is tried. If a better expansion plan cannot be obtained from backward path, the forward path proceeds to the next time stage and the procedure goes on. When the backward path succeeds a new better expansion plan is obtained, a new forward path planning is started from the first stage to the time stage where the backward planning started and this new plan is compared with the backward plan to find the better plan. This procedure is done recursively until finding the best solution. Unlike in [4], [11] starts backward pull-out planning from the last time stage and proceeds to the first time stage. Then forward fill-in planning is carried out from the first time stage to the next using the expansion plan obtained for the first stage from the backward planning and this forward planning is continued until the last time stage. The total cost of the backward and forward paths are then compared and if the difference in cost between two plans is below a value predetermined by the planner the optimization stops, otherwise backward planning is redone from the last time stage using the expansion plan of the last stage obtained from forward planning. This procedure continues until convergence. The forward-backward approach used by [5] for multi-year active distribution network planning is different to the above two approaches. All possible forward backward multi-planning scenarios are compared to find the best expansion plan. Possible multi-year planning scenarios includes forward fill-in from the first year to the final year, backward pull-out from the last year to the first year, and

from each intermediate year backward pull-out planning until the first year and forward fill-in planning to the last year.

This paper presents an efficient forward-backward approach for MSDEPP including energy storage sizing and placement, capacitor and voltage regulator placement, transformer and line upgrade. The results are compared in terms of precision and computational efficiency for IEEE 13 bus system.

### THE PROPOSED FORWARD-BACKWARD TO MSDEPP

The flowchart of proposed forward-backward approach is shown in Fig.1.

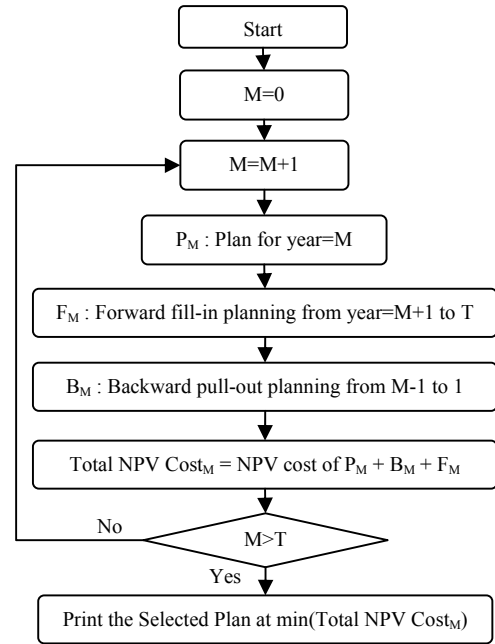


Figure 1. The flowchart of proposed forward-backward approach

The objective of MSDEPP is to design a distribution system that can economically supply the load over the planning stages. Therefore, the objective function to be minimized is as:

$$\begin{aligned}
 \text{Objective Function} &= \min(\text{Fix Installation NPV} + \\
 &\quad \text{Variable Installation NPV} + \text{Operation NPV} - \text{Salvage NPV}) \\
 \text{Subject to: } &\begin{cases} 0.95 \leq |V_i| \leq 1.05 & i = 1 \dots N \\ |I_i| \leq I_{i, \text{Max}} & i = 1 \dots B \\ \text{Discretized Transformer, Battery, Capacitor Size} \\ \text{4 Type of conductor, doubling and tripling} \\ \text{Regulator Numbers} \leq 3 \end{cases} \quad (1)
 \end{aligned}$$

Where NPV is net present value; Fix Installation NPV is fix NPV cost of equipment installation; Variable Installation NPV is variable NPV cost of equipment installation; Operation NPV is operation NPV cost of equipment + energy and power loss; Salvage NPV is salvage NPV cost of variable Installation NPV;  $|V_i|$  is the magnitude of voltage at bus  $i$ ;

$|I_i|$  is the magnitude of current at branch  $i$ ;  $I_{i,Max}$  is the current rating of branch  $i$ ;  $B$  is number of branches and  $N$  is number of buses.

The MSDEPP in modern power systems is a large-scale, mixed-integer, and non-linear problem and exact mathematical methods can hardly be applied to solve it. Heuristic approaches are most commonly used to solve MSDEP problems. Due to the nature of Particle Swarm Optimization (PSO) as an optimization tool proven to be capable of handling highly non-linear and mixed integer problems, PSO has been widely applied to solve DEPPs [10, 12-15]. However, in some cases, premature convergence and poor fine-tuning of the final solution can occur in PSO. Therefore, in this study, we use a modified version of PSO (MPSO) by combining the strengths of PSO and GA to increase the diversity of variables and thereby to escape from local minima. Our hybrid algorithm combines standard PSO particle update rules with idea of mutation from GA as in [16]. In addition, the constriction factor approach for PSO is applied in this algorithm because it has better performance compared to the inertia weight approach [17].

The decision variables in each particle of MPSO to MSDEP of active distribution networks include the size, location and size batteries, capacitors, conductors and voltage regulators as presented in Fig. 2.

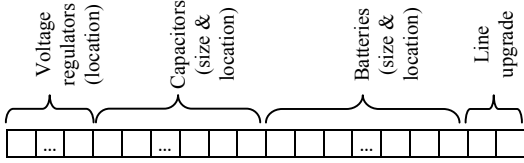


Figure 2. Particle structure in MPSO

In the initialization process, a population of particles is randomly generated. Each particle represents a candidate solution for expansion planning. The fitness calculation involves two steps in our problem. First we calculate the total capital and operating cost for each given particle, then we carry out AC load flow based on the bus-injection to branch-current (BIBC) matrix[18] for the network configuration given by the particle to check if all constraints are met. If a constraint is violated we penalized the particle by adding a high cost value to the objective. In addition, the load flow gives all necessary information for calculating the total loss for each particle. The required transformer upgrades for both distribution transformers and sub-transmission transformer are calculated for each corresponding particle after finding the loading of transformer by including the new voltage regulators, capacitors, batteries, and line upgrades given in the particle. The estimated objective function value or fitness value of each particle is used to locate the individual best particle and global best particle. Then we proceed to the next iteration, where a new population is generated by updating the velocity of each particle based on the best solution seen so far by that particle and on the global best particle. This procedure is continued until convergence.

## SIMULATION RESULTS

In order to compare the different approaches of solving the MSDEPP, they are applied to the IEEE 13 bus network which is a 4.16 kV radial distribution network including one voltage regulator, 13 nodes (9 load nodes), and two capacitor banks in buses 10 and 6 whose capacities are 600 and 100 kvar, respectively as shown in Fig. 3 [6]. The test system consists of both overhead and underground cables. The load characteristics and sizes of distribution transforms obtained from initial network design are given in Table I.

TABLE I. LOADS AND TRANSFORMER SIZES OF IEEE 13 BUS

Bus no.	3	4	6	7	9	10	11	12	13
kw	100	1255	170	128	170	843	170	230	400
kvar	58	718	80	86	151	462	125	132	290
Trans. (kVA)	63+	315+	200	315	100+	1000	10+0	315	315+
	100	3*500			200		200		500

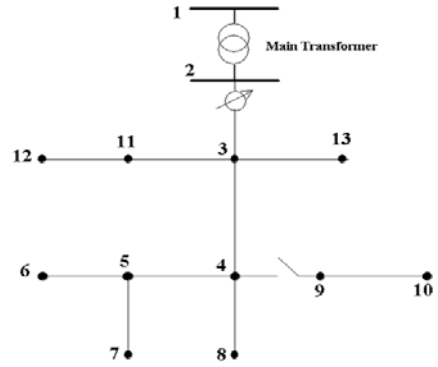


Figure 3. IEEE 13 bus test System

The cost parameters of the candidate equipment are shown in Table II. Note that in this case study, storage units and capacitors are used for both peak clipping and voltage control. The analysis here assumes a load growth of 7% per each time stage and discount rate of 5%. The MPSO parameters used in the simulation are particle population = 50, maximum iterations = 100,  $\psi_{max} = 4.05$ ,  $K = 0.99$  and the mutation probability = 80%. The mutation operator is applied to 10% of particle population.

We develop the network expansion plans using four different approaches and expansion plans obtained from different approaches are compared from the point of view of their precision and their computational efficiency. The four different cases considered in the study are:

- Dynamic planning case (at-once planning):
- Planning case with forward fill-in approach
- Planning case with backward pull-out approach
- Planning with proposed forward-backward approach

In case A, we consider all time stages together and optimal expansion plan is outlined for whole planning period by minimizing overall present value of all planning periods. Network expansion plans are developed for these four scenarios for two different planning horizons, 3 and 5 years.

Therefore, altogether we developed eight network expansion plans to examine the effectiveness of different pseudo-dynamic planning approaches in terms of their precision and their computational efficiency. The MPSO technique described above was used to develop these eight MSDEPs.

TABLE II. COST PARAMETERS OF CANDIDATE EQUIPMENT

Candidate equipment type		Fix Inst. cost (k\$)	Variable inst. cost (k\$)	Life time (yr)
OH conductors	Pluto	75	91 k\$/km	40
	Saturn		98 k\$/km	
UG cables	Al Triplex	100	511 k\$/km	40
	Cu 1 core		525 k\$/km	
Overhead trans.	25kVA	75	30	40
	63kVA		31	
	100kVA		35	
	200kVA		42	
	315kVA		50	
Pad mounted substation	500kVA	125	65	40
	315kVA		63	
	500kVA		77	
	750kVA		98	
Sub Trans.	1000kVA	125	119	40
	1500kVA		140	
Regulator		25	-	40
Capacitor		3	0.025/kVAR	20
Battery storage		0.7	1/kWh	7

The choice of approach for solving multi-stage dynamic DEPPs mainly depends on the precision of the approach and its computational efficiency. Table III gives the comparison of computational time and total NPV cost of expansion plans for the different cases for two different planning horizons (3 and 5 years). Although we expect that the dynamic planning procedure can lead to the better optimal solution, our results show that in both 3 and 5 year planning case, the total NPV cost of expansion plan obtained from this case is higher than that of other cases. This is because in case *A*, the optimization algorithm may trap in local optima due to the problem complexity and size. Furthermore, the computational time of case *A* is a multiple of that of other cases for both 3 and 5 years planning horizons.

TABLE III. COMPARISON OF DIFFERENT APPROACHES OF MSDEPP

Approach	3 year planning horizon		5 year planning horizon	
	time (min)	Total NPV (k\$)	time (min)	Total NPV (k\$)
A	676	199.9	1296	1435.0
B	4.2	198.1	52	925.3
C	5.8	195.5	75	805.7
D	14.5	195.5	256	795.5

Tables IV and V present the optimal network expansion plans for planning horizon of 3 years and 5 years respectively for all four approaches considered. As shown in Table IV, existing network equipment can supply the load growth in first year without violating any constraints. Therefore there is no difference in optimal expansion plans obtained from forward fill-in approach that starts from the first year and forward-backward planning that starts from the 2nd year. Hence, in

applying forward-backward approach for 3 year planning horizon case we effectively need to compare the optimal plans obtained from case *B* and case *C*. As expected backward pull-out approach gives lower cost expansion plans than that obtained using forward fill-in approach.

A 7% annual growth in load demand for 3 years would overload the distribution transformers located in bus 10 and 12 in year 2, and transformers located in bus 4 in year 3 respectively. Therefore, in 3 year optimal expansion plan (in the backward pull-out approach) we expect that new transformers would be added in these three buses. However, as can be seen from Table IV, in the lowest cost plan obtained from backward pull-out approach, only one new distribution transformer of capacity 1000 kVA is added at bus 10. It would be economical to install battery storage units and capacitors in year 2 and 3 to defer the transformer upgrades at bus 4 and 12. The results show that these battery storage units and capacitors not only defer the transformer investments but also help to keep voltages at load buses and line current within acceptable limits.

TABLE IV. MSDEPP RESULTS FOR 3 YEAR PLANNING HORIZON

Approach	Upgrades	Planning Years			Total	Time (min.)
		1	2	3		
A	Trans. (kVA)	0	0	0	0	676
	Line (no.)	0	0	0	0	
	Regul. (no.)	0	0	0	0	
	Batt. (kVA)	0	29	234	263	
	Cap.(kVAR)	0	678	0	678	
	Sub.Tr(kVA)	0	0	0	0	
	NPV (K\$)	0	43.6	156.3	199.9	
B	Trans. (kVA)	0	0	1000	1000	4.2
	Line (no.)	0	0	0	0	
	Regul. (no.)	0	0	0	0	
	Batt. (kVA)	0	32.5	50	82.5	
	Cap.(kVAR)	0	450	1350	1800	
	Sub.Tr(kVA)	0	0	0	0	
	NPV (K\$)	0	48.1	150.0	198.1	
C	Trans. (kVA)	0	1000	0	1000	5.8
	Line (no.)	0	0	0	0	
	Regul. (no.)	0	0	0	0	
	Batt. (kVA)	0	5	81.5	86.5	
	Cap.(kVAR)	0	325	1400	1725	
	Sub.Tr(kVA)	0	0	0	0	
	NPV (K\$)	0	136.2	59.3	195.5	
D (optimal plan obtained for starting year =3)	Trans. (kVA)	0	1000	0	1000	14.5
	Line (no.)	0	0	0	0	
	Regul. (no.)	0	0	0	0	
	Batt. (kVA)	0	5	81.5	86.5	
	Cap.(kVAR)	0	325	1400	1725	
	Sub.Tr(kVA)	0	0	0	0	
	NPV (K\$)	0	136.2	59.3	195.5	

As can be seen from the Table V, The load growth for 5 years at a 7% annual growth rate would overload the transformers at bus 6 and bus 9 in year 4 and transformers at bus 3 in year 5 respectively, in addition to transformers at bus 10 and 12 in year 2 and transformers at bus 4 in year 3. In this case, as shown in Table V, the optimal planning is obtained from case *D*, proposed forward-backward, starting from year 3. As seen in Table V, the amount of batteries and capacitor for case *D* is more than that for cases *B* and *C*.

In addition, there will be one line upgrade to keep line currents within their maximum allowable limits. As in the 3 year planning case, battery storage and capacitors not only defer transformer upgrades but also keep load voltages within limits. It should be noted here that, in both 3 and 5 year planning horizon, no any voltage regulator is selected. This is because existing voltage regulator can provide the voltage support for a five year load growth as lines are not very long in this 13 bus test system.

TABLE V. MSDEPP RESULTS FOR 5 YEAR PLANNING HORIZON

Approach	Upgrades	Planning Years					Total	Time (min.)
		1	2	3	4	5		
A	Trans. (kVA)	0	315	0	0	315	315	1296
	Line (no.)	0	0	0	1	0	1	
	Regul. (no.)	0	0	0	0	0	0	
	Batt. (kVA)	0	41	271	0	340	652	
	Cap.(kVAR)	0	338	186	147	252	923	
	Sub.Tr(kVA)	0	0	0	0	0	0	
	NPV (K\$)	0	182.1	210.9	228.9	164.6	1435.0	
B	Trans. (kVA)	0	100	1000	200	100	1400	52
	Line (no.)	0	0	0	1	0	1	
	Regul. (no.)	0	0	0	0	0	0	
	Batt. (kVA)	0	18	117.5	0	182.5	318	
	Cap.(kVAR)	0	300	975	0	225	1500	
	Sub.Tr(kVA)	0	0	0	0	0	0	
	NPV (K\$)	0	125.8	353.0	276.1	170.4	925.3	
C	Trans. (kVA)	0	1000	0	0	415	1415	75
	Line (no.)	0	1	0	0	0	1	
	Regul. (no.)	0	0	0	0	0	0	
	Batt. (kVA)	0	5	18.5	34	260	317.5	
	Cap.(kVAR)	0	25	100	600	775	1500	
	Sub.Tr(kVA)	0	0	0	0	0	0	
	NPV (K\$)	0	444.8	36.7	46.1	278.1	805.7	
D (optimal plan obtained for starting year =3)	Trans. (kVA)	0	1000	0	100	100	1200	256
	Line (no.)	0	1	0	0	0	1	
	Regul. (no.)	0	0	0	0	0	0	
	Batt. (kVA)	0	5	18.5	5	296	324.5	
	Cap.(kVAR)	0	25	100	675	650	1450	
	Sub.Tr(kVA)	0	0	0	0	0	0	
	NPV (K\$)	0	444.8	36.7	74.4	239.7	795.5	

### CONCLUSION

This study proposes an effective forward-backward approach to solve MSDEPP and compares different approaches in terms of their computational efficiency and precision. The different approaches including proposed algorithm (Case D) are applied to obtain the expansion plans for MSDEP of IEEE 13 bus test system. It is observed that pseudo-dynamic approaches are more effective in solving MSDEPP compared to full dynamic planning because they are computationally efficient and give acceptable results. Since the proposed forward-backward approach develops series of expansion plans starting from every year of planning horizon, this approach would give better results than that obtained by other cases. The simulation results show that using batteries and capacitors can defer the installation of transformer and line upgrades and presents more cost effective planning for distribution networks.

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