Optimal Distributed Generation Placement in Radial Distribution Systems

Ali Arefi, IEEE Member
Deptartment of Electric Power Engineering, South Tehran Branch, Islamic Azad University
a_arefi@azad.ac.ir

M.R. Andervazh
Deptartment of Electric Power Engineering, South Tehran Branch, Islamic Azad University
mr.andervazh@gmail.com

Akbar Yavartalab
TAVANIR Company
yavartalab@tavanir.org.ir

Javad Olamaei, IEEE Member
Deptartment of Electric Power Engineering, South Tehran Branch, Islamic Azad University
j_olamaei@azad.ac.ir

Abstract— **Distributed generation (DG) resources are commonly used in the electric systems to obtain minimum line losses, as one of the benefits of DG, in radial distribution systems. Studies have shown the importance of appropriate selection of location and size of DGs. This paper proposes an analytical method for solving optimal distributed generation placement (ODGP) problem to minimize line losses in radial distribution systems using loss sensitivity factor (LSF) based on bus-injection to branch-current (BIBC) matrix. The proposed method is formulated and tested on 12 and 34 bus radial distribution systems. The classical grid search algorithm based on successive load flows is employed to validate the results. The main advantages of the proposed method as compared with the other conventional methods are the robustness and no need to calculate and invert large admittance or Jacobian matrices. Therefore, the simulation time and the amount of computer memory, required for processing data especially for the large systems, decreases.**

Keywords- Energy Loss Reduction (ELR); Grid Search Algorithm; Power Loss Reduction (PLR); Loss Sensitivity Factor (LSF); Optimal DG Placement (ODGP); Radial Distribution system (RDS).

I. INTRODUCTION

A new identity known as "distributed generation" (DG) appeared in the electric power systems after deregulation in the electric power sector. Distributed generations generally refer to small-scale electric power generators near to customers or are connected to an electric distribution system.

Employing DGs in electric distribution systems have several advantages such as Grid reinforcement, reducing power losses and on-peak operating costs, improving voltage profiles and load factors, relieved T&D congestion, deferring or eliminating system upgrades. In addition, improving system integrity, increasing overall energy efficiency and reducing fuel costs, enhancing system reliability, reducing emissions of pollutants and health care costs, and improving power quality are some benefits of using DGs in distribution systems [1]-[5].

In order to achieve the mentioned goals, proper location and size of distributed generation resources known as optimal DG placement is of great importance to obtain their maximum potential benefits. On the other hand, studies have shown side effect of inappropriate selection of location and size of DG (increasing in the losses) [6], [7]. Reducing the losses by the proper selection of DGs has been an important subject of study conducted by distribution engineers. Therefore, it is important to find optimal location and size of DGs required to minimize feeder losses.

Analytical approaches for optimal placement of DG with unity power factor is to minimize the power loss of the system. In a radial distribution network with uniformly distributed load, a "2/3 rule" is used to place DG on the network. Based on this rule, it is suggested a DG of approximately 2/3 capacity of incoming generation is to be installed at approximately 2/3 of the length of line [8].

As mentioned earlier, in order to minimize line losses of the electric systems, determining the size and location of DGs to be placed in the RDS is very important. There is the number of studies to define the optimum size and location of DG. Some mathematical approaches in this field are as: loss sensitivity factors for determining the near optimal [4], optimal load flow with second order algorithm method [9], genetic algorithm and Hereford Ranch algorithm [10], Fuzzy-GA method [11], tabu search approach [12], 2/3 rule [13], and an analytical approach in radial as well as networked systems [14].

By investigating the approaches presented by the authors, the solution techniques for loss minimization by optimal placement of DG in the electric power system can be categorized as Analytical, numerical programming, heuristics, and artificial intelligence based methods [15]. It should be noted that although the heuristic methods are intuitive, easy to understand and simple to implement as compared with the analytical and numerical programming methods, the results presented by the heuristic algorithms are not guaranteed to be optimal [15].

In this paper, a method is presented for ODGP in RDSs based on the analytical method. In the proposed method, the optimum size and location of DG will be defined to minimize total power losses using loss sensitivity factor (LSF) based on bus-injection to branch-current (BIBC) matrix, without the use of impedance or Jacobian matrices for radial systems. In this study, DG is capable of supplying both active and reactive power.

The solution of ODGP problem for loss minimization based on the proposed method is validated against the results obtained based on the classical grid search algorithm, which is implemented by successive load flow for two test RDS.

The proposed method is suitable for radial distribution systems and is fast, accurate and easy to implement. since the proposed method is an analytical method and exploits the topological characteristics of a distribution system, there is no need for the Jacobian, bus admittance, Y_{bus}, or the bus impedance, Z_{bus}, matrices. Therefore, It is more suitable for RDSs of considerable sizes than the earlier analytical methods,.

The advantageous of the proposed method is its capability in finding optimal solution with maximum computation time reduction for solving the problem of ODGP because there is no need to compute and invert any matrices, which get even more increased in size as the size of the RDS becomes larger. Therefore, the proposed method can achieve the advantages of computation time reduction and accuracy improvement.

The paper is organized as follows: the proposed method and problem formulation for solving the ODGP problem in radial distribution system is presented in section II. Section III gives the flowchart of the proposed method. The simulation results obtained by applying the proposed method on 12 and 34-bus test RDSs are presented in section IV. Section V presents some conclusions.

II. The Proposed Method

In this section, a detailed problem formulation using an analytical method based on equivalent bus current injection technique, distribution load flow and LSF is presented. The main constraint is to keep bus voltages within acceptable range of $0.95 \le V_i \le 1.05$. The values are in per unit and V_i is the voltage at bus *i* in the RDS.

A. Equivalent Bus Current Injection Formulation

In this section, the equivalent bus current injection for each bus is presented based on complex active and reactive power and voltage at each bus. Assuming the load as constant power and the system in its three phase steady state operation mode then the complex load S_i and the equivalent bus current injection at bus i can be expressed by (1) and (2) respectively:

$$S_i = P_i + jQ_i \qquad \text{i=1, 2,…, n} \qquad (1)$$

$$I_i = \left(S_i / |V_i| \angle \theta_i\right)^* \qquad (2)$$

where S_i, P_i, Q_i are the equivalent complex, active and reactive powers at bus *I*, respectively, and $|V_i|$ and θ_i are the voltage magnitude and angle at bus *i*, respectively, and *n* is the total number of buses in the system. Symbol * stands for the complex conjugate operator.

By substituting (1) in (2), the corresponding equivalent current injection at bus *i* is obtained as (3):

$$I_i = \left(S_i / |V_i| \angle \theta_i\right)^* = \frac{(P_i - jQ_i)(\cos\theta_i + j\cos\theta_i)}{|V_i|}$$
$$= \frac{P_i \cos\theta_i + Q_i \sin\theta_i}{V_i} + j\frac{P_i \sin\theta_i - Q_i \cos\theta_i}{V_i} = \mathrm{Re}(I_i) + j\,\mathrm{Im}(I_i) \qquad (3)$$

where *Re(.)* and *Im(.)* stand for the real and imaginary part of a complex quantity, respectively.

B. Branch Current Formulation

The power injections at each bus can be rewritten in respect to the equivalent current injections using (2). In addition, the branch currents can be formed as a function of the equivalent current injections by using of Bus-injection to Branch-current (BIBC) matrix. The elements of BIBC matrix are “0”s or “1”s and its dimension is m×(n– 1) for a system with *m* branch and an *n* bus. The relationship of bus current injections and branch currents can be expressed in a compact matrix form as:

$$[B] = [BIBC]_{nb\times(n-1)} \cdot [I]_{(n-1)\times 1} \qquad (4)$$

where *nb* and *n* are the number of branches and buses, respectively, *[I]* is the equivalent bus current injection vector, *[B]* is the branch current vector, and *[BIBC]* is the relationship matrix between the branch currents and the bus current injections.

In other words BIBC matrix is responsible for the relationship between the branch currents and bus injection currents. Since for distribution feeder with many laterals, it may not be practical to build the matrix BIBC by hand, this matrix should be calculated from the matrix *A*, node to branch incidence matrix.

By using (4) and the set of equations obtained by KCL at each bus, the *[BIBC]* matrix for the 12-bus test system shown in Fig. 2 (one of the test RDSs) is obtained as (5). This BIBC matrix is an upper triangular matrix, which contains values of 0 and +1 only.

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \end{bmatrix} = \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1 \\ 0&1&1&1&1&1&1&1&1&1&1 \\ 0&0&1&1&1&1&1&1&1&1&1 \\ 0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&1&1&1&1 \\ 0&0&0&0&0&1&1&1&1&1&1 \\ 0&0&0&0&0&0&1&1&1&1&1 \\ 0&0&0&0&0&0&0&1&1&1&1 \\ 0&0&0&0&0&0&0&0&1&1&1 \\ 0&0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} \cdot \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \\ I_{10} \\ I_{11} \\ I_{12} \end{bmatrix} \qquad (5)$$

where B_i, I_i are the branch and bus current at bus *i*, respectively.

C. Power Loss Formulation

Total power loss is obtained by the sum of individual line losses at each branch. The total power loss is written in terms of the branch current injections as:

$$Ploss = \sum_{j=1}^{nb} Ploss_j = \sum_{j=1}^{nb} R_j \cdot |B_j|^2 \qquad \text{i=1, 2,… ,nb} \qquad (6)$$

where $|B_j|$ and R_j are the branch current magnitude and resistance of the j^{th} branch, respectively, and *nb* is the number of branches in RDS.

By substituting (3) and (4) in (6), the total power loss expressed in (6) is rewritten in terms of bus current injections in its extended matrix form after some mathematical calculation as:

$$Ploss=\sum_{j=1}^{nb} \begin{matrix} R_j\left(\left[\sum_{k=2}^{n} BIBC(j,k-1)\frac{P_k\cos\theta_k+Q_k\sin\theta_k}{|V_k|}\right]^2\right. \\ \left.+\left[\sum_{k=2}^{n} BIBC(j,k-1)\frac{P_k\sin\theta_k-Q_k\cos\theta_k}{|V_k|}\right]^2\right) \end{matrix} \quad (7)$$

D. ODGP Formulation Based on LSF

The derivative of the total power losses with respect to i^{th} bus injected reactive power gives the sensitivity factor that can be expressed as:

$$\begin{aligned}\partial Ploss/\partial Q_i &= 2\sum_{j=1}^{nb} R_j\left(\left[\sum_{k=2}^{n} BIBC(j,k-1)\cdot \mathrm{Re}(I_k)\right]BIBC(j,i-1)\frac{\sin\theta_i}{|V_i|}\right. \\ &\left.-2\sum_{j=1}^{nb} R_j\left[\sum_{k=2}^{n} BIBC(j,k-1)\cdot \mathrm{Im}(I_k)\right]BIBC(j,i-1)\frac{\cos\theta_i}{|V_i|}\right)\end{aligned} \quad (8)$$

It should be noted that if the i^{th} bus is not connected to the j^{th} branch, the elements of BIBC matrix is zero, that is, *BIBC (j, i − 1) = 0*, and the derivative of the corresponding element is equal to zero, $\partial Ploss/\partial Qi = 0$. So, the expression for the sensitivity factor, (8), can be rewritten as:

$$\begin{aligned}\partial Ploss/\partial Q_i &= 2\sum_{j=1}^{nb} R_j \times \\ &\sum_{k=2}^{n} dBIBC_i(j,k-1)\times\left[\frac{\sin\theta_i}{|V_i|}\mathrm{Re}(I_k)-\frac{\cos\theta_i}{|V_i|}\mathrm{Im}(I_k)\right]\end{aligned} \quad (9)$$

where $[dBIBC_i]$ matrix is constructed by a simple procedure as follows:

- Read BIBC matrix and the bus number of DG.
- Set $[dBIBC_i]=[BIBC]$
- Find the row with zero elements for the $(i-1)^{th}$ column of $dBIBC_i$ matrix (*zero_row = find(dBIBC*$_i$*(:, i−1)=0)*).
- Set all non-zero elements of these rows to zero (*dBIBC*$_i$*(zero_row,:) = zeros(length(zero_row),n−1)*).

Substituting real and imaginary parts of the equivalent bus current injections from (3) in (9) yields:

$$\begin{aligned}\partial Ploss/\partial Q_i &= 2\sum_{j=1}^{nb} R_j\sum_{k=2,k\neq i}^{n} dBIBC_i(j,k-1)\left[\frac{\sin\theta_i}{|V_i|}\mathrm{Re}(I_k)-\frac{\cos\theta_i}{|V_i|}\mathrm{Im}(I_k)\right]+ \\ &2\sum_{j=1}^{nb} R_j\times \\ &\sum_{k=2}^{n} dBIBC_i(j,i-1)\times\left[\frac{\sin\theta_i}{|V_i|}\frac{P_i\cos\theta_i+Q_i\sin\theta_i}{|V_i|}-\frac{\cos\theta_i}{|V_i|}\frac{P_i\sin\theta_i-Q_i\cos\theta_i}{|V_i|}\right]\end{aligned} \quad (10)$$

By rearranging (10), derivative of the total power loss with respect to i^{th} bus injection reactive power is as:

$$\begin{aligned}\partial Ploss/\partial Q_i &= 2\sum_{j=1}^{nb} R_j\sum_{k=2}^{n} dBIBC_i(j,k-1)\left[\frac{\sin\theta_i}{|V_i|}\mathrm{Re}(I_k)-\frac{\cos\theta_i}{|V_i|}\mathrm{Im}(I_k)\right] \\ &+2\sum_{j=1}^{nb} R_j\left[dBIBC_i(j,k-1)\cdot\frac{Q_i}{|V_i|^2}\right]\end{aligned} \quad (11)$$

To achieve minimum power loss, (11) should be equal to zero. That is:

$$\partial Ploss/\partial Qi = 0 \quad (12)$$

By substituting (11) in (12), at the minimum total power loss, the reactive power injection at bus *i*, Q_i, is as follows:

$$\begin{aligned}Q_i &= -\frac{|V_i|\cdot}{\sum_{j=1}^{nb} R_j dBIBC_i(j,i-1)}\times \\ &\sum_{j=1}^{nb} R_i\sum_{k=2,k\neq i}^{n} dBIBC_i(j,k-1)\cdot(\sin\theta_i[\mathrm{Re}(I_k)]-\cos\theta_i[\mathrm{Im}(I_k)])\end{aligned} \quad (13)$$

Equation (13) can be expressed in matrix form as:

$$Q_i=\frac{|V_i|\cdot[R]^T[dBIBC_i](\sin\theta_i\cdot[\mathrm{Re}\,dI_i]-\cos\theta_i\cdot[\mathrm{Im}\,dI_i])}{[R]^T dBIBC_i(:,i-1)} \quad (14)$$

where $[\mathrm{Re}\,dI_i],[\mathrm{Im}\,dI_i]$ are obtained by equating i^{th} elements of the injection current vector to zero.

The optimal reactive power supplied by DG corresponding to minimum power loss added at bus *i* can be calculated as:

$$Q_{DG_i}=Q_i+Q_{load_i} \quad (15)$$

where Q_{load_i} is the reactive load at i^{th} bus.

In order to minimize power losses, the derivative of the total power losses with respect to i^{th} bus injected active power should be equal to zero, that is:

$$\partial Ploss/\partial P_i = 0 \quad (16)$$

After performing similar mathematical approach, equivalent active power injection at bus *i* corresponding to minimum power loss can be obtained as:

$$P_i=\frac{|V_i|\cdot[R]^T[dBIBC_i](\cos\theta_i\cdot[\mathrm{Re}\,dI_i]+\sin\theta_i\cdot[\mathrm{Im}\,dI_i])}{[R]^T dBIBC_i(:,i-1)} \quad (17)$$

Therefore, the optimum active power of DG at bus *i* is as:

$$P_{DG_i}=P_i+P_{load_i} \quad (18)$$

III. THE FLOWCHART OF THE PROPOSED METHOD

As depicted in Fig. 1, the proposed method to determine ODGP in RDSs proposed in this paper is implemented step by step as follows:

Step1. Input line and load data and system constraints including voltage limits.

Step2. Run the distribution load flow (DLF) program for the base case of RDS.

Step3. Calculate optimum size of DG for each bus using (15), (18), except the reference bus.

Step4. Calculate total system power losses after adding optimal size DGs calculated in step 3 to each bus.

Step5. Select the bus resulting in minimum power loss.

Step6. The bus voltages after placement of optimal size DG whose location is calculated in step 5 is within acceptable limits? If yes, go to the next step, otherwise, neglect DG from that bus and go to the **step5** (examine the next optimal DG calculated in step 3).

Step7. The size and location is the optimal case for DG placement.

In the **Step5**, one or more number of buses can be selected for installing DGs and in the next step, all voltages should be checked.

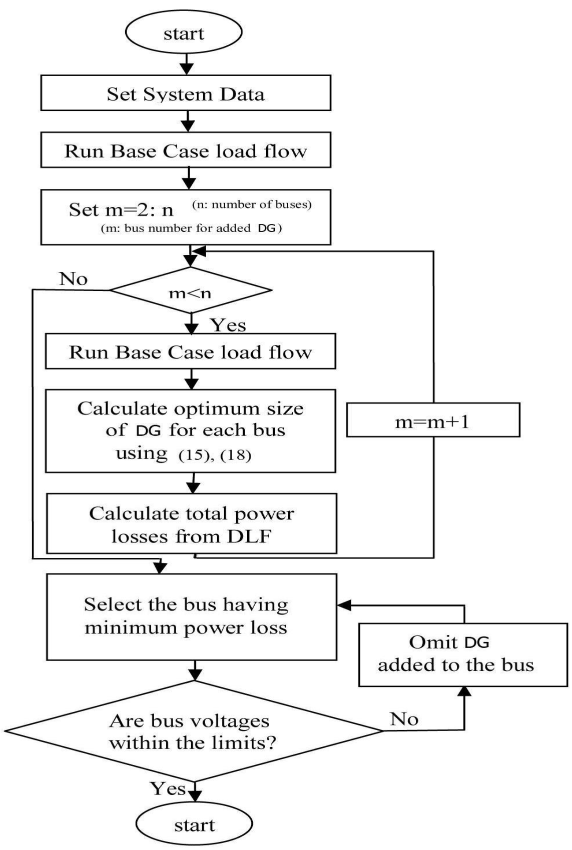

Figure 1. Flowchart of the proposed method.

IV. Test Results

The proposed method is applied to 12 and 34-bus RDS without and with lateral branches. The line and load data of these two test systems are from [16] and [17], respectively. Since the maximum power loss occurs at the peak load condition, the optimal DG placement is performed with the data at peak load to achieve minimum power loss. The classical grid search algorithm is employed with MATPOWER by adding DG to each bus, changing the size of DG from 0% to 100% of the total system active and reactive load power with the step size of 0.01 MW and 0.01 MVAr for the active and reactive power supplied by DG for each case to validate the results. The results obtained from the method are presented in the following sections.

A. Test Results for 12-Bus RDS

The first test case for the proposed method is a 12-bus single feeder RDS [16]. This system has no laterals. The single phase diagram of this system is shown in Fig. 2. The rated line voltage of this system is 11 kV.

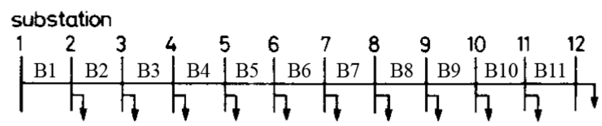

Figure 2. The diagram of a 12-bus radial distribution test system.

The total active and reactive load of this test system is 0.4350 MW and 0.4050 MVAr respectively. Total power losses of this system before DG installation is 0.0207 MW equal to 4.76% of the total load power.

The size of optimal DG placed on each bus determined based on the proposed method presented in this paper and the classical grid search algorithm are depicted in Fig. 3 and Fig. 4, respectively.

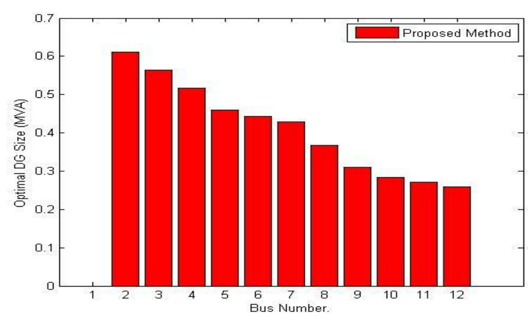

Figure 3. The optimal size of DG for each buses in 12-bus test RDS obtained by the classical grid search algorithm method.

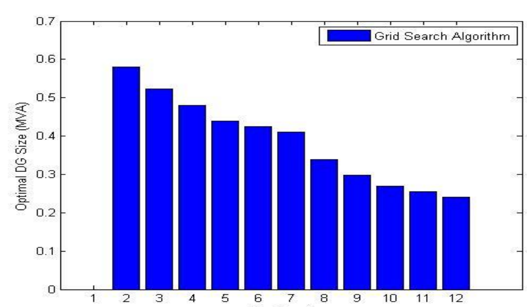

Figure 4. The optimal size of DG for each buses in 12-bus test RDS obtained by the proposed method.

Total power losses for each bus, where the optimum DG size is added, are presented in Fig. 5. As shown in Fig. 5, minimum power loss equal to 3.7927 KW can be achieved when optimal DG (0.3671MVA) is added to bus 8. In this case, the maximum PLR equal to 17 KW and the maximum ELR is 148.9 MWh/year.

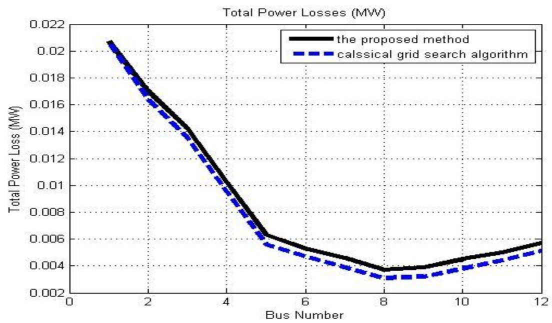

Figure 5. Total power losses after ODGP per each bus for 12-bus test RDS.

The comparison between the results obtained by the proposed method and the classical grid search algorithm for 12-Bus RDS is shown in Table I.

TABLE I. COMPARISON BETWEEN THE RESULTS OF ODGP FOR 12-BUS RDS

Method	Optimum location	Optimum size		CPU time (s)
		MW	*MVAr*	
proposed	Bus.no.8	0.2693	0.2495	0.025
Classical grid search algorithm	Bus.no.8	0.26	0.24	4

In addition, system voltage profile for the optimum case when DG of the optimal size is added to the optimum location at bus 8 is presented in Fig. 6. As shown in this figure, system voltage profile is also improved.

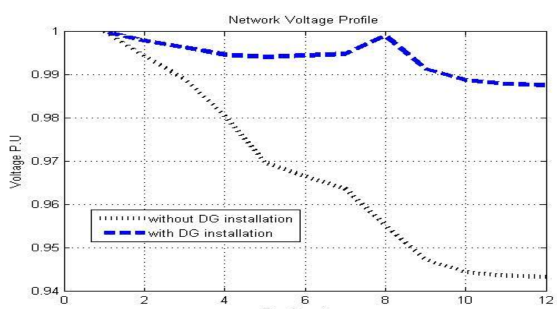

Figure 6. Voltage profile of the 12-bus test system after ODGP at bus 8.

B. Test Results for 34-Bus RDS

The second test case for the proposed method is a 34-bus RDS [17]. This system has a main feeder and four laterals (sub-feeders). The single line diagram of this system is shown in Fig. 7. The rated line voltage of this system is 11 kV.

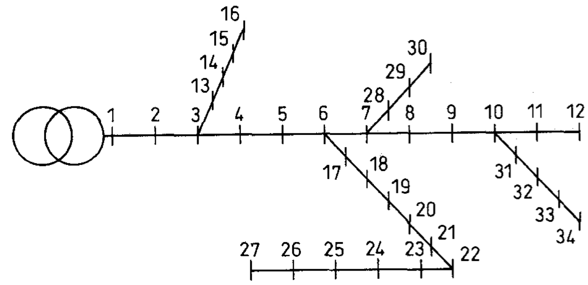

Figure 7. The diagram of a 34-bus radial distribution test system.

The total active and reactive load of this test system is 4.6365 MW and 2.8735 MVAr, respectively. Total power losses of this system before DG installation is 0.2217 MW equal to 4.78% of total load power.

The size of optimal DG placed on each bus determined based on the proposed method presented in this paper and the classical grid search algorithm are depicted in Fig. 8 and Fig. 9, respectively.

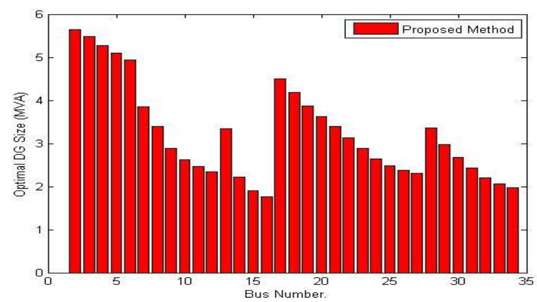

Figure 8. The optimal size of DG for each buses in 34-bus test RDS obtained by the proposed method.

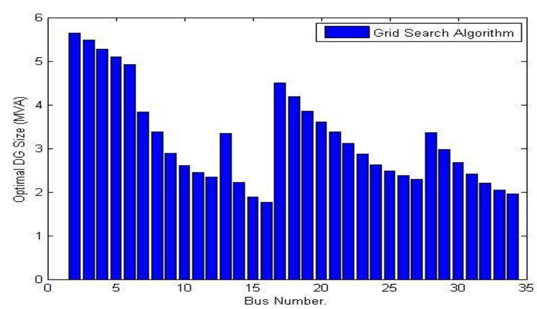

Figure 9. The optimal size of DG for each buses in 34-bus test RDS obtained by the classical grid search algorithm method.

In addition, total power losses for each bus, where the optimum DG size is added, are presented in Fig. 10. As shown in Fig. 10, minimum power loss equal to 0.054126 MW can be achieved when optimal DG (3.6163 MVA) is placed at the bus 20. In this case, the maximum PLR equal to 167.574 KW and maximum ELR is 1468 MWh/year.

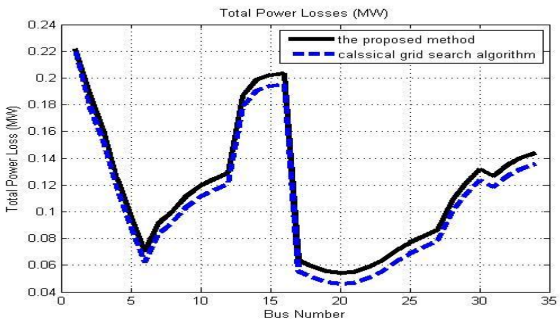

Figure 10. Total power losses after ODGP per each bus in 34-bus test RDS.

The comparison between the results obtained by the proposed method and the classical grid search algorithm is shown in Table II.

TABLE II. COMPARISON BETWEEN THE RESULTS OF ODGP FOR 34-BUS RDS

Method	**Optimum location**	**Optimum size**		**CPU time (s)**
		MW	***MVAr***	
proposed	Bus.no.20	3.0752	1.9028	0.035
Classical grid search algorithm	Bus.no.20	3.05	1.9	140

System voltage profile for the optimum case when DG of the optimal size is added to the best location at bus 20 is also depicted in Fig. 11. As shown in this figure, system voltage profile is also improved.

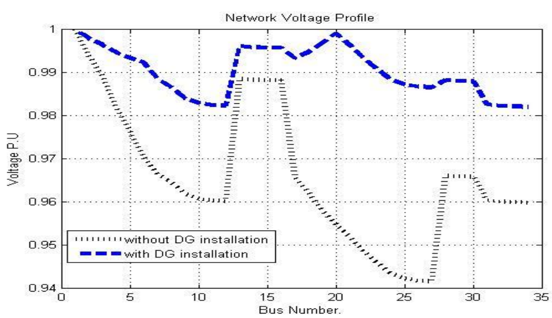

Figure 11. Voltage profile of the 34-bus test system after ODGP at bus 20.

V. CONCLUSION

Distributed generation resources are recently widely installed in distribution systems to achieve loss reduction, improving voltage profile and other operational benefits. The achievement of such benefits depends highly on how and where these resources are to be located in the power system. In this paper, the distributed generation optimal placement and sizing problem is formulated and solved through an analytical method based on loss sensitivity factor and equivalent bus current injection technique with the objective of minimum electric power loss. The proposed optimal DG placement method is tested on 12 and 34-bus RDSs. The classical grid search algorithm based on successive load flows has been employed to validate the results. Examining and comparing the results obtained by the two approaches demonstrates the effectiveness and speed of the proposed method. The future works are the applying the proposed method for the DGs with variable outputs and the variable loads (both three and single phase).

REFERENCES

[1] N. S. Rau and Y.-H.Wan, "Optimum location of resources in distributed planning," IEEE Trans. Power Syst., vol. 9, pp. 2014–2020, Nov. 1994.

[2] K.-H. Kim, Y.-J. Lee, S.-B. Rhee, S.-K. Lee, and S.-K. You, "Dispersed \generator placement using fuzzy-GA in distribution systems," in Proc. 2002 IEEE Power Engineering Soc. Summer Meeting, vol. 3, Chicago, IL, July 2002, pp. 1148–1153.

[3] N. Hadjsaid, J. F. Canard, and F. Dumas, "Dispersed generation impact on distribution networks," IEEE Comput. Appl. Power, vol. 12, pp. 22–28, Apr. 1999.

[4] T. Griffin, K. Tomsovic, D. Secrest, and A. Law, "Placement of dispersed generation systems for reduced losses," in Proc. 33rd Annu. Hawaii Int. Conf. Systems Sciences, Maui, HI, 2000.

[5] M. H. Nehrir, C.Wang, and V. Gerez, "Impact of wind power distributed generation on distribution systems," in Proc. 17th Int. Conf. Electricity Distribution (CIRED), Barcelona, Spain, May 2003.

[6] N. Mithulananthan, Oo Than, Van Phu Le, "Distributed generatorplacement in power distribution system using genetic algorithm to reduce losses," Thammasat Int. J Sci. and Tech. vol. 9, no. 3, pp. 55–62, 2004.

[7] T. Griffin, K. Tomosovic, D. Secrest, A. Law, "Placement of dispersed generations systems for reduced losses," In: Proceedings of the 33rd Hawaii international conference on sciences, Hawaii, 2000.

[8] C. Wang and M. Hashem Nehrir, "Analytical approaches for optimal placement of distributed generation sources in power systems" IEEE Trans. Power Syst.,vol.19, no. 4, Nov. 2004.

[9] S. Rau and Y.H. Wan, "Optimum location of resources in distributed planning", IEEE Trans. Power Syst., vol. 9, pp.2014–2020, Nov. 1994.

[10] K. H. Kim, Y. J. Lee, S. B. Rhee, S. K. Lee, and S.-K. You, "Dispersed generator placement using Fuzzy-GA in distribution systems", IEEE PES Summer Meeting, vol. 3, July 2002, pp.1148–1153

[11] J. O. Kim, S. W. Nam, S. K. Park, and C. Singh, "Dispersed generation planning using improved hereford ranch algorithm", Electric Power System Research , vol. 47, no. 1, Oct. 1998, pp.47–55

[12] K. Nara, Y. Hayashi, K. Ikeda, and T. Ashizawa, "Application of tabu search to optimal placement of distributed generators", IEEE PES Winter Meeting, 2001, Volume: 2, pp.918 – 923

[13] H. L. Willis, "Analytical methods and rules of thumb for modeling dg-distribution interaction", IEEE PES Summer Meeting, vol. 3, Seattle, WA, July 2000, pp. 1643–1644

[14] C. Wang, M. H. Nehrir, "Analytical approaches for optimal placement of dg sources in power systems", IEEE Trans. Power Syst., vol. 19, no. 4, Nov. 2004, pp. 2068–2076

[15] T. S. Abdel-Salam, A. Y. Chikhani, and R. Hackam, "A new technique for loss reduction using compensating capacitors applied to distribution systems with varying load condition," IEEE Trans. Power Del., vol. 9, no. 2, pp. 819–827, Apr. 1994.

[16] D. Das, H.S. Nagi, D.P. Kothari, "Novel method for solving radial distribution networks," IEE Proc. -Gener. Transm. Dist., vol. 141, no. 4, pp. 291– 298, July 1994.

[17] M. Chis, M.M.A. Salama, S. Jayaram, "Capacitor placement in distribution systems using heuristic search strategies," IEE Proc. -Gener. Transm. Distrib. vol. 144, no. 3, pp. 225–230, May 1997.