Identifying sources of uncertainty in groundwater recharge estimates using the biophysical model WAVES

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Abstract

A biophysical model WAVES, developed by CSIRO, has recently been implemented as the modelling engine to provide recharge input and discharge output to an integrated regional groundwater model for the Perth region, namely, the Perth Regional Aquifer Modelling System (PRAMS). Application of the WAVES model over such a large area requires an extensive set of data that are not known with certainty. This paper describes a method based on the mean value first order reliability analysis method to determine the parameters that significantly affect uncertainty in the recharge estimates. The fraction of model output (recharge) variance (FOV) contributed by each basic parameter is determined using the sensitivity coefficients and uncertainty (measured by the variance or the coefficient of variation) of the model parameters. The FOV provides a quantitative means to rank the order of importance of parameters that affect the reliability of the recharge estimate and can be used to prioritise data collection to improve the estimate. Application of the proposed method to the local conditions on the Gnangara Mound found that the critical parameters for WAVES in the modelling framework of PRAMS are: rainfall, vegetation density measured by leaf area index, soil water holding capacity of the soil rootzone, maximum root depth and maximum carbon assimilation rate.

1. INTRODUCTION

The Gnangara Mound groundwater system is the most valuable source of fresh water in the Perth region. It currently provides up to 60% of public water supply for the Perth Integrated Water Supply Scheme (IWSS). It also provides a significant amount of private water use for horticulture, industry and domestic gardens, and supports a variety of groundwater dependent ecosystems (GDEs), including native woodlands and numerous wetlands and lakes. Groundwater levels on the Gnangara Mound have declined over the last four decades in response to the increased abstraction for public and private water supply and reduction in recharge due to the continued drying climate, maturation of pine plantations and changed fire regime of native woodlands.

To aid management of the regional groundwater system, a new regional groundwater model, the Perth Regional Aquifer Modelling System (PRAMS) was developed (CyMod 2004, Davidson and Yu 2004, Silberstein et al. 2004, Xu et al. 2004). The PRAMS model consists of two coupled components: a Vertical Flux Model (VFM) package which calculates the net recharge/discharge into/from the watertable and a saturated groundwater model based on MODFLOW (McDonald and Harbaugh 1988) for flows in the multi-layer aquifer system below. The VFM depends on land-use, and for vegetated areas, is based on WAVES (Zhang and Dawes 1998), a detailed biophysical model linking transpiration and soil-water uptake to climate and soil conditions. To avoid running a WAVES simulation for each vegetated cell in the model domain and to reduce computing time, the VFM employs the concept of Representative Recharge Units (RRUs) (Barr et al. 2003), being spatial units with similar conditions. Essentially, the approach first groups the cells within the modelling domain into a number of designated RRUs based on climate, landuse (including vegetation characteristics), soil profile and watertable depth. WAVES simulations are only required for the RRUs and all cells that share the same RRU will have same net recharge.

WAVES is a one-dimensional, daily time step model that simulates the fluxes of water and energy between the atmosphere, vegetation, and soil systems. It is a process-based model that couples these systems by modelling the interaction and feedback between them. WAVES uses an efficient numerical solution to solve Richards equation for unsaturated water flow. Daily transpiration is estimated by the Penman-Monteith equation and is extracted from the soil profile using weighting factors determined by the modelled root density and a normalised weighted sum of the matric and osmotic soil water potentials of each layer. WAVES accounts for antecedent moisture conditions in the root zone and deep unsaturated zone, for different plant species, extent of plant development, root zone depth, and the physical characteristics of soil type (soil-moisture characteristic). This model has been shown to accurately simulate water dynamics and vegetation growth for a wide range and combinations of climate, soil and vegetation type (Zhang et al. 1996), including conditions existing on the Swan Coastal Plain (Hatton et al. 2001, Silberstein et al. 2004).

The WAVES model requires inputs of daily climatic data, parameters that characterise the vegetation type and soil parameters describing the water holding characteristics and hydraulic properties of soil layers. Collection of these data is very expensive, and model calibration is usually undertaken with uncertain data. The uncertainty in the model parameters will result in inaccuracy in the modelling results.

This paper presents a probabilistic approach based on the mean value first-order reliability analysis method (MFORM) to evaluate how uncertainty in model input parameters to WAVES in the current modelling framework of PRAMS may affect the accuracy of recharge estimates. The proposed probabilistic model also provides the facility for computing the proportional contribution of each individual parameter to the uncertainty in the recharge estimates. This capability enables the priorisation of resources in data collection to improve the model accuracy and reliability.

2. UNCERTAINTY ANSLYSIS AND MFORM

Uncertainty analysis is finding the relation between given uncertainty of the model input/parameters and uncertainty of model output. It usually involves three major steps: (a) estimation of uncertainties in model inputs and parameter (characterization of input uncertainties), (b) estimation of the uncertainty in model outputs resulting from the uncertainty in model inputs and model parameters (uncertainty propagation), (c) sensitivity analysis to identify the critical parameters contributing to the variance (sensitivity analysis of uncertainty).

Uncertainties in model parameter estimates and input data can stem from a variety of sources, for example measurement error, values inferred from indirect measurement or scaled up from limited local data, or simply difficulties in taking adequate measurements over the domain. Uncertainty of input data and model parameters is commonly represented by probabilistic measures and/or fuzzy set theory.

The problem of how uncertainty in the input data and model parameters is propagated to the modelling results in the recharge estimates is not well understood due to the complexity and high nonlinearity of the process-based recharge models such as WAVES. Traditionally, the effect of a particular model parameter on the recharge estimates is examined via the sensitivity analysis (Xu et al. 2003). In sensitivity analysis, basic variables are perturbed around the baseline values one at a time and sensitivity of model output to the perturbation in each variable is assessed. Although sensitivity analysis allows identification of the parameters that have greatest impact on recharge, it doesn't account for the likelihood that the parameter is different from the baseline value. A highly sensitive parameter that is known with low uncertainty may have a much lower impact on the uncertainty of model output than a less sensitive parameter that is highly uncertain (Melching and Bauwens 2004). Uncertainty analysis allows consideration of the combined effects of parameter sensitivity and parameter uncertainty in the determination of the key parameters affecting the uncertainty of model prediction. There are many methods that can be used for assessing the propagation of uncertainty, such as the Monte Carlo simulation method and first order reliability analysis method (FORM).

This study used a simple probabilistic approach based on the mean value first-order reliability analysis method (MFORM) (Yen et al. 1986) to investigate how uncertainty associated with the model parameters is propagated in the recharge estimate. The concept underlying the probabilistic modelling is that the recharge R = F(**X**) is a function of uncertain model input **X**=[x_1, x_2, …, x_n]. In turn, uncertainty in **X** results in a corresponding uncertainty in the recharge R. The uncertainties in the recharge and input parameters are measured by their variances or the coefficient of variation (CoV) defined as the standard deviation divided by the mean value. The MFORM uses the first-order terms of the Taylor series expansion of F(**X**) about the mean value of input variables, $\overline{X}$.

$$R = F(\overline{X}) + \sum_{i=1}^{n} (x_i - \overline{x}_i)\left(\frac{\partial F}{\partial x_i}\right)_{\overline{X}} \tag{1}$$

The mean, $\overline{R}$, and variance, σ_R^2, of recharge can be estimated by the following expression:

$$\overline{R} = F(\overline{X}) \tag{2}$$

and

$$Var(R) = \sigma_R^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)_{\overline{X}}\left(\frac{\partial F}{\partial x_j}\right)_{\overline{X}} E\left[(x_i - \overline{x}_i)(x_j - \overline{x}_j)\right] \tag{3}$$

where $\frac{\partial F}{\partial x_i}$ is the derivative of the recharge with respect to random variable x_i evaluated at the mean values $\overline{X}$, σ_{x_i} is the standard deviation of random variable x_i.

If the basic variables are statistically independent, the variance of R becomes

$$\sigma_R^2 = \sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)_{\overline{X}}^2 \sigma_{x_i}^2 \tag{4}$$

Sensitivity analysis is then performed to determine ranking of the parameters' contribution to uncertainty in the modeling results. The fraction of the model output (recharge) variance (FOV) contributed by each basic variable can be determined by

$$FOV_{x_i} = \left(\frac{\partial F}{\partial x_i}\right)_{\overline{X}}^2 \sigma_{x_i}^2 \,/\, \sigma_R^2 \tag{5}$$

As shown in eq.(5), the importance of a particular parameter to the uncertainty in the results of the recharge estimate depends on two factors: the actual uncertainty in the parameter value and the sensitivity of the estimated recharge to the parameter value. The importance of a parameter will be the greatest when the value of the parameter is relatively uncertain and the recharge estimated by the model is sensitive to the parameter's value. FOV provides an objective measure to rank the importance of each individual parameter to the uncertainty in modelling results and can hence be used to identify the key sources of uncertainty affecting the recharge estimates.

In this study, a forward difference was used to estimate the value of $\frac{\partial F}{\partial x_i}$ with an incremental value, Δx_i , of 10% applied to all basic variables. The variance, $\sigma_{x_i}^2$, of random variable x_i is derived from the range of the parameter values using the six sigma rule, i.e., σ_{x_i} is equal to the range of x_i divided by six.

3. APPLICATION

MFORM was applied to identify the critical parameters/input data in the recharge estimates using WAVES in the modelling framework of PRAMS. Since PRAMS has more than 700 RRUs for WAVES simulations, it is not practical to undertake uncertainty analysis for each RRU. In this paper, two RRUs, which represent the typical conditions on the Gnangara Mound were analysed. These two RRUs have the same climatic conditions, soil properties and vegetation type (banksia woodland) but with different density (measured by leaf area index (LAI)). The model parameters for WAVES were initially calibrated using plot scale data (Hatton et al. 2001, Silberstein et al. 2004) and were further fine tuned to get better fittings in the regional watertable response during the calibration of the integrated PRAMS model (CyMod 2004, Xu et al. 2004).

WAVES is a complex process-based model involved many parameters, some of which are set in the code. As a result, it is not feasible to examine effects of variability in all model parameters and input data. This study focuses on three sets of model parameters and data: climate forcing that drives the model, vegetation parameters that define the water use, and soil hydraulic properties that regulate water movement in the soil. The uncertainty analysis is applied to the total recharge accumulated over a period of 24 years (1980-2003).

3.1. Climate

Climate data required for WAVES include daily rainfall, maximum and minimum temperature, vapour pressure (need to convert into vapour pressure deficit) and radiation. In this study daily meteorological data [Point Patched Data (PPD) from SILO] for the period 1980 to 2003 at the Perth Region Office (Station 9023) were used to drive the model simulations. The climate in Perth can be characterised as Mediterranean with mild wet winters and hot dry summers. About 90% of the rain falls between April and October. The long-term average rainfall for Perth is 870 mm/yr but rainfall over last forty years has reduced significantly due to climate variability. The mean annual rainfall during 1980-2003 is 800 mm/yr and class ‘A’ Pan evaporation for the same period is 1760 mm/yr.

Uncertainty in the climate data at a climate station is considered to be low as the data is sourced from the Commonwealth Bureau of Meteorology (BoM). However, uncertainty in the climate at a particular location within the model domain can be very high due to the current approach to delineating the climate zones (namely assumption of homogeneous climatic conditions within a large area). In the current implementation of PRAMS only five climate zones were used over an area of about 9000 km^2. To quantify the potential errors, analysis was undertaken comparing the climate data between the Perth station and the Wanneroo station (Station 9105) located about 25 km further north but still within the same climate zone for the PRAMS model. The difference in annual rainfall and other climate data between the two stations is first calculated and error ranges are then estimated by the minimum and maximum values over the 24 year period. These error ranges are used to approximately define the uncertainty in the climate data using the six sigma rule. Annual averaged data is used in this analysis to smooth out the seasonal and daily variation (effects of climate data with different temporal patterns on recharge have not been examined in this study). Table 1 gives the potential uncertainty arising from spatial variability in climate.

Table 1 Difference in climate data between the Perth and Wanneroo stations

Parameters	Unit	Mean (Perth)	Error range	σ	CoV
Rainfall	mm/a	800	296.00	49.33	0.062
Maximum temperature	ºC	24.08	0.77*	0.13	0.005
Minimum temperature	ºC	13.56	1.85*	0.31	0.023
Vapour pressure	Hpa	13.1	1.09*	0.18	0.014
Radiation	MJ/m^2	18.83	0.20*	0.03	0.002

*Error ranges were estimated based on the difference in the annual mean

3.2. Vegetation

WAVES uses 26 vegetation parameters to fully describe canopy energy and carbon balance, canopy and root growth, and interactions between soil and vegetation. The plant growth component of WAVES is inactive for the current implementation of the VFM, and so the rooting density with depth and the leaf area index (LAI) of the vegetation were predetermined as defined model inputs. This reduces the number of vegetation parameters to 16 (Table 2).

Baseline values and range as shown in Table 2 for most of these parameters were taken from plant physiological literature (references indicated in the user manual (Dawes et al. 1998) and Hatton et al., 2001 and Silberstein et al., 2004), with only a few requiring fitting or adapting to local conditions, e.g., LAI, litter load, root depth etc. Maximum root depth was estimated based on a previous study using chloride balance method on the Gnangara Mound (Farrington and Bartle 1991). Litter loads were estimated based on personal communication with ecologists in the Department of Environment and Conservation (DEC) on ground litter in the banksia woodlands near Perth (Clayton Sanders and Mark Garkaklis, personal communications). LAI for the low and medium density of banksia woodlands were measured by Hodgson et al. (2005) but nominal values for the WAVES modelling were refined during the PRAMS calibration. The banksia woodlands were classified into three types (low density, medium density and high density) based on LAI maps, which were generated from a regression model based on ground measurements and the correlation with a vegetation density index consisting of Landsat TM bands (3+5)/2 (Hodgson et al. 2005). Whilst the LAI map is adequate for RRU classification, there is greater uncertainty in the actual mean values of LAI for each of the RRUs classified. Uncertainties in these data are combination of measurement errors and spatial variation but are difficult to quantify. Ranges given in Table 2 are best guesses based on experiences (Geoff Hodgson and Richard Silberstein, personal data). Note that the data range is specified for the average conditions of the plants and land-use classes, not for a particular site which may have more variability than those specified in the table; for example, LAI may vary between 0 and 3 at a site.

Table 2 Vegetation parameters and their variability

Parameter	Unit	Baseline	Low	High	σ	CoV
1 - albedo of the canopy	—	0.8	0.75	0.85	0.017	0.021
1 - albedo of the soil	—	0.7	0.65	0.9	0.042	0.060
Rainfall interception	m d-1 LAI-1	0.0007	0.0005	0.001	0.0001	0.119
Light extinction coefficient	—	-0.45	-0.4	-0.5	0.017	0.037
Max carbon simulation rate	kg C -2 d-1	0.022	0.015	0.03	0.003	0.114
Slope of the conductance	—	0.9	0.8	1	0.033	0.037
Max available water potential	M	-300	-150	-350	33.333	0.111
IRM weighting of water	—	2.1	1	2.5	0.250	0.119
IRM weighting of nutrients	—	0.3	0.2	0.5	0.050	0.167
1/2 optimum Temperature	ºC	13	10	25	2.500	0.192
Optimum Temperature	ºC	24	15	25	1.667	0.069
Saturation light intensity	mmoles m-2d-1	1200	800	1500	116.667	0.097
Aerodynamic resistance	s d-1	10	5	20	2.500	0.250
Maximum rooting depth	M	10	8	15	1.167	0.117
Litter (low density)	t/ha	1	0.5	3	0.417	0.417
LAI (low density)		0.66	0.5	0.8	0.050	0.076
Litter (medium density)	t/ha	2.5	1	5	0.667	0.267
LAI (medium density)		1.08	0.8	1.2	0.067	0.062

3.3. Soil Characteristics

A typical soil profile for Bassendean sand is used for this study, which is defined by three soil layers: two top soils and one subsoil. The soil hydraulic properties were derived from data in Salama et al. (1999) for the topsoil and data sets in Vermooten (2002) and Smettem (2003) for the subsoil. These data show that the Bassendean sand has very little water holding capacity and has very high saturated hydraulic conductivity. In this study, a baseline value of 10 m/d is used to be consistent with the average hydraulic conductivity of the regional aquifer. Modified Campbell's soil hydraulic model has been fit to the field data to generate soil retention functions for the model (Table 3). Campbell's soil hydraulic model was used because it fits better in situ measurements of unsaturated hydraulic conductivity (Smettem 2003, Xu et al. 2004).

Table 3 Soil profile, soil hydraulic properties and fitted Campbell model parameters

Soil layer	Depth (m)	K_s (m/d)	θ_s *	b	ψ_e (m)
Topsoil A	0-0.15	1.63	0.38	0.9	-0.12
Topsoil B	0.15-0.5	3.59	0.35	0.8	-0.15
Subsoil C	0.5-30	10	0.33	0.9	-0.12

θ_s is effective saturated moisture content, b is Campbell's shape parameter, and Ψ_e the pressure potential at air entry.

Topsoils A and B represent only a small proportion of the soil column and a previous study (Xu, et. al. 2003) indicated that groundwater recharge is relatively insensitive to the change in the two topsoil layers. Uncertainty in properties in the two soils is ignored in this analysis. The uncertainty in the parameters for the subsoil is given in Table 4.

Table 4 Uncertainty in subsoil properties

Soil parameter	Unit	baseline	Low	High	σ	CoV
Hydraulic conductivity (K)	m/d	10	5	30	4.167	0.417
Soil water holding capacity	% (v/v)	0.03	0.02	0.06	0.007	0.222

3.4. Uncertainty Analysis

Uncertainty analysis was undertaken for the recharge estimates for the two RRUs, considering 23 model parameters and input data listed in Tables 1, 2 and 4 as random variables with the specified uncertainties. The effect of the variation of a particular model parameter on the groundwater recharge is examined by repeatedly running the model over the period 1980 to 2003 with the value of a single parameter altered by 10% while holding all other parameters constant. Sensitivity coefficients required in MFORM were determined through simply dividing the change in the recharge by the change in the parameters value. In this analysis, all the uncertain basic variables were assumed to be statistically independent.

For the base cases, the averaged annual recharge for the low and medium banksia woodlands for the period 1980–2003 is 38% and 18% of rainfall respectively. By applying equation (4) to the dataset, the standard deviations for recharge under low and medium banksia can be determined. It was found that both RRUs have similar standard deviation of about 7% of rainfall. These results imply that the recharge estimate for the medium banksia woodlands has higher uncertainty than the low banksia woodlands since it has a coefficient of variation (CoV) of 0.4 against a CoV of 0.2 for the low banksia woodlands. Results indicate that although the current PRAMS model gives reasonable results, there is still fair uncertainty in the recharge estimate using the VFM in its current implementation.

Table 5 Results of uncertainty analysis and ranking of important parameters

Parameters	Low Banksia		Medium Banksia	
	FOV (%)	Ranking	FOV (%)	Ranking
Climate				
Rainfall	61.47	1	54.08	1
Maximum temperature	0.15		0.15	
Minimum temperature	1.04		1.11	
Vapor pressure	0.69		0.87	
Radiation	0.00		0.00	
Vegetation				
1 – albedo of the canopy	0.01		0.01	
1 – albedo of the soil	0.12		0.01	
Rainfall interception	1.14	6	2.77	
Light extinction coefficient	2.92	5	2.55	
Max carbon simulation rate	8.42	3	6.60	4
Slope of the conductance	0.89		0.70	
Max available water potential	0.00		0.00	
IRM weighting of water	0.00		0.00	
IRM weighting of nutrients	0.00		0.00	
1/2 optimum Temperature	0.00		0.09	
Optimum Temperature	0.00		0.01	
Saturation light intensity	0.00		0.16	
Aerodynamic resistance	0.07		0.20	
Maximum rooting depth	0.05		6.15	6
Litter	0.87		0.15	
LAI	15.49	2	11.37	2
Soil				
Hydraulic conductivity (K)	0.35		6.83	3
Soil water holding capacity	6.32	4	6.17	5
Total	100		100	

Uncertainty in the recharge estimates explained by individual variables was calculated as FOV according to equation (5) and results are given in Table 5. For both RRUs, rainfall stands out as the most important parameter, contributing to 61.5% and 54.1% of uncertainty in recharge estimates for the low banksia woodlands and medium banksia woodlands respectively. The next most important parameter is the vegetation density measured by LAI, which accounts for 15.5% and 11.4% of the uncertainty in the recharge estimate for respective RRUs. For the low banksia woodland, maximum carbon assimilation rate, soil water holding capacity and light extinction coefficient are also relatively important, contributing about 8.4%, 6.3% and 2.9% respectively of the uncertainty. For medium banksia woodland, hydraulic conductivity, carbon assimilation rate, soil water holding capacity and maximum root depth are important parameters, contributing between 6.1 - 6.8% to the uncertainty, respectively. (Note that with this implementation model vegetation growth is not active and carbon assimilation rate is effectively a scaler for maximum canopy conductance.) Results indicate that the first six ranked parameters contribute to 95% of the uncertainty for the low banksia woodlands and 91% of uncertainty for the medium banksia woodland. However, note that as the vegetation density increases and competition for water becomes more intense, recharge becomes more sensitive to the maximum root depth and soil hydraulic conductivity.

Results given in Table 5 also demonstrate that most of the vegetation parameters have insignificant effect on the uncertainty in the recharge estimate. This doesn't imply these basic variables are unimportant for accurate modelling of recharge. A good estimate for these parameters is still required to obtain accurate simulation results. Furthermore, some parameters that are not important for a particular RRU, may be significant in other RRUs. For example, maximum root depth and hydraulic conductivity are not important for RRUs with the low LAI banksia woodland but they are relatively important in modelling RRUs for the medium banksia woodland.

On the basis of the MFORM analysis, it is clear that an improved modelling result can be achieved by reducing the uncertainty in rainfall. This may require increasing the number of climate zones over the modelling domain. However, an increase in the number of climate zones will significantly increase the number of RRUs required for WAVES simulation, with a commensurate increase in simulation time. A trade-off is needed to balance the accuracy of modelling results with computing requirements. Other important parameters are vegetation density measured by leaf area index, soil water holding capacity, hydraulic conductivity, maximum root depth and maximum carbon assimilation rate. Further field work would be required to refine estimates for those parameters to improve the VFM modelling result.

Above analysis assumed that the basic variables are statistically independent. In practice, some of the parameters may be correlated, e.g. LAI may have good correlation with soil water holding capacity. Also use of 10% change in the basic variable to derive the sensitivity coefficient may be too coarse for these variables with small variability. Further work may be required to refine the approach described in this paper.

4. CONCLUSIONS

This paper has illustrated that a methodology based on the mean value first order reliability analysis method may be applied to determine key sources of uncertainty affecting uncertainty in recharge estimates for a complex recharge model, WAVES, in the current modelling framework of PRAMS. Quantitative uncertainty analysis presented in this paper is not only able to quantify the degree of confidence in the modelling results based on the available information for input data and model parameters but also to provide an effective means to identify key parameters to focus on for data collection to improve the confidence and accuracy of model prediction. Results from this study indicate that the most important parameter contributing to the uncertainty of recharge estimate in PRAMS is rainfall, which accounts for 50-60% of uncertainty in the recharge estimate. To improve accuracy of recharge estimates, it may be necessary to increase the resolution of climate zonation. An increase of the number of climate zones will, however, significantly increase the number of RRUs for WAVES simulations. The trade-off between the reliability of recharge estimates and computing time for PRAMS modelling needs further investigation. Other important parameters include vegetation density measured by leaf area index; soil water holding capacity; hydraulic conductivity; maximum root depth and maximum carbon assimilation rate. Field work to collect further information on these parameters is necessary to improve the confidence in the PRAMS modelling results.

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