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## The initial surface absorption test (ISAT): an analytical approach

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**Contribution** by D. Lockington*, J.-Y. Parlange†, R. Haverkamp‡, K. Smettem§ and P. Ross||

Wilson *et al.*[1] analyse absorption of water from a circular source into concrete, on the basis of the approach of Hall,[2] to obtain sorptivity values in the absence of gravity. This fundamental problem is appropriate for the ISAT and has been studied before in a different context by Haverkamp *et al.*[3] The following discusses some limitations of Wilson *et al.*'s solution and offers an improved analytical formula.

Hall[2] estimated absorption from a strip source (i.e. two-dimensional radial flow). His solution is exact in the long-time limit but, as he noted, in error at short times. Wilson *et al.*[1] adapt Hall's result to the fundamental problem of absorption from a circular source to obtain for the cumulative absorption, $i$,

$$i = 0{\cdot}3086\, f L(\mathrm{e}^{7\pi S^2 t/2 f^2 L^2} - 1)^{1/2} \tag{1}$$

where $L$ is the source diameter, $f$ is the available porosity and $S$ the sorptivity. When the time $t$ is very small their result for the cumulative absorption $i$ per unit area of the source is

$$i = \sqrt{\frac{\pi}{3}} S t^{1/2} \tag{2}$$

instead of the exact result $St^{1/2}$, giving a 2·3% error. Owing to the difference in geometry, Hall[2] had in fact a larger error in the short-time limit.

As noted above, Hall's solution does indeed yield the exact long-time absorption for a strip source. He also points out the exact result that in *three* dimensions the flux per unit area approaches a constant. This is quite different from the result of Wilson *et al.*,[1] equation (1), which suggests that $i$, and the flux, increase exponentially for long times. The reason for this discrepancy is that their method extending Hall's two-dimensional approach to the three-dimensional case does not hold.

The best result for the absorption of water from a circular source into concrete in the absence of gravity is, for all practical times,[3]

$$i = St^{1/2} + \frac{2gS^2}{Lf} t \tag{3}$$

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where $g$ is a function of time which increases slowly from about 0·6[4] to about 0·8 for long times.[3] Further discussions and extensions (including the effect of gravity) can be found in Haverkamp *et al.*[3] and applications can be found in Smettem *et al.*[5]

Wilson *et al.*[1] presented their experimental results with reduced variables in their Fig. 6. The very small scatter of the data for the four experiments confirms the excellent quality of the data. With their reduced variables equation (3) becomes

$$I = \frac{2}{\sqrt{\pi}} T^{1/2} + \frac{4g}{\pi} T \tag{4}$$

Fig. 1 shows the theoretical results for $g = 0{\cdot}6$ (smallest value), $g = 0{\cdot}7$ (average value) and $g = 0{\cdot}8$ (largest value). The agreement with their data is obviously excellent, especially for $g = 0{\cdot}8$, although the average value, $g = 0{\cdot}7$, can also be used with confidence.

## **Reply** by the authors

We are grateful to Lockington *et al.* for their valuable contribution, which gives an improved analysis of this absorption problem.

The approximation inherent in our analysis arises from our extension of Hall's analysis of absorption from a line source to cover the three-dimensional case. Although we have calculated the volume of the hemiellipsoid produced by rotation of the semiellipse defined by Hall's conformal mapping of a line source, this does not take into account all the flow lines that are possible in the three-dimensional case. As far as the physics of absorption is concerned, effectively our equations estimate the quantity of water which would be absorbed if the hemiellipsoid were built up of a series of semielliptical slices to which the flow lines are confined. We have therefore not taken account of the volume of liquid spreading perpendicular to these slices and consequently our analysis underpredicts the absorption somewhat. Thus the contribution of Lockington *et al.* provides a better fit to our experimental data, particularly at long times.

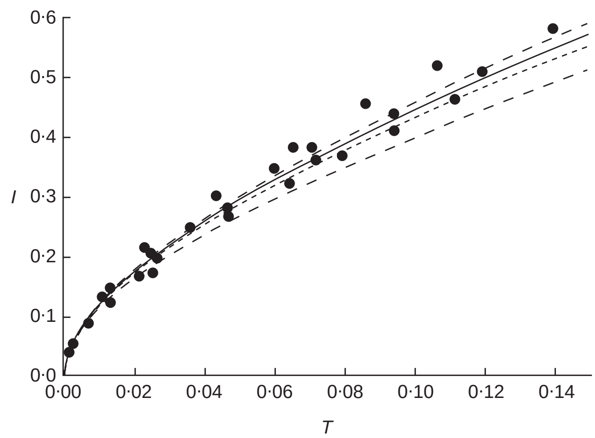


*Fig. 1. Comparison of equation (4) for* $g = 0{\cdot}6$ *(lower dashed curve),* $g = 0{\cdot}7$ *(solid curve), and* $g = 0{\cdot}8$ *(upper dashed curve) with experimental data of Wilson et al.*[1] *as transcribed from their Fig. 6. Wilson et al.'s approximation, equation (1) in reduced variables, is also shown (dotted line)*

Our results show that, within the normal timescale of an ISA test, our analysis gives values of sorptivity which are within the range of accuracy that is required in practice. It should be noted that even this level of accuracy could probably not be approached *in situ*. It is interesting to reanalyse our experimental data using both the two-term and the three-term versions of the analysis of Haverkamp *et al.* A selection of results is given in Table 1. These show that the three-term equation gives slightly more accurate values of sorptivity than the two-term equation.

An important aspect of the contribution by Lockington *et al.* is that it is consistent with the development of a unified approach to capillary absorption from a range of sources of different, radially symmetrical geometries. Thus Wilson *et al.*[6–8] have derived equations for capillary absorption from a cylindrical source, from a hemispherical source and from a drilled hole with a hemispherical end. The relevant equations are

$$i = St^{1/2} + \frac{S^2}{3fr} t \tag{5}$$

for a cylindrical source,

$$i = St^{1/2} + \frac{2}{3}\frac{S^2}{fr} t \tag{6}$$

for a hemispherical source and

$$i = St^{1/2} + \frac{S^2}{3fr}\left(\frac{\alpha + 2}{\alpha + 1}\right) t \tag{7}$$

for a source consisting of a drilled hole with a hemispherical end. (In these equations $f$ is the porosity, $r$ is the source radius and $\alpha = h/r$, where $h$ is the depth of the cylindrical part of the drilled hole.)

The result of Haverkamp *et al.* for a circular source can be written

*Table 1. Comparison of sorptivity values obtained by fitting circular-source absorption data to equation (12) Wilson et al.*[1] *and equation (3)*

| Material | Sorptivity derived from circular-source absorption data: mm/min$^{1/2}$ | | |
|---|---|---|---|
| | Equation (12)[1] | Equation (3) | |
| | | Three-term fit | Two-term fit |
| Jaumont limestone, | 0·38 | 0·37 | 0·35 |
| 58 mm dia. source, | 0·35 | 0·33 | 0·32 |
| 1-D $S = 0{\cdot}34$ mm/min$^{1/2}$ | 0·34 | 0·32 | 0·33 |
| Mean | 0·36 | 0·34 | 0·33 |
| Standard deviation | 0·02 | 0·03 | 0·02 |

$$i = St^{1/2} + \frac{g}{rf} S^2 t \quad (8)$$

Combining all of these results suggests that, in general, absorption from any radially symmetrical source will be given by an equation of the form

$$i = St^{1/2} + GS^2 t \quad (9)$$

where $G$ is a geometrical factor.

For complex geometries where an analytical expression for $G$ is not readily determined, values of sorptivity can still be determined by using this equation.

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