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Scale Space Clustering Evolution for Salient Region Detection on 3D Deformable Shapes

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Abstract

Salient region detection without prior knowledge is a challenging task, especially for 3D deformable shapes. This paper presents a novel framework that relies on clustering of a data set derived from the scale space of the auto diffusion function. It consists of three major techniques: scalar field construction, shape segmentation initialization and salient region detection. We define the scalar field using the auto diffusion function at consecutive time scales to reveal shape features. Initial segmentation of a shape is obtained using persistence-based clustering, which is performed on the scalar field at a large time scale to capture the global shape structure. We propose two measures to assess the clustering both on a global and local level using persistent homology. From these measures, salient regions are detected during the evolution of the scalar field. Experimental results on three popular datasets demonstrate the superior performance of the proposed framework in region detection.

Keywords: Deformable Shape Segmentation, Salient Region Detection, Diffusion Geometry, Clustering Algorithm, Persistent Homology

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1. Introduction

Salient feature detection and description on three dimensional shapes is a fundamental problem in the field of computer vision and graphics. It has wide applications in surface matching, surface registration, shape recognition and shape retrieval \[1,2\]. Early research mainly focus on rigid shapes \[3,4,5\]. The problem is much more challenging for non-rigid shapes due to the large degree of local deformations \[6,7,8\]. In the past decade, a large number of point based feature detectors and descriptors have been proposed, such as the MeshDOG and MeshHOG \[8\], Harris 3D \[9\], heat kernel signature \[10\], wave kernel signature \[11\], stable topological signature \[12\], local binary descriptor based on heat diffusion \[13\], heat propagation contours \[14\] and descriptors based on machine learning methods \[15,16\]. In recent years, an increasing interest has been given to the detection and description of the stable regions on 3D shapes \[17,18,19,20\]. These methods achieved a remarkable success in several applications \[21,22,23\], such as shape correspondence and retrieval, due to their higher robustness compared to local surface based descriptors.

Existing region detection approaches focus mainly on the property of stability under different deformations of a shape. As a result, some of their resulting detected shape components do not usually contain informative features \[14\]. Furthermore, not many works consider the relationship between region detection and human perception. According to cognitive theory, a salient region is a perceptually related subset of a shape determined by its relative size and degree of protrusion in comparison with the neighboring surfaces \[24\]. Salient region detection on three dimensional shapes is very useful, because it provides more insights and facilitates the interpretation of a shape in terms of both geometric and semantic information.

In this paper, we present a novel framework for the detection of salient regions on 3D deformable shapes, which combines two main approaches used for 3D shape analysis \[25\], i.e., diffusion geometry and persistent homology. This
framework first defines a scalar field using the auto diffusion function \[ \text{24} \] at consecutive time scales. Being a local surface descriptor derived from the Laplace-Beltrami decomposition, the auto diffusion function is robust to isometric transformations and is able to describe a surface at multiple scales. Therefore, during the evolution of the scalar field, all of the salient regions will appear in the scale space. Based on the constructed scalar field, we exploit the method of persistent homology to extract salient regions. Thereby, we take advantage of the capabilities of the method in multivariate data analysis, i.e., persistence-based clustering \[ \text{27} \] and clustering assessment \[ \text{28} \]. Persistence-based clustering is performed to produce an initial segmentation of the shape. Persistent homology is then calculated to assess the clustering and discover the newly emerged salient regions during the process. Comparative experiments were performed on three popular datasets to demonstrate the superiority of the proposed method over the existing ones.

The rest of this paper is organized as follows. Section 2 provides a brief literature review of region detection algorithms on deformable shapes. Section 3 analyzes the limitations of the existing persistence-based shape segmentation method, which sets the stage for our approach. Section 4 describes the proposed salient region detection framework. Section 5 presents all evaluation results and analysis of our method. Section 6 concludes the paper.

2. Related work

Salient region detection is a non-trivial problem, especially for shapes under isometric transformations. It has been well studied by the shape analysis community in the recent years.

One of the first notable works was proposed by Litman et al. \[ \text{17} \], which formulated the problem as seeking the maximally stable components on a shape. This approach exploited the diffusion geometry to derive weighting functions in order to achieve invariance to isometric transformations, and proposed both vertex and edge weighted graph representations of the mesh. The edge weighted
graph representation is more general than the vertex weighted representation and shows superior performance. This framework was extended to handle volumetric shapes in [29]. Sipiran et al. [18] considered the salient regions as the “Key-components” on a shape, and assumed that they contain rich discriminative local features. This approach was inspired by the cognitive theory on saliency of visual parts. According to this theory, these salient parts correspond to the regions with a high protrusion, and are detected by a clustering process in the geodesic space. However, this method produces an incomplete decomposition of a shape [18].

Diffusion geometry based methods have achieved a remarkable success as they exploit the intrinsic properties of a shape [11]. Reuter et al. [30] used eigenvectors of the Laplace Beltrami operator, due to their invariance to isometric transformations, and exploited the persistence diagram to perform a hierarchical segmentation of a shape. In [31], a global point signature is calculated for each point, and the shape is mapped into an intrinsic space. A clustering algorithm is then applied in this space to segment the shape. However, these methods use eigenfunctions of the Laplace-Beltrami operator for segmentation. The use of eigenfunctions suffers from the problems of sign flipping and eigenvectors switch, especially when the difference between corresponding eigenvalues is small [32]. Rodola et al. [19] introduced the idea of consensus clustering into this area to achieve a stable segmentation. They generated a heterogeneous ensemble of segmentations by applying multiple clusterings in the global point signature space. Their work assumes that a robust segmentation can be obtained by gathering statistical information from these segmentations. This approach produces the state-of-the-art results on a wide range of transformations.

More stable variants based on the heat kernel from the theory of diffusion geometry have also been introduced. The persistence-based shape segmentation approach was proposed in [32, 53]. This framework first computes the prominence of the basins of attractions that are associated with the local maxima, which are then grouped in the form of a component tree. A persistence threshold is then selected to merge the components and to produce a stable segmentation.
of the shape. This framework is stable under isometric transformations. However, it uses a vertex weighted function and is relatively less robust to noise compared to the edge weighting scheme \[24\]. In addition, as discussed in this paper (Sec. 3.3), the method depends heavily on the selection of the merging parameter, which is hard to derive. Heat walk \[23\] was proposed to derive a salient and stable segmentation by making full use of the information contained in the heat kernel. It achieves a robust performance and can be considered as an edge weighted method \[19\].

As noted in \[19\], the aforementioned approaches focus on the robustness property of a region detector under different transformations of a shape. They are unable to automatically determine the optimal number of components to be extracted. As a result, some of the regions produced by these methods are lacking in features, which limits their applications, e.g., for dense correspondence and shape retrieval. In this paper, we propose a region detector, which is stable and at the same time captures the salient regions on a shape. We demonstrate that the regions detected by our method are distinctive and closely related to human perception.

The main contributions of this paper include:

First, we analyze the limitations of the existing persistence-based segmentation methods (Section 3), and propose a method to detect the salient regions on deformable shapes. Second, we take advantage of the recent progress in persistence homology to achieve a salient shape segmentation (Section 4). Third, we provide a comprehensive evaluation of the proposed method to demonstrate its saliency and robustness (Section 5).

3. Deformable shape segmentation using persistence-based clustering

In this section, we give a brief overview of the basic concepts behind our method.
3.1. Persistent homology

Persistent homology has originated from computational topology. It describes a data set using topological features with various dimensions. The topological features consist of connected components (dimension 0), holes (dimension 1) and voids (dimension 2). In this paper, we focus on the 0-dimensional persistent homology because of its significant effectiveness for data clustering, which can be directly applied to shape segmentation. The fundamental idea of persistent homology is to provide a framework for data set characterization, which incorporates both topological information and the geometrical properties measured by a function.

Given a manifold $M$ and a scalar function defined on it $f : M \to \mathbb{R}$, the function $f$ is assumed to have a finite number of critical points, i.e., points where the gradient of the function values vanishes. Figure 1 gives an example of $M$ and its associated height function $f$. Since $M$ is a 2-dimensional manifold, the critical points include both local extrema and saddles.

Sub-level sets of the scalar function $\mathcal{L}_\alpha(f) = f^{-1}([\alpha, +\infty))$ induces a filtration of the manifold $M$, i.e., a family of subspaces nested by inclusion. Persistent homology encodes the evolution of connectivity information when the value of function $f$ changes from large to small. The topology of the space created by

Figure 1: Computation of 0-dimensional persistent homology. **Left**: a manifold $M$ defined with a height function $f$. **Right**: derived persistence diagram $D_f$. 
$\mathcal{L}_\alpha(f)$ changes only at the critical points. At a local maximum, a new connected component emerges in the space. At a local minimum or a saddle, two connected components are merged. As $\alpha$ decreases from $+\infty$ to $-\infty$, a hierarchy of components of the manifold is generated. To be consistent with the construction of the component tree, the component with a smaller maximum is merged into the larger one.

Let $\mathcal{C}_\alpha(f,u)$ be a connected component of the space created by $\mathcal{L}_\alpha(f)$ with $f(u)$ be the global maximum. $\mathcal{C}_\alpha(f,u)$ is claimed to be born at $u$ and the infimum of $\alpha$, i.e., $f(v)$, with which $f(u)$ remains to be the largest value of the component is called the death value of $\mathcal{C}_\alpha(f,u)$. Thus, each component can be represented by a couple $(f(v), f(u))$, where $f(u)$ and $f(v)$ are its birth and death values, respectively.

Hence, the manifold $\mathcal{M}$ is decomposed into a hierarchy of components and projected onto a 2D plane, producing a persistence diagram $\mathcal{D}_f$. As shown in Fig. 1, each component corresponds to a point in the persistent diagram $(a, b) \in \mathcal{D}_f$. Persistence of a component is defined as its lifespan $\text{pers}(a, b) := a - b$.

3.2. Scalar field definition

Deformable shape segmentation using persistence-based clustering is related to the shape description method [34], which requires a scalar function for data description. The auto diffusion function [26], also known as the heat kernel signature [10], is adopted because it is informative for isometric surface description [2].

We model a shape as a compact Riemannian manifold $\mathcal{M}$ endowed with the standard measure induced by the volume form. The heat diffusion process over $\mathcal{M}$ is controlled by the heat equation, defined as:

$$\left(\Delta_{\mathcal{M}} + \frac{\partial}{\partial t}\right)u(x,t) = 0. \quad (1)$$

Here, $u(x,t)$ describes the heat distribution on $\mathcal{M}$ at time $t$ and $\Delta_{\mathcal{M}}$ is the Laplace-Beltrami operator of $\mathcal{M}$, a generalization to the Riemannian manifold. The fundamental solution to Eq. 1 is called heat kernel $h_t(x, y)$, which measures
the amount of heat transferred from \( x \) to \( y \) at time \( t \) with an initial unit heat source located at \( x \). According to the spectral decomposition theorem, the heat kernel has the following eigen-decomposition:

\[
h_t(x, y) = \sum_{i \geq 1} e^{-\lambda_i t} \Phi_i(x)\Phi_i(y),
\]

where \( \lambda_i \) and \( \Phi_i \) are the \( i \)th eigenvalue and eigenfunction of \( \triangle_M \), respectively. By restricting the heat kernel to the temporal domain:

\[
h_t(x, x) = \sum_{i \geq 1} e^{-\lambda_i t} \Phi_i(x)^2,
\]

the kernel function \( h_t(x, x) \) measures the amount of heat remaining at a point \( x \) after time \( t \), and is referred to as the auto diffusion function \( ADF_t(x) \). To make it scale invariant, the function is divided by its second eigenvalue \( \lambda_2 \), that is:

\[
ADF_t(x) = \sum_{i \geq 1} e^{-\lambda_i t/\lambda_2} \Phi_i(x)^2.
\]

As formalized in \([10][25]\), the heat kernel has several desirable properties for shape analysis. Its isometric invariance property makes it suitable for the handling of articulated shapes. Its stability under the local perturbations of the underlying manifold enables it to be insensitive to noise to some extent. Particularly, the auto diffusion function \( ADF_t(x) \) is a smooth scalar function, which describes the geometric information around a point \( x \) at multiple scales. For a small \( t \), it is determined by a small neighborhood of \( x \), and the scale grows larger as \( t \) increases. In addition, the auto diffusion function is closely related to the surface curvature for small values of \( t \) according to:

\[
ADF_t(x) = (4\pi t)^{-\frac{d}{2}} \sum_{i \geq 0} a_i t^i,
\]

where \( a_0 = 1 \) and \( a_1 = s(x) \) with \( s(x) \) denoting the Gaussian curvature for 2-dimensional manifolds. Thus, the auto diffusion function can be interpreted as a multi-scale notion of curvature of the local surface around \( x \) with its scale implicitly defined by \( t \). Its maxima have the ability to capture tips of the salient regions on a model. Its gradients and level sets follow and encircle the salient regions, respectively, as shown in Fig. 2.
3.3. Deformable shape segmentation using persistence-based clustering

Persistence-based clustering is based on the assumption that the components, induced by the computation of persistent homology, provide a clustering of the data. More importantly, it provides each cluster with a measure of saliency or stability, i.e., persistence. The components with large persistence are considered to be salient features, while topological noise has small persistence. Originated from Morse theory \[^{34}\], persistence has been proven to be stable under perturbations of the function. This is exploited by persistence-based clustering to guide the merging of clusters, producing clustering with a stability measure \[^{27}\].

Clustering algorithms support shape segmentation by grouping inputs that share similar features. The stability property of persistence-based clustering makes it a natural candidate for robust deformable shape segmentation. Generally, it consists of three steps. First, a scalar field is defined by the auto diffusion function at a fixed time scale, which provides a characterization of the shape. Second, persistent homology of the scalar function is computed and persistences of the shape components are summarized in a persistence diagram. Third, given a threshold of persistence (i.e., a merging parameter \(\tau\)), the components are classified as salient features or topological noise. Components corresponding to topological noise are merged into salient regions, yielding a stable segmentation.

This approach depends on the selection of the merging parameter to determine the salient regions to be detected. However, persistence of a component varies at different time scales, according to Eq. \[^{4}\]. There is no guarantee of existence of a consistent merging parameter for the algorithm. In fact, the parameter \(\tau\) should be adjusted with respect to different time values and models. An example of this approach is shown in Fig. \[^{2}\], which is conducted at different time scales. The instability of this method is demonstrated in Fig. \[^{2}\]a, b, c by the different segments which are generated by different choices of \(\tau\).

Another important criterion to evaluate the quality of a segmentation algorithm is saliency. A salient segmentation decomposes a shape into salient
regions considering their relative size and degree of protrusion. It is closely related to human perception [24]. Due to the instability of the scalar field, which is induced by the multi-scale property of the auto diffusion function to describe the local surface, the saliency of this approach is determined by the time parameter, where all the salient regions on a shape are generated by the auto diffusion function with a large persistence, as shown in Fig. 2c.

Our key insight is that there exists an evolution of the scalar field across time scales, and that all the salient regions are exposed during this process. In
the next section, we show how to obtain a salient region detection on deformable shapes based on our observation.

4. Salient region detection using persistent homology

In this section, we focus on the problem of deriving a salient segmentation of deformable shapes by analyzing the scalar field of the auto diffusion function at different time scales using persistence homology. Our intuition is that the evolution of the clustering derived from the scalar field can be exploited to extract the salient regions on a shape. Our approach is inspired by recent work on multi-variant data analysis \[28\] and uses persistent homology to generate and evaluate clustering. An illustrative example of the proposed method, Scale Space Clustering Evolution (SSCE), is given in Fig. 3. Its pseudo code is given in Algorithm 1.

Given a shape \(M\), a clustering \(S = \{s_i|i = 1, ..., k\}\) generated by persistence-based clustering produces a partition of \(M\). It constructs a scalar field from the auto diffusion function \(f_{t_j}\) at a particular time scale \(t_j\) and calculates persistent homology of the scalar function, producing a persistence diagram \(D_{f_{t_j}}\). Finally, it groups points \(p \in M\) in their image \(f_{t_j}(p)\), i.e., \(S: f_{t_j}(p) \rightarrow i\), to form a finite number of clusters \(s_i \in S\). These clusters are pair-wisely disjoint and their union makes up the whole shape. The computation of persistent homology provides each cluster with a persistence measure \(\text{pers}(s_i)\), which is used as a measure of saliency. Clusters with a large persistence correspond to shape components with a high protrusion, while clusters with a small persistence are the relatively flat regions. That is because the scalar field reflects the property of curvature of the local surface. However, the segmentation produced by the clustering \(S\) lacks stability and saliency due to the instability of the scalar field. To solve this issue, we explore the clustering process together with the evolution of the scalar field across scales.
Figure 3: Block diagram of our proposed salient region detection framework. (a) The input cat model. (b) The segmentation of a model is initialized by clustering of the scalar field at scale $T_0$. It is derived using the method of persistence-based clustering from the scalar field, which is defined by the auto diffusion function at $T_0$. (c) During the evolution of the scalar field with time scales decreasing from $T_1$ to $T_n$, clusterings are evaluated and updated, and all the salient regions are extracted. (d) Unstable regions are removed from the primitive segments to refine the segmentation. (Figures are best viewed in color.)

4.1. Clustering assessment

In this framework, we propose two measures, i.e., $\sigma_{\text{Global}}$ and $\sigma_{\text{Local}}$, to assess the clustering of the scalar field defined on a shape. They are developed based on the persistent homology and are capable of evaluating clustering both on a global and a local level. They are exploited to indicate if the salient features on the scalar field are captured by the clustering process.

4.1.1. Scalar field normalization

Since persistent homology summarizes the geometrical and topological behaviors of functions, *persistence* describes the degree of function variations within a connected space. The persistence of a region is considered as a measure of saliency which characterizes the region in terms of its deviation from featureless or flat regions. Our method is based on the key assumption that although
Algorithm 1 Scale Space Clustering Evolution (SSCE)

Input:

X: A 3D shape M;
T: The set of time samples for the construction of scalar field;

Output:

Y: Final set of salient regions detected on M;

1: The number of scales: scales ← count(T)
2: for j = 0 → (scale − 1) do
3: Construct normalized scalar field \( \hat{f}_{t,j} \) using Equation (1)
4: Calculate persistent diagram \( \mathcal{D}_{\hat{f}_{t,j}} \) of the scalar field, resulting in a cluster \( S_j \leftarrow M \)
5: if j = 0 then
6: Initial segmentation of M using persistence-based clustering \( S_0 \)
7: else
8: Refine the segmentation using Equations (11) (12)
9: Calculate \( \sigma^j_{Global} \) using Equation (8)
10: if \( \sigma^j_{Global} > 0 \) then
11: The number of components: components ← count(\( S_j \))
12: for i = 1 → components do
13: Calculate \( \sigma^j_{Local}(s_i) \) using Equation (9)
14: if \( \sigma^j_{Local}(s_i) < 1 \) then
15: \( s_i(1) \cup \ldots \cup s_i(n) \leftarrow s_i \)
16: end if
17: end for
18: end if
19: end if
20: end for
21: Refine the segmentation using Equation (13)

the persistence of the salient features on a scalar field varies significantly across scales, they are of the same magnitude.
Thus, we scale the function values \( f_{t_j} \) in the range of 0 to 1:

\[
\hat{f}_{t_j}(x) = \frac{f_{t_j}(x) - \min(f_{t_j}(x))}{\max(f_{t_j}(x)) - \min(f_{t_j}(x))},
\]

(6)

where \( \min(f) \) and \( \max(f) \) are the extremal values of \( f \).

With the scaled function \( \hat{f}_{t_j} \), the persistence diagram \( \mathcal{D}_{\hat{f}_{t_j}} \) summarizes the persistence of the components in a normalized space, which describes their relative prominence on a shape. Based on our key assumption (above), we consistently set the merging parameter \( \tau \) to be 0.1. This makes the evaluation of components consistent across scales and stable for different shapes.

4.1.2. Total persistence

As shown in Fig. 1, persistent homology provides a coarse characterization of the function behaviors on a manifold with a persistent diagram. A straightforward statistic of the persistence diagram is total persistence, which measures the amount of fluctuations in a function.

Given a persistence diagram \( \mathcal{D}_f \) of a function \( f \), the total persistence \( \text{pers}(\mathcal{D}_f) \) is defined as the sum of all squared persistence values of points in the persistence diagram, that is:

\[
\text{pers}(\mathcal{D}_f) = \sum_{(u_i, v_i) \in \mathcal{D}_f} \text{pers}(u_i, v_i)^2.
\]

(7)

Intuitively, the total persistence of a function is similar to the measure of “total variation” in statistics.

4.1.3. Global assessment of a segmentation

Following the previous definition, we evaluate the quality of a clustering globally by computing:

\[
\sigma_{\text{Global}} = 1 - \frac{\sum \text{pers}(u_{s_{i_j}}, v_{s_{i_j}})^2}{\text{pers}(\mathcal{D}_f)},
\]

(8)

where \((u_{s_{i_j}}, v_{s_{i_j}})\) represent the persistent points in the persistence diagram \( \text{pers}(\mathcal{D}_f) \) and correspond to the salient clusters \( s_{i_i}, s_{i_j} \in S \).

This equation measures the ratio of function variations which are not captured by the clustering \( S = \{s_j\}_{j = 1, \ldots, k} \). The value of \( \sigma_{\text{Global}} \) ranges from 0
to 1 with 0 meaning that the topological features have been completely captured by the clustering.

4.1.4. Local assessment of a segment

To assess a clustering locally, we propose the measure $\sigma_{Local}$. It calculates the ratio of variations within each cluster $s_i$ captured by the clustering $S = \{s_j|j = 1, ..., k\}$. It is defined as follows:

$$
\sigma_{Local}(s_i) = \frac{\sum \text{pers}(u_{s_i,j}, v_{s_i,j})^2}{\text{pers}(u_{s_i}, v_{s_i})^2},
$$

(9)

where $\text{pers}(u_{s_i}, v_{s_i})$ denotes the persistence of a cluster $s_i$, and $(u_{s_i,j}, v_{s_i,j})$ correspond to the shape features in the cluster $s_i$, including the salient features and the topological noise.

The value of $\sigma_{Local}$ is not smaller than 1, where 1 means that the cluster $s_i$ fully captures the function behaviors within it. The larger values of $\sigma_{Local}$ indicate the presence of more geometrical features that are not captured in the cluster $s_i$.

4.2. Deformable shape segmentation by clustering exploration

As shown in Fig. 4, there exists an evolution of the salient regions on the scalar field at different time scales, because of the multi-scale characteristic of the auto diffusion function. According to Eq. 4, the auto diffusion function $ADF_t(x)$ is a linear combination of eigenfunctions of the Laplace-Beltrami operator. While $t$ is large, the eigenfunctions corresponding to large eigenvalues make less contributions to $ADF_t(x)$ and the auto diffusion function captures the main shape features which are reflected by the small eigenvalues. As $t$ decreases, more features emerge, and evolve from global to local and from large scales to small ones [26].

Our intuition is that the evolution of the scalar field across time scales exposes all the salient regions on a shape. Note that, we focus on shape features in large scales, which are reflected by $ADF_t(x)$ at large $t$. Thus, we trace the evolution of the auto diffusion function computed at large time scales with consecutively decreasing values $T = \{t_j, j = 0, 1, ..., n_t\}$. 

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At the first step $t_0$, a primitive segmentation of a shape is initialized with the clustering $S_0 = \{ s^i_0 | i = 1, ..., k_0 \}$ obtained from the persistence-based clustering. Particularly, the persistent homology operates on the normalized scalar field $\hat{f}_{t_0}$, which results in a classification (that is consistent across different shapes) of components into salient features or topological noise.

As $T$ decreases, the persistent homology of the scalar field $\hat{f}_{t_j}$ is calculated at each time scale $t_j$ to evaluate the validity of the clustering $S_j$. We compute $\sigma^i_{\text{Global}}$ and $\sigma_{\text{Local}}(s^i_j)$ on the basis of the produced persistence diagram $D_{\hat{f}_{t_j}}$. If $\sigma^i_{\text{Global}}$ is equal to 0, the clustering $S_j$ succeeds in detecting all the salient shape features on the current scalar field, and the algorithm continues to explore the clustering at the following time scale. Otherwise, the clustering has to be updated. We examine all the individual clusters $(s^i_j \in S_j)$ to find those clusters with $\sigma_{\text{Local}}(s^i_j)$ larger than 1. These clusters contain salient features which need to be further detected. Therefore, they are decomposed according to the hierarchy of components which is induced by the computation of persistent homology.

When the algorithm finishes, the salient regions on deformable shapes are detected in the form of the final clustering.

### 4.3. Segmentation refinement

The clustering evolution procedure provides a primitive segmentation of a shape. In many cases, there exist a number of plateaus in the scalar field and they are assigned randomly to clusters, resulting in segments of a shape with an arbitrary size $\mathcal{U}$. As shown in Figs. 2a, b, the chest is relatively featureless and wrongly grouped into different segments. In the next step, we propose two methods to refine the segmentation.

**First**, regions attached to different clusters during the clustering evolution process are considered as unstable regions. That is:

$$s^i_j(\text{unstable}) = s^i_j \setminus s^i_{j-1},$$

where $s^i_j(\text{unstable})$ is the unstable region corresponding to the $i$th component.
They are isolated and then grouped into the unstable segment. That is:

\[ s^i_j = s^i_j \setminus s^i_j(\text{unstable}), \]

\[ s_j(\text{unstable}) = s^1_j(\text{unstable}) \cup \ldots s^i_j(\text{unstable}) \ldots \cup s^{i-1}_j(\text{unstable}), \]

where \( s_j(\text{unstable}) \) denotes the unstable segment with \( s_0(\text{unstable}) = \emptyset \). Note that, the unstable segment is not included in the final set of salient regions, because it contains less features.

**Second**, since these unstable regions are featureless, they can be separated from their respective salient regions using the isocontours of the scalar function at small values. At the same time, the persistence of the salient regions is decreased. That is because the gradients and the level sets of the auto diffusion function follow and encircle the salient regions. We introduce the retained ratio of persistence \( \gamma_p \) to control the refinement of the segments. That is:

\[ \hat{s}^i_j = \{ x_k, x_k \in s^i_j \text{ and } x_k \leq \Theta \} \]

where \( \Theta = \max(f(s^i_j)) - \gamma_p(\max(f(s^i_j)) - \min(f(s^i_j))) \). Intuitively, as \( \gamma_p \) is decreased, more unstable regions are removed and the scale of the detected regions decreases accordingly.

With featureless regions removed from the segments, the combination of the two segmentation refinement methods improves the saliency of our framework.

The use of persistent homology makes the segmentation stable and produces salient regions. An initial segmentation of the shape is produced by persistence-based clustering, which is stable due to the stability property of persistence \[34\]. Two measures based on persistent homology are then computed to guide the search of all the salient regions on the scalar field. The stability of the segmentation is further enhanced through a segmentation refinement process using the retained ratio of persistence.

5. Experimental Results

We conducted several experiments to demonstrate the effectiveness of our salient region detection method, named Scale Space Clustering Evolution (SSCE).
The first set of experiments were performed on the TOSCA dataset \cite{11} to evaluate the repeatability of our method and to investigate the influence of different parameterizations. The second experiment was conducted on the Princeton Segmentation benchmark \cite{11} to test the saliency of our approach, that is its relation with human perception. Our approach was also evaluated on the SHREC’10 feature detection and description benchmark \cite{12} to show its robustness with respect to a wide range of deformations, and compared with several state-of-the-art methods. Finally, we investigated its application to shape matching as a region descriptor.

The most widely used criteria for a region detector is repeatability \cite{17} \cite{18} \cite{19}. It is defined as follows: let \( M \) and \( N \) be a transformed and its corresponding null shape, respectively, and let \( \{M_1, ..., M_k\} \) and \( \{N_1, ..., N_k\} \) denote the regions which are detected on the two shapes. Since the ground-truth correspondence is available, for each detected region \( M_i \), we can obtain its corresponding region \( \hat{N}_i \) on the null shape. Given two regions \( M_i \) and \( N_j \), their overlap \( O(M_i, N_j) \) is calculated as the area ratio:

\[
O(M_i, N_j) = \frac{\text{Area}(\hat{N}_i \cap N_j)}{\text{Area}(\hat{N}_i \cup N_j)},
\]

where \( \text{Area}(Q) \) denotes the area of a region \( Q \). Given an overlap value \( o \in \mathbb{R} \), the repeatability of a region detector is defined as the percentage of regions detected on a transformed shape, whose corresponding regions on the null shape are detected and their overlap is larger than \( o \).

In discrete settings, several discrete Laplace-Beltrami operators for meshes have been proposed, such as the cotangent weight scheme \cite{13} and the finite element method \cite{35}. For a fair comparison, we adopted the cotangent weight scheme to compute the eigenvalues and eigenvectors of the Laplace-Beltrami operator, which were then used to derive the auto diffusion function. Because persistent homology belongs to the category of topological methods, its computation only depends on the connectivity of the meshes. Therefore, differential geometry was not used in the computation of persistent homology, and preprocessing techniques were not applied. Persistent Homology Algorithms Tool-
box (PHAT) is developed as an open source library for the computation of persistent homology. In this paper, we adopted the Union-Find algorithm as an alternative, which is easy to implement and widely used in this field. Since shapes can be represented as triangular meshes, our salient region detection algorithm works directly on the faces, in the same way as in [13]. The auto diffusion function $f$ is projected onto mesh faces using the linear combination $f(tr) = \frac{1}{3} \sum_{i} f(tr_{i})$, where $tr$ is a mesh triangle and $\{tr_{i}, i = 1, 2, 3\}$ denotes its vertices. As shown in the following experiment, this face weighting function improves the robustness of our method.

5.1. Performance on the TOSCA dataset

The TOSCA dataset consists of nine shape classes (i.e., "cat", "centaur", "david", "dog", "gorilla", "horse", "michael", "victoria" and "wolf") under isometric transformations. These shapes are represented as triangular meshes. The number of vertices on each shape ranges from about 40,000 to 50,000. The ground-truth point-to-point correspondences between shapes are available. For a fair comparison with the Consensus Segmentation method [19], we downsampled all the meshes to at most 10,000 vertices.

Our salient region detection method SSCE has three parameters: (i) the time samples $\{T_{i}, i = 0, 1, ..., n_{t}\}$, (ii) the number of eigenfunctions $n_{E}$, (iii) the ratio of persistence retained $\gamma_{p}$. The performance of SSCE with different settings of these parameters was tested on the dataset using the repeatability criterion.

5.1.1. The Time Samples

The selection of the time samples $\{T_{i}, i = 1, ..., n_{t}\}$ plays an important role in our method. It determines the regions to be detected. We sampled 6 time scales uniformly distributed in the logarithmic scale over the time interval $[t_{\text{max}}, t_{\text{min}}]$, where $t_{\text{max}} = \frac{4\ln(10)}{\lambda_{2}}$ and $t_{\text{min}} = \frac{4\ln(10)}{\lambda_{100}}$. As suggested in [13], there is no noticeable variations in the function values when $t > T_{1}$, because they are mainly influenced by the small eigenvalues and their corresponding eigenvectors. We
empirically chose the first 4 time samples \( \{T_i, i = 1, ..., 4\} \). The minimum time value \( T_{\text{min}} \) was set as \( T_4 \), which achieved the best results in our experiments. Note that, the eigenvalues \( \lambda_i \) are normalized to achieve scale invariance according to Eq. \( 4 \). \( T_1 \) and \( T_4 \) were set equal to 9.20 and 0.15, respectively. The number of time samples \( n_t \) was set to 4, which is sufficient for our method to detect all of the salient regions. We tested the performance of our method with respect to different values of \( T_{\text{min}} \), while the other two parameters were set to \( n_E = 300 \) and \( \gamma_p = 0.7 \). The segmentation results are shown in Fig. 4.

It can be learned that when the minimum time sample is large, some prominent shape features cannot be detected (as shown in Fig. 4a). As \( T_{\text{min}} \) decreases, a salient segmentation of the shape was obtained in Fig. 4b. When \( T_{\text{min}} \) was further decreased from Fig. 4b to Fig. 4c, scales of the salient regions declined. This is because of the multi-scale property of the auto diffusion function to describe the local surface. At a large \( T_{\text{min}} \), some relatively small scale shape features were not persistent enough to be detected, because they are smoothed by a large neighboring surface. As \( T_{\text{min}} \) decreases, the auto diffusion function describes the geometric information more locally, and its isocontours are accumulated closer to the tips of the salient regions, as shown in Fig. 2. Therefore, with \( T_{\text{min}} \) decreasing, all of the salient regions are detected and the scale (area) of the detected regions is decreased.
Figure 5: Performance of our method SSCE on the TOSCA dataset. (a) Influence of the number of eigenfunctions on the repeatability of SSCE. (b) The magnified version of the region indicated by the rectangle in the plot (a). (c) Influence of the ratio of persistence retained on the repeatability of SSCE. (d) The magnified version of the region indicated by the rectangle in the plot (c). (e) Comparison with the best result of the Consensus Segmentation method [65]. (f) The magnified version of the region indicated by the rectangle in the plot (e). (Figures are best seen in color.)

5.1.2. Number of Eigenfunctions

The number of eigenfunctions determines the accuracy of the auto diffusion function to describe a local surface. Since our approach depends on the scalar function to characterize a shape, in theory we can expect a higher repeatability with more eigenfunctions included. We tested the influence of the number of eigenfunctions on the performance of the repeatability of our method. The ratio of persistence retained $\gamma_p$ was set to 0.7. The results are shown in Fig. 5a.

It can be observed that as the number of eigenfunctions increases, the repeatability of our region detector improves, which confirms our assumption. To
be specific, its performance improves slightly as the number of eigenfunctions increases from 50 to 100. The performance remains almost the same with the number of eigenfunctions increasing from 100 to 300. That is because there is a limit on the number of eigenfunctions that are required to compute the auto diffusion function for a particular time scale. According to Eq. 4, the influence of an eigenfunction $\Phi_i$ decreases exponentially with respect to its index $i$. Therefore, a further increase in the number of eigenfunctions will not enhance the descriptiveness of the scalar function in order to improve the performance. On the other hand, a larger number of eigenfunctions will be computationally costly and requires more memory resources. Therefore, we set the number of eigenfunctions to 100 throughout our experiments.

5.1.3. Ratio of Retained Persistence

The ratio of retained persistence $\gamma_p$ is used to refine the segments. When corresponding regions are detected on two shapes, $\gamma_p$ determines the repeatability of our SSCE region detector. We tested the performance of our method with respect to varying ratios of retained persistence while keeping the other parameters fixed. The results are shown in Fig. 5.

The figure shows that the performance of our method improves as the ratio of persistence retained $\gamma_p$ decreases. Especially when the overlap is at 0.9, the repeatability of our method can achieve as high as 1.0 with $\gamma_p$ equal to 0.6, in comparison with the repeatability of about 0.7 with 0.9 of $\gamma_p$. This is because when $\gamma_p$ is decreased, more unstable regions are removed from the segments, leading to its higher repeatability. Note that reducing the value of $\gamma_p$ will decrease the scale of the detected regions. Therefore, we set $\gamma_p$ to 0.7 in the rest of our experiments.

5.1.4. Comparison

We compared our approach with the method of Consensus Segmentation [19] on this dataset, which is the state-of-the-art at present. We used the parameters as described above, and the best results from [19] are reported. The results are
given in Fig. 5c. It can be seen from Fig. 5c that our approach achieves a better performance in terms of repeatability compared to [19]. The performance of Consensus Segmentation begins to decrease at an overlap of 0.6, while our approach declines at approximately 0.9.

5.2. Performance on the Princeton dataset

As noted in [19], existing methods [17][19][32] in this field fail to determine the optimal number of components for detection, and this number is often given as an input. In contrast, our method has the capability to automatically detect all of the salient regions on a shape, which are extracted based on the analysis of the scalar field across scales. Due to the multi-scale property of the auto diffusion function to describe the local surface, there is no guarantee of existence of a particular time value for all the salient regions to be captured, but they will be revealed during the evolution process. In this section, we investigated the saliency of the detected regions obtained by our method, that is their relation with human perception.

This experiment was performed on the Princeton Segmentation (PS) Benchmark [41]. We tested our algorithm on eleven categories of non-rigid 3D models from the PS Dataset, in a similar way as in [24]. Four metrics including Cut
Table 1: Comparison of the performance between our salient region detection method and the ground-truth (Humans) [31], Heat Walk [23] and methods tested in the benchmark [31]. Four metrics are used, including Cut Discrepancy (CD), Hamming Distance (HD), Rand Index (RI) and Consistency Error (CE).

<table>
<thead>
<tr>
<th>Method</th>
<th>CD</th>
<th>RI</th>
<th>HD</th>
<th>CE1</th>
<th>CE2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Truth</td>
<td>0.140</td>
<td>0.081</td>
<td>0.108</td>
<td>0.082</td>
<td>0.055</td>
</tr>
<tr>
<td>Heat Walk</td>
<td>0.267</td>
<td>0.148</td>
<td>0.234</td>
<td>0.221</td>
<td>0.136</td>
</tr>
<tr>
<td>Rand Cuts</td>
<td>0.150</td>
<td>0.093</td>
<td>0.127</td>
<td>0.149</td>
<td>0.083</td>
</tr>
<tr>
<td>Shape Diam</td>
<td>0.221</td>
<td>0.143</td>
<td>0.177</td>
<td>0.144</td>
<td>0.094</td>
</tr>
<tr>
<td>Core Extra</td>
<td>0.272</td>
<td>0.159</td>
<td>0.177</td>
<td>0.144</td>
<td>0.094</td>
</tr>
<tr>
<td>Rand Walks</td>
<td>0.297</td>
<td>0.164</td>
<td>0.215</td>
<td>0.222</td>
<td>0.123</td>
</tr>
<tr>
<td>Fit Prim</td>
<td>0.253</td>
<td>0.145</td>
<td>0.249</td>
<td>0.265</td>
<td>0.174</td>
</tr>
<tr>
<td>Kmeans</td>
<td>0.288</td>
<td>0.161</td>
<td>0.268</td>
<td>0.286</td>
<td>0.190</td>
</tr>
<tr>
<td>our method</td>
<td>0.149</td>
<td>0.090</td>
<td>0.118</td>
<td>0.124</td>
<td>0.065</td>
</tr>
</tbody>
</table>

Discrepancy (CD), Hamming Distance (HD), Rand Index (RI), and Consistency Error (CE) were used to evaluate its performance (refer to [31] for more details of these metrics). The parameters of our algorithm were set as follows: $n_E = 300$ and $\gamma_p = 0.6$, which achieved the best results in our experiments. We can observe from Table 1 that our method achieves the best performance on this benchmark. It is worth pointing out that both our proposed approach (SSCE) and the heat walk [23] use diffusion geometry, and they are more specifically based on the heat kernel. However, our method differs from the heat walk in two aspects. **First**, SSCE constructs the scalar field using the auto diffusion function at consecutive time scales. Consequently, all the salient regions on the shape are revealed during the scalar field evolution process. In contrast, the heat walk approach computes the heat kernel function at a particular time scale, and its saliency is limited. **Second**, persistent homology is applied to extract salient regions on a shape in SSCE, which provides the method a guarantee of stability regarding the mesh quality. Figure 6 shows the segmentations of a wide range of models using our method. These experimental results show that the regions
obtained by our method are closely related with human perception.

5.3. Performance on the SHREC’10 feature detection and description benchmark

In this section, we tested the repeatability of our salient region detector (SSCE) under different surface perturbations. The experiment was carried out on the SHREC’10 feature detection and description benchmark [12]. There are three shape classes in this dataset (people, dogs, and horses), which were subject to nine different types of deformations, i.e., isometry, noise, shot noise, holes,
micro holes, sampling, scale, local scale, and topology. Each transformation has 5 intensity levels. The performance was compared to three state-of-the-art methods, including Litman et al. [17], Sipiran et al. [18] and Consensus Segmentation [19]. We set the parameters based on the experiments in Sec. 5.1, and the best results from [17] [18] [19] were reported. Comparative results are given in Fig. 7. Visual results on all of the deformations of a human model from the SHREC’10 feature detection and description benchmark are illustrated in Fig. 8.

We can learn from Fig. 7a that our proposed region detector outperformed all the other methods by a large margin under isometric transformations. SSCE can achieve almost full scores at the overlap of 0.9 while the performance of the existing methods declines at 0.6. The pose invariance property of the approach is induced by the isometry invariance of the auto diffusion function. In addition, the computation of the persistent homology exploits the connectivity information, which is stable under rigid and non-rigid transformations.

As shown in Figs. 7b, c, SSCE achieved the best performance in terms of scaling and local scaling as compared to the other three methods. Its repeatability is as high as 1.0 even when the overlap is at 0.9. According to Eq. 4, this is because the scalar field is transformed to a scale invariant scalar field.

It was found in Fig. 7d that SSCE is robust to micro holes and it can detect almost all the regions of the corresponding shapes at the overlap of 0.9. Its performance declines in case of holes (Fig. 7f), because the introduction of holes decreases overlaps between corresponding regions significantly.

As shown in Fig. 7g, the regions detected by SSCE are highly repeatable in the case of shot noise with a repeatability of 1.0 at the overlap of 0.9. Our method also achieved a comparative result on meshes with additional noise. This is because our method is performed on the mesh faces, which makes it more robust in comparison with the vertex and edge weighting methods. Consequently, outliers are smoothed by points within the same triangle and the robustness of our method is increased.

Our region detector is unable to deal well with topology changes, because
the computation of persistent homology is dependent on the connectivity information. However, as shown in Fig. 7h, the performance of SSCE is comparable to the best result achieved by Consensus Segmentation [19]. We believe the reason is that during the evolution of the scalar field, some unstable regions are removed from the segments, which partly improves its robustness to topology changes.

Figure 8: Salient regions detected using our method on a human model from the SHREC’10 feature detection and description benchmark. The figure illustrates the detected regions on the model with eight transformations, i.e., isometry, noise, shot noise, holes, micro holes, sampling, scale, local scale and topology. Note that, the strongest strength is used for each transformation. (Figures are best viewed in color.)
It can be found in Fig. 7 that SSCE achieved the best performance under sampling. This is due to the fact that the sampling of a shape does not influence the accuracy of the computation of the auto diffusion function. Besides, the connectivity of the model is not changed by sampling, which has no effect on the calculation of persistent homology.

Overall, our method achieved the best performance on the SHREC10 dataset in comparison with three state-of-the-art methods. This is mainly due to the stability property of the auto diffusion function, which ensures that the scalar field is stable under different mesh deformations. The robustness of our method is further enhanced by the face weighting scheme. In addition, the use of persistence homology comes with the guarantee of stability regarding the mesh quality.

5.4. Shape Matching

As noted in [42], the main applications of a salient region detector within a feature-based approach are shape matching and retrieval. In this section, we evaluated the performance of our method on the SHREC’10 feature detection and description benchmark and compared it with two existing methods [17] [19]. For a fair comparison, we defined a region descriptor \( \hat{\alpha}(C) \) for a component \( C \in \mathcal{M} \) by calculating the area-weighted average of the q-dimensional point-wise descriptors \( \alpha : \mathcal{M} \rightarrow \mathbb{R}^q \), that is:

\[
\hat{\alpha}(C) = \sum_{p \in C} \alpha(p) \text{Area}(p),
\]  

where \text{Area}(p) denotes the area on \( \mathcal{M} \) belonging to \( p \).

The discriminativity of a region descriptor was evaluated following the same matching pipeline as in [19]. Given a detected region \( \mathcal{N}_i \) on the null shape, its first match \( \mathcal{M}_{i*} \) on the transformed shape \( \mathcal{M} \) is defined as its nearest neighbor in the descriptor space, as follows:

\[
\mathcal{M}_{i*} = \arg\min_{\mathcal{M}_j} ||\hat{\alpha}(\mathcal{N}_i) - \hat{\alpha}(\mathcal{M}_j)||_2.
\]
Figure 9: Performance of our region descriptor on the SHREC’10 feature detection and description benchmark. Results are compared with two existing methods, i.e., Consensus seg. and Litman et al.

The matching score at overlap $o$ is defined as the percentage of successful first matches, that is:

$$ \text{score}(o) = \frac{|\{O(M_j, N_i) \geq o\}|}{k_n}. $$

(17)

For a fair comparison with [17][18], the scale invariant heat kernel signature (SI-HKS) [19] was exploited to compute the region descriptor. The heat kernel signature was computed at six time scales $t = 16, 22.6, 32, 45.2, 64, 90.5, 128$ and the first six discrete frequencies were adopted. Our experimental results are shown in Fig. 9.

As shown in Fig. 4, our method consistently achieved superior performance...
compared to the methods of Litman et al. [17] and Consensus segmentation [19]. This demonstrates the effectiveness of our approach and its potential use for deformable shape analysis.

6. Conclusion

In this paper, we proposed a robust salient region detection method for 3D deformable shapes. The salient regions are detected based on the analysis of the clustering derived from the scale space. The scale space is defined on a shape by the auto diffusion function at several time scales. The segmentation of the shape is initialized with clusters produced by persistence-based clustering at a large time scale. Persistent homology is then exploited to assess the clustering across scales and to extract the salient regions. The evolution of the scale space reveals all of the salient regions on the shape at multiple scales, and it does not require any prior on the number of segments to achieve a salient segmentation.

We performed several experiments on the TOSCA dataset and SHREC’10 feature detection and description benchmark to assess the repeatability of our salient region detector. Comparative results show that our method outperforms the state-of-the-art methods with respect to a set of shape deformations, including isometry, scale, local scale, holes, micro holes, noise, shot noise, topology and sampling. In addition, we examined the relation of our salient region detection method with human perception to demonstrate its saliency. The results on the Princeton Shape Segmentation benchmark show a superior performance in comparison with the existing algorithms. Moreover, we investigated the application of our method as a region descriptor on the SHREC’10 feature detection and description benchmark. Experimental results demonstrate its potential application to deformable shape matching and retrieval.

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