THE IMPORTANCE OF DISTINGUISHING BETWEEN ZENO'S ORIGINAL PARADOXES AND MODERN MATHEMATICAL ARGUMENTS INSPIRED BY THOSE PARADOXES

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I, Rowan McCullough declare that this dissertation is my own account of my research and contains as its main content work which has not previously been submitted for a degree at any University.

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Abstract

In arguments made about the paradoxes of Zeno of Elea there is not always a clear distinction between *attempts to determine what Zeno was originally trying to argue* and *how Zeno's arguments have influenced, or been used in, modern mathematics*.

In this Dissertation it is argued that if one is interested in determining the original purpose of Zeno's arguments, then it is not helpful to either express the paradoxes as modern mathematical problems or pose modern mathematical solutions to them. Doing either of these things will adversely effect the way in which Zeno's paradoxes can be interpreted. To the extent that Zeno's paradoxes are discussed in modern mathematics they are usually used merely as analogies, where the question of what Zeno was originally trying to argue is largely irrelevant. This is not to say that modern mathematical argument about Zeno should not be discussed, simply that they are a different set of arguments. If one is trying to determine Zeno's original argument then one should focus instead on analysing the arguments made against Zeno by other ancient thinkers. Ancient thinkers are likely to have had a far better understanding of Zeno's original argument simply because they are historically closer to him.

This dissertation discusses two articles in which this issue arises. In neither article is it claimed that Zeno's paradoxes were simple statements of mathematical fallacy. However, both authors insist on comparing the arguments which Zeno and his ancient opponent appear to make, with modern mathematical arguments. This obscures certain ways of interpreting Zeno's paradoxes. An interpretation of Zeno's argument, which is likely to be overlooked if a modern approach to Zeno is taken, will be discussed in this dissertation.

A mathematical approach to Zeno can obscure the possibility that Zeno might have been more interested in the question of how it is possible that an extension can have the infinite number of extended parts which it appears to have. A mathematical approach to Zeno tends to focus more on infinite summation and infinite amounts in a purely abstract and non-physical sense.

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Table of Contents

Introduction

Zeno's paradoxes are a set of arguments posed by Zeno of Elea in the fifth century BCE. The paradoxes, to the extent which we know them, describe various assertions about the physical world which appear to be true but, according to Zeno, lead to absurd conclusions.

I will primarily be discussing Zeno's Dichotomy and Achilles paradoxes. Both these paradoxes involve a runner attempting to run a finite distance. It would seem these runners should be able to complete their respective runs, however according to Zeno they cannot. I will elaborate on the details of these paradoxes later.

Because we have access to little, if any, of Zeno's original arguments it is difficult to determine what Zeno was really trying to argue. It is this question of *what was Zeno trying to argue* which I am interested in in this paper.

In my research of this topic, however, I encountered a particular problem. Zeno's paradoxes are also used in various modern mathematical arguments. The issue is that there is not always a clear distinction made between how Zeno's paradoxes have been used by modern mathematicians and the task of trying to determine what Zeno was originally trying to argue.

In this paper I will argue that assuming a modern mathematical perspective when evaluating the fragments of Zeno's argument, and the ancient argument against them, can be detrimental to the task of determining Zeno's original purpose. If Zeno's paradoxes, and particularly Aristotle's arguments against them, are described in modern mathematical terms then people are likely to treat these arguments as if they are purely mathematical arguments. I will argue that this encourages the view that Zeno only uses physical examples in the paradoxes as analogies for describing these purely mathematical problems.

My argument is significantly influenced by a brief comment made by Palmer in an article he wrote about Zeno in the *Stanford Encyclopaedia of Philosophy* (2016). Palmer mentions that it is important to distinguish between the act of trying to determine Zeno's original paradoxes and the act of trying to solve the paradoxes

(2016). He suggests that if one is too focused on trying to solve the paradoxes this will affect the interpretation which one can make of the paradoxes (2016). The purpose of my dissertation is to elaborate on this line of argument and discuss particular instances where, I will argue, this had occurred.

For instance, the Dichotomy and Achilles paradoxes are often described as involving the false statement that *all* infinite series of values sum to an infinite total. This modern way of describing the paradoxes can give the impression that the physical circumstance described in the paradoxes is only an analogy for a purely arithmetical problem.

This is an issue because there is no reason why one should discount the possibility that Zeno might have been primarily interested in the physics of the problem. In this paper I will discuss an interpretation of Zeno which I suggest is likely to be overlooked if Zeno is discussed in the context of modern mathematics. I will argue that Zeno might have posed his paradoxes because he was concerned that an extension could not be considered to have many extended parts without leading to absurd conclusions. Interpretations of this kind, and any other which does not presuppose modern mathematical concepts, are more likely to be overlooked if Zeno's paradoxes are summarised and explained in modern mathematical terms.

This, however, does not mean that there is no value in discussing how Zeno’s paradoxes have been appropriated and used by modern mathematicians to argue about modern mathematical problems.

Zeno paradoxes have been taken on as challenges by various mathematicians and have been used to explain modern mathematical views on the concept of infinity. Bertrand Russell, for instance, discusses Zeno’s Achilles paradox in his *Principles of Mathematics* (1937 348). However, Russell explicitly notes, when he discusses the Achilles paradox, that he is not interested in what Zeno was originally trying to argue with the paradoxes and that he is only using it as an analogy for his arguments about mathematics (1937 348 (footnote)).

I will argue that such modern mathematical treatments should not be considered to solve Zeno's *original* paradoxes. This is simply because nobody is entirely certain what Zeno was *originally* trying to argue. The modern mathematical solutions instead pertain to modern problems which have been inspired by Zeno's paradoxes.

Among classical scholars there is a broad range of arguments made regarding what Zeno might have been originally trying to argue. Cajori (1920) provides a good summary of these kinds of arguments. For instance, it is sometimes argued that Zeno was specifically arguing against Pythagorean mathematics (Cajori 1920). It is also sometimes argued that Zeno might have been purely a sophist or someone who was was skilled in making a weak argument stronger (Cajori 1920). But, as Cajori points out, all of these arguments suffer from the fact that there is very little information about Zeno or his paradoxes in the known Greek fragments (1920). These theories remain plausible speculations about Zeno's arguments but cannot be anything more. The article of Cajori to which I refer is a dated article which does not take into account some modern theories about Zeno. However, his basic point still stands because the newer theories suffer from the same lack of source material as do the ones which Cajori discusses[1].

Plato indicates that Zeno had written a book of 40 paradoxes and Plato suggests Zeno had specifically written this book to defend the views of his teacher Parmenides (Plato's *Parmenides*). We do not have access to Zeno's book. We only have access to fragments of Zeno's Paradoxes and some counter arguments made against them by Aristotle and others. Additionally, the arguments of Simplicius, a Neoplatonist from the 6th century CE, are used in discussions of Zeno, because it is thought that Simplicius had access to some of Zeno's original arguments when he wrote his commentaries on Aristotle (Graham 2010 266).

What this means is that to the extent that we might have an understanding of or theories about what Zeno was arguing, no modern thinker is in a position to show why Zeno was wrong – because we cannot be sure of what he was trying to show.

1 Palmer's article (2016) about Zeno provides a more up to date and thorough discussion of the theories and arguments made about Zeno. However, I have referred to Cajori's article (1920) because he deals more directly with the issue that there is very little evidence to work with when discussing Zeno.

There is, however, value in simply assuming that ancient arguments against Zeno *did* solve the paradoxes. If we assume that they did, then it provides us with a way of inferring what Zeno might have been arguing. Aristotle and Simplicius[2] are two thinkers who, for argument's sake, we can assume did solve Zeno's paradoxes or had insight into what Zeno was trying to argue.

Again this does not however mean that we should refrain from trying to solve modern arguments which use Zeno's paradoxes as analogies. What it does mean is that there should be a clear distinction made between Zeno's original argument and modern arguments.

If a scholar who is interested in the modern mathematical significance of Zeno does not make this distinction then problems will arise. I suggest that a reader of the work of such a scholar would be left with the impression that Zeno's arguments are mathematical arguments primarily about whether or not it is possible to sum up infinite sets of magnitudes. This kind of interpretation of Zeno would indicate that the physical circumstances described in the paradoxes are largely irrelevant: it would indicate that Zeno is merely using a physical circumstance to give an analogy of a purely mathematical problem.

The problem with this mathematical reading of Zeno is that it is entirely possible that Zeno's argument was an argument about physics, not about pure mathematics. This is because there is not enough evidence in the fragments we have of Zeno's arguments to indicate that his primary concern was that mathematicians would be unable to complete a certain kind of sum. I do not entirely discount a mathematical interpretation of Zeno's original intended argument, however it is worthwhile considering other possibilities. For instance, I will be arguing that Zeno posed his paradoxes because he was concerned that extensions appeared to be merely collections of smaller extensions.

If this distinction is not made, then Zeno's paradoxes will be confused with modern arguments which are inspired by them, and Zeno will be treated as a mathematician

2 Because he is thought to have had access to Zeno's original arguments.

who unfortunately lacked various modern mathematical concepts. I will argue that this kind of interpretation of Zeno and his paradoxes does not reflect the arguments which Aristotle makes against Zeno.

I will suggest there is not a clear distinction made between Zeno's original argument and modern arguments in Hugget's article in the *Stanford Encyclopaedia of Philosophy* (2010) or Dowden's article in the *Internet Encyclopaedia of Philosophy* (2016), both titled *Zeno's Paradoxes*. By not making this distinction both Hugget and Dowden obscure possible interpretations of Zeno's paradoxes, and Aristotle's solutions to them, which do not rely on modern mathematical concepts.

Both Hugget and Dowden appear to recognise that modern discussions of Zeno are different from what Zeno might have been trying to argue. However, neither Hugget nor Dowden clearly identify that the modern solutions to Zeno's paradoxes do not necessarily have anything to do with Zeno's original argument. Both scholars move between discussion of modern solutions and ancient solutions without clearly identifying that they do not necessarily refer to the same paradoxes.

Dowden notes early in his article that he is not going to discuss the arguments which are made about what Zeno had originally intended to show with the paradoxes (2016 section 1c). He appears to be specifically interested in what he calls the standard solutions to the paradoxes (2016 section 2). These standard solutions appear to revolve around modern mathematical conceptions of continuity (2016 section 2). The issue with Dowden's argument is that he does not clearly identify that these standard solutions cannot be assumed to have solved Zeno original paradoxes.

Dowden does not clarify the difference between modern arguments which are inspired by Zeno and Zeno's original intended argument. I will argue that this is evident when Dowden characterises Aristotle's solutions to Zeno's paradoxes as being insufficient specifically because Aristotle did not have access to calculus and because his solution used the concept of "potential infinities" which is not used in modern mathematics (Dowden 2016 section 2). The fact that Dowden identifies that Aristotle's solutions are different to modern solutions is not an issue. But to claim that Aristotle's solution is insufficient because he lacked certain modern concepts is absurd. This is because it is not known what Zeno had intended to argue in the paradoxes that Aristotle set out to solve. If there was consensus about what Zeno was trying to argue then, and only then, one could ask the question of whether modern mathematics is required to solve the paradoxes or whether those modern solutions make Aristotle's solutions redundant. This problem can be alleviated by removing the claim that Aristotle's solutions are made redundant by modern solutions and arguing instead that Aristotle's solutions involve different assumptions or concepts to those which occur in modern mathematics.

Hugget, in his article, is unclear as to whether he is interested in discussing the effects which Zeno's paradoxes have had on the developments of modern mathematics or whether he is interested in determining the original purpose of the paradoxes. Hugget states that he is not interested in discussing the role which Zeno's paradoxes have served in the development of modern mathematics (Hugget 2010 section 1) but still argues that the solutions provided by Aristotle would not satisfy a modern mathematical thinker (2010 section 1)[3].

I will suggest that the only reason one would discuss how modern thinkers would not be satisfied with Aristotle's solution to Zeno would be if one were interested in the influence of Zeno's arguments on modern mathematical thinking. The question of *whether or not modern mathematical thinkers would be satisfied by Aristotle's solution to Zeno* is not a question one should ask if one is interested in trying to determine what Zeno had intended to argue.

To give an example of how I would suggest Hugget ought to have discussed Aristotle – if he were trying to determine Zeno's original intended argument – I will attempt to infer what Zeno might have been arguing based on the arguments which Aristotle makes against Zeno.

In summary, no modern thinker can legitimately claim to have solved Zeno's *original* paradoxes.

3 Hugget suggests that Aristotle's solutions might have been entirely sufficient to solve the problems which Zeno was originally trying to show with the paradoxes.

There are some fragments of the paradoxes and some ancient and medieval commentaries about the paradoxes. Also there are various modern theories posed by classical scholars about what the purpose of them might have been. However the ancient commentaries are very brief and the modern theories are often criticised for being very speculative and having little grounding in the known Ancient Greek texts.

If one is interested in Zeno’s original arguments then one might want to discuss Aristotle’s solution to Zeno’s paradoxes. However one is not in the position to judge whether or not Aristotle’s solution sufficiently solves Zeno’s paradoxes. The best that one can hope to do is to assume that Aristotle does solve Zeno’s paradoxes and then try to infer what Zeno might have been arguing from the arguments which Aristotle poses against him.

This, however, is complicated by the fact that Zeno’s paradoxes, and Aristotle’s solutions to them, have been used by many mathematicians over the course of history to discuss various mathematical concepts such as *infinite summation* and *actual infinite* sets. So long as these topics are treated as completely independent from Zeno’s original argument then there is no problem with discussing how Zeno’s paradoxes are used in arguments about these concepts.

Chapter One:
Detailed Summary of Argument

The Paradoxes and the Ancient Arguments About Them

Before I critique Hugget and Dowden's articles about Zeno's paradoxes I will first list and discuss some of the fragments which describe and argue against the paradoxes. I will be discussing the Dichotomy, Achilles and Plurality paradoxes. I have taken these passages from Graham's collation of the Presocratic fragments (2010)[4], Hardie and Gaye's translation of Aristotle's *Physics* and Joachim's translation of Aristotle's *On Generation and Corruption*.

Aristotle provides us with the most information about the Dichotomy and Achilles paradoxes. He describes the Dichotomy paradox in the following section of the *Physics*:

> The first [the Dichotomy paradox] asserts the non-existence of motion on the ground that that which is in locomotion must arrive at the half-way stage before it arrives at the goal.
> (Aristotle *Physics* 239b11)

The Dichotomy paradox, from this passage, can be seen as describing the following circumstance. Suppose that there is a runner running some finite distance. Before the runner travels the entire distance the runner will reach a "halfway stage". Before the runner can complete the last half of the run, the runner will reach another "halfway stage" between the mid point of the whole run and the end point of the whole run. The division can be repeated ad infinitum, and so there is an infinite set of half distances for the runner to run.

Alternatively, before the runner reaches the halfway point of the whole run the runner must first reach an earlier halfway point between the start and the halfway point of the whole run. There is an infinite number of these "halfways" involved in the motion.

4 It is from here that I have taken the passages from Simplicius.

Aristotle describes the Achilles paradox in this passage of the *Physics*:

> The second is the so-called 'Achilles', and it amounts to this, that in a race the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead. This argument is the same in principle as that which depends on bisection, though it differs from it in that the spaces with which we successively have to deal are not divided into halves. (Aristotle *Physics* 239b14)

The Achilles paradox, described here, is considered, by Aristotle, to be the same basic argument as the Dichotomy. The Achilles paradox describes a race between a slower runner and a faster runner. The slower runner gets a head start. Before the faster runner can over take they must reach the point which the slower runner started from when the race began. By that time the slower runner will have moved forward. This catchup process must be repeated an infinite number of times before the faster runner over takes the slower. These catchups are considered to be equivalent to the half runs in the Dichotomy paradox.

Aristotle also provides an in depth argument against these two paradoxes which I will discuss at length later in my dissertation.

As for Zeno's Plurality paradox, Simplicius reiterates parts of this paradox when commenting on the argument which Aristotle makes in *On Generation and Corruption* regarding the infinite divisibility of extended things. The passage from Simplicius which I will be focusing on is as follows:

> …if there exists [many things], each thing must have some size and solidity, and one part must stand out from the other. And the same consideration applies to what projects from this. For it will have size and a part will project from it. And it is the same to say this once and always. For there will be nothing which can serve as a final part of this nor will one part be different from another. Thus if there are many things, they

> must be both small and great: so small as to have no size, so great as to be unlimited. (Simplicius *Physics* 140.34-141.8)

The argument here appears to have some similarities in structure to the Dichotomy paradox. Specifically, both arguments seem to involve the assertion that there is an infinite set of smaller parts within a larger thing. The Argument which Simplicius describes in this passage is suggested to be the origin of the argument which Aristotle makes in this passage from *On Generation and Corruption* (Hugget 2010):

> Hence the same principle will apply whenever a body is by nature divisible through and through, whether by bisection, or generally by any method whatever: nothing impossible will have resulted if it has actually been divided – not even if it has been divided into innumerable parts, themselves divided innumerable times. Nothing impossible will have resulted, though perhaps nobody in fact could so divide it.
>
> Since, therefore, the body is divisible through and through, let it have been divided. What, then, will remain? A magnitude? No: that is impossible, since then there will be something not divided, whereas *ex hypothesi* the body was divisible *through and through*. But if it be admitted that neither a body nor a magnitude will remain, and yet division is to take place, the constituents of the body will *either* be points (i.e. without magnitudes) *or* absolutely nothing. (Aristotle *On Generation and Corruption* 316a 20-30)

I use this passage from Aristotle to construct an alternative interpretation of what Aristotle was arguing against Zeno to those given by Hugget and Dowden.

Discussions of the Paradoxes in Modern Terms

The first part of my argument will be dedicated to identifying what I suggest is an ambiguity in Hugget's and Dowden's articles. Neither scholar clearly identifies that modern solutions to Zeno's paradoxes ought not be treated as solutions to Zeno's original paradoxes and that they should instead be treated as solutions to problems which Zeno's paradoxes have led modern thinkers to consider.

It is not that either Hugget or Dowden explicitly or intentionally depict Zeno's paradoxes as modern problems with modern solutions and consider Aristotle’s solutions fundamentally flawed. My concern with these articles is that they do not separate modern arguments from ancient arguments as much as, I will suggest, they should be.

To show the issues with Hugget and Dowden’s articles I will discuss Zeno’s Dichotomy and Achilles paradoxes focusing primarily on Aristotle’s solution to these particular paradoxes. I will first discuss how Hugget and Dowden describe Aristotle’s solution. I will argue that both Hugget and Dowden misrepresent Aristotle’s solution because they are more interested in showing why modern thinkers would not be satisfied with Aristotle’s solution or would not be willing to make some of the assumptions which Aristotle’s appears to make in his arguments.

Aristotle treats both the Dichotomy and Achilles paradoxes as presenting fundamentally the same argument. Aristotle’s solution to them is split into two main sections.

Aristotle’s first discussion of the Dichotomy and Achilles paradoxes spans 233a and 233b in the *Physics*. In this argument Aristotle argues that because the time taken to travel a half of distance will be half the time taken to travel the whole distance the time taken to traverse the complete motion in the Dichotomy will always be finite because each successive run will will take proportionally less time to complete.

Aristotle revisits the Dichotomy and Achilles paradox later in the *Physics*, at 263a to 263b10, and suggests that his initial solution, the argument about the proportionality of the size of each successive run in the paradoxes, did not satisfactorily address the problem of the paradoxes.

Hugget and Dowden both claim that neither of Aristotle's solutions would satisfy a modern thinker because he does not provide a method for summing the size of each of the infinite set of runs described in the paradoxes. Hugget and Dowden both argue that the second part of his solution is a way of avoiding (and not addressing) this problem by positing that there is only ever a finite amount of divisions made of the runs (in the Dichotomy and Achilles paradoxes) at any one time and thus the sum would always be of a finite number of things and thus the sum can always be performed. The run is still considered to be infinitely divisible because, although it can only be divided a finite number of times at any one time, it can always be further divided up. This kind of infinity is called a *potential infinity* because it is infinite in regard to what it can *potentially be* – as opposed to what it *actually is*[5].

Hugget and Dowden argue that Aristotle's solution would not be seen as satisfactory by modern thinkers. The reason for this is two fold. Firstly we now have methods for summing infinite sets of values. Secondly the concept of a potential infinity is not used by modern mathematicians given that we have Set theory which allows mathematicians to work with complete infinite sets.

Aristotle identifies a related issue with infinities in *On Generation and Corruption* where he says that an extended thing cannot be completely divided an infinite number of times (316a-316b)[6]. This is related to Zeno's Dichotomy and Achilles paradoxes because both those paradoxes describe extensions, in the form of finite motions, which are infinitely divided.

Aristotle's argument is that the smaller elements which result from the complete infinite division of an extended thing are points without extension. Thus the results of a complete division of an extended thing, a set of points, cannot be said to be the constituent elements of the thing which had been divided. This is because an extended thing cannot be made of non-extended things (316b 5).

According to Hugget, this has been overcome by Set theorists. Hugget refers to an argument posed by Grünbaum (1967) to show this. The particular part of Grünbaum's argument in question proceeds as follows. Grünbaum's discussion starts from the idea that a continuous line is an "assemblage of points" where each point has no size (Grünbaum 1967 122). If this is the case then it would seem impossible that the line could have a size (Grünbaum 1967 122). This is not a problem,

5 An example of an actually infinite thing would be a collection or set which had an infinite number of elements or an extent which is infinitely large.

6 Hugget notes that this argument is considered to have originated from Zeno's plurality paradox (section 2.3 2010) which I will discuss later in of this dissertation.

according to Grünbaum, because a line is not defined (in mathematics) as a set of points. Instead a line is defined as *"the union of unextended unit point sets"* (Grünbaum 1967 126). A *unit point set* is a set which contains only one element and that element specifically being a geometric point. A *unit point set* differs from a geometric point in that it can be put into union with other *unit point sets* to define a region which has a size. It is within that region which the points are distributed, but it is the region which provides the size and not the points. Thus a line or any extension, when defined as a union of *unit point* sets, does in fact have a size. Hugget's interpretation of Grünbaum's argument is that an extension is a set of points with a size function (2010). I will suggest this is to say that an extension is a region within which the set of points are located.

I agree that Aristotle does not demonstrate how an infinite set of values can be summed and also that he does not allow for extensions to be infinitely divided up. However this is no reason to conclude that Aristotle's solution was inadequate given that we do not yet know what the paradox was intended to show[7].

I will be assuming that Aristotle's full[8] argument is entirely adequate as it stands. I will do this for two reasons. Firstly because we do not have Zeno's original Dichotomy and Achilles paradoxes to compare it against – all we have are the brief descriptions which Aristotle himself gives of the two paradoxes[9]. Secondly by assuming Aristotle does solve the paradoxes I ultimately intend to show, in the last part of this dissertation, how one might infer what Zeno might have argued from the

7 I am unsure what Hugget's view on this is because he suggests that Aristotle's solutions might have been sufficient to satisfy Zeno, but he makes no claim about the influence which Zeno has had on the development of modern mathematics (2010). As I mentioned earlier I suggest that there is no reason to ask if Aristotle's solutions satisfy modern thinkers unless one is interested in the influence which Zeno's paradoxes and Aristotle's solutions to them have had on the development of modern mathematics. I suggest that Hugget needs to be more clear as to whether or not he is claiming that Aristotle's solutions solve Zeno's original paradoxes.

8 By "Aristotle's full argument" against the Dichotomy and Achilles paradoxes I am including the second part of Aristotle's argument which I have not discussed yet.

9 I am separating the descriptions which Aristotle gives of the Dichotomy and Achilles paradoxes from the arguments (Aristotle *Physics* 239b11 and Aristotle *Physics* 239 b14) which he makes against them. While Aristotle does provide brief arguments against them after he describes them, the bulk of his counter arguments are in other parts of the *Physics*.

particular arguments which Aristotle makes against him. I will argue that Aristotle was of the view that an extension, like the run described in the paradoxes, is not fundamentally collections of smaller extensions. I will argue that Aristotle's view as to what an extension is revolves around his definition of a continuous extension. I will return to this argument in more detail after I discuss the second part of Aristotle's solution to the Dichotomy and Achilles paradoxes.

I will be further suggesting that the modern understanding of the terms *actual* and *potential* infinity to which Hugget and Dowden refer do not necessarily reflect the ways in which Aristotle was using the terms in his solution to Zeno. I will suggest that definitions which Hugget and Dowden give to these terms reflect how they were understood when Set theory was first being considered by Cantor and others who set out to show that mathematicians could work with complete infinite sets. The efforts of Cantor and other early Set theorists in convincing other mathematicians that they could work with complete infinite sets was significant in the development of modern mathematics. However, as I will argue, Aristotle might not have meant the same thing as modern set theorists do when he discussed an actual infinite set. Specifically, I will argue that Aristotle was more interested in how infinites manifest in the world and that he was of the view that there was never an actually infinitely divided thing nor were there things of actual infinite size; none the less, Aristotle required that there be an actual infinity of potential points at which an extended thing could be divided. But first I will discuss the traditional understanding of Aristotle's views on infinity.

As Hankinson notes, in the Cambridge guide to Aristotle, Aristotle is traditionally believed to have denied that there were complete (or actual) infinities (1995 140). It is suggested that Aristotle argued that an infinity was instead something which could always increase and was never complete – hence the term *potential infinity* was used because there was an infinite potential to increase but not an infinite amount or magnitude (Hankinson 1995 140). It is suggested, from this, that Aristotle was a Finitist or someone who believed that mathematics only ever work with finite numbers of things each being finite in magnitude (Hankinson 1995 140). Hugget and Dowden seem to take a similar position.

I will propose an alternative interpretation of Aristotle's arguments about infinities, *actual* and *potential*, and how he uses them to solve the Achilles and Dichotomy paradoxes. I will suggest that Aristotle was solely interested in how infinities manifested physical and temporal extensions as we encounter them in the world. I will argue that the second part of Aristotle's solution to the Dichotomy and Achilles paradoxes is the argument that there is an actual infinity of potential divisions which can be found in any given continuous extension but that the extension itself cannot be reduced to that set of divisions. I will suggest that this is similar to Grünbaum's claim that an extension is not merely a collection of points; the difference of course is that Aristotle is talking about divisions rather than points and that he does not go on to define a continuous extension as a union of *unit point sets*[10].

Even though Aristotle does not give the neat definition of a continuous extension which Grünbaum does (1967 126) he does still highlight that a continuous extension is not a collection of points and neither is it a collection of the divisions which can be made of it. Aristotle specifically criticises Zeno for confusing infinite extent with infinite divisibility (Aristotle *Physics* 233a20-25). I will suggest from this that Aristotle's main argument against Zeno was that the number of divisions which can be made of an infinitely divisible thing has no relation to that things size. Aristotle argues that an infinitely divisible extension (a continuous extension) is not fundamentally an infinite collection of smaller extensions. The fact that there is an infinite set of half distances within any distance (as the Dichotomy paradox indicates) is merely accidental property of a continuous extension and does not represent what that extension essentially is (Aristotle *Physics* 263b9-10).

But just as much as Hugget and Dowden can only speculate that modern thinkers can solve Zeno's paradoxes, I can only speculate as to whether Aristotle did solve the paradoxes and how he might have done so. There is however value in simply assuming that Aristotle did solve the paradoxes. Aristotle is much more historically proximate to Zeno and it is possible that Aristotle had access to Zeno's book which Plato alludes[11] to. Thus it is reasonable, for practical purposes, to assume that

10 Aristotle's argument in *On Generation and Corruption* from 316a-316b, discussed earlier, is expressed in terms of points. The second part of Aristotle's solution to the Dichotomy and Achilles paradoxes is expressed in terms of divisions.

11 Refer to Plato's *Parmenides.*

Aristotle did solve the paradoxes because his arguments are the most suitable, currently known[12], arguments for scholars to infer Zeno's arguments from.

An Alternative Interpretation of Zeno

The last part of my argument will be an example of how one could go about inferring what Zeno might have been trying to argue from Aristotle's argument against him in the *Physics*, or at least my interpretation of Aristotle's argument. The interpretation which I will make of Zeno is similar to the interpretation which G.E.L. Owen posed in his article *Zeno and the Mathematicians.* Owen argued that Zeno had assumed that it was problematic to assume that extension was a plurality of parts. This is specifically because if it is assumed that all extensions are composed of smaller extensions, including all of those smaller extensions, then it is impossible to locate the units of this plurality. There appears to be an infinite number of smaller extensions. If they have a finite size then the larger extension which is defined as a collection of those smaller extensions will be infinite in size. If they have no size then nothing will have size.

I will argue for this interpretation in two stages.

Firstly, I will summarise my interpretation of Aristotle's argument. I will infer Zeno's argument from the arguments which Aristotle uses against him.

Secondly, I will argue that this interpretation can be seen to reflect the Simplicius reconstruction of a part of Zeno's plurality paradox. For the sake of simplicity I will assume that Zeno's Dichotomy and Achilles paradoxes are more particular and complicated versions of Zeno's Plurality paradox.

I will finish my discussion by arguing that someone who used Hugget or Dowden's article to get a summary of what arguments are made about Zeno would be unlikely to consider the kind of interpretation of Zeno which I have discussed. This is not necessarily because Hugget and/or Dowden would not consider it a fair interpretation, but because both of these scholars insist on framing their discussion of Zeno in terms of modern mathematics.

12 Allowing for the possibility that Zeno's book might yet be located.

Chapter Two: Issues with Hugget's and Dowden's Discussions of Zeno's Paradoxes and Aristotle's Solutions

To begin with I will discuss how Hugget describes the Achilles and Dichotomy paradoxes.

Hugget discusses the Dichotomy paradox in chapter 3.1 and the Achilles paradox in chapter 3.2 of his article on Zeno. In this article Hugget's main claim appears to be that the problems that Zeno's paradoxes have raised could not be completely solved until we had developed modern mathematical methods regarding infinite summation and infinite sets (2010). I argue that this misrepresents the original paradoxes in that such modern solutions are solutions to modern problems, not those originally posed by the paradoxes.

Hugget notes that the paradoxes have given rise to new problems.

> That said, it is also the majority opinion that—with certain qualifications—Zeno's paradoxes reveal some problems that cannot be resolved without the full resources of mathematics as worked out in the Nineteenth century (and perhaps beyond). This is not (necessarily) to say that modern mathematics is required to answer any of the problems that Zeno explicitly wanted to raise; arguably Aristotle and other ancients had replies that would—or should—have satisfied Zeno. (Nor do I wish to make any particular claims about Zeno's influence on the history of mathematics.) However, as mathematics developed, and more thought was given to the paradoxes, new difficulties arose from them; these difficulties require modern mathematics for their resolution. These new difficulties arise partly in response to the evolution in our understanding of what mathematical rigor demands: solutions that would satisfy Aristotle's standards of rigor would not satisfy ours. Thus we shall push several of the paradoxes from their common sense formulations to their resolution in modern mathematics. (Another qualification: I will offer resolutions in terms of 'standard' mathematics, but other modern formulations are also capable of dealing with Zeno.) (Hugget 2010 section 1)

In this passage Hugget acknowledges that these *new problems* are not necessarily what Zeno was originally trying to show with the paradoxes and he also suggests that Aristotle perhaps satisfactory solved the original arguments. However, Hugget then goes on to claim that Aristotle's solution would no longer be considered satisfactory because it does not deal with the *new problems* which have since been found. More generally Hugget claims that Zeno's paradoxes require a modern modern mathematical solution.

It does not make sense to say that Aristotle's solutions to Zeno's paradoxes are not satisfactory because they do not answer these *new problems.* The paradoxes as posed by Zeno are *not* about problems which modern thinkers have raised. Thus, it is not Zeno's paradoxes which are being resolved by modern mathematics, as Hugget appears to suggest. Instead, the problems which are are being resolved are modern problems which have been inspired by or based on the fragments of Zeno's paradoxes.

To explain this in another way, consider Hugget's claim that "Aristotle's standards of rigor would not satisfy ours (our modern mathematical standards)". What modern mathematics demands is only relevant to the modern versions of the paradoxes. It is not relevant to the ancient paradoxes. If one requires that Aristotle's solutions to Zeno must satisfy the "rigor" of modern mathematics, then it is assumed that Zeno's paradoxes and Aristotle's solutions were simply primitive mathematical arguments. This ignores the possibility that Zeno and Aristotle were not arguing about mathematics, but instead about the constitution of extended things in the physical world. Over the course of this chapter I will be arguing this in greater detail.

Hugget's starting point for his discussion of both the Dichotomy and Achilles paradoxes is that both paradoxes can be reduced down to infinite series which sum to a finite total. This is particularly clear in the Dichotomy paradox.

To reiterate the circumstance in the Dichotomy paradox, a runner is trying to run some particular distance. Before the runner can complete that distance the runner needs to run half that distance. Hence the first term of a series is ½. Then before the runner can complete the second half of the run the runner must complete half of the remaining distance, which is ¼ of the whole distance. This series of half-runs proceeds indefinitely, The paradox can be reduced to the set of values "1/2, 1/4, 1/8, 1/16, … and so on" which in a more general form is $1/2^n$ for n=1 to infinity.

The Achilles paradox can also be reduced to an infinite series of shorter and shorter runs.

If we were to sum the terms in these series, the more terms that we include in the sum, the closer the total will get to the value of one. The sum of any finite number of terms of this series cannot exceed the value of one. Thus it is inferred that when *all* the terms have been summed the total will be 1. I will explain the reasoning behind how this infinite sum can be performed later, but first I will discuss how Aristotle initially discusses the paradoxes.

Hugget notes that Aristotle initially makes an argument similar to this (Aristotle *Physics* 233a-233b) but does not use a method of infinite summation (Hugget 2010 section 3)[13]. In a simple sense, Aristotle agues that the sum of all the divisions of a finite extension will always sum to the original finite magnitude of that extension, but he does not initially discuss summing an infinite number of them. Hugget does not discuss Aristotle's argument in much detail – his article was more a broad overview than an in-depth analysis – so I will elaborate on his line of argument. Consider the following section from Aristotle's *Physics*:

> … if time is continuous, magnitude is continuous also, inasmuch as a thing passes over half a given magnitude in half the time taken to cover the whole: in fact without qualification it passes over a less magnitude in less time; for the divisions of time and of magnitude will be the same . (Aristotle *Physics* 233a10-20)

The term "continuous" in this section refers to the idea that the extension – the distance covered in the Dichotomy and Achilles paradoxes – is infinitely divisible[14].

13 Dowden also briefly mentions that Aristotle makes an argument of this sort (2016 section 4)

14 Aristotle's definition of a continuum is more sophisticated than just that a thing is infinitely divisible. But, for the time being, all that is important is that it is infinitely divisible.

This term is used to reflect the structure of the argument used by Zeno in the Dichotomy and Achilles paradoxes. Both these paradoxes appear to involve the assumption that there is no smallest kind of division and hence that extension is infinitely divisible. This is what allows for there to be an infinite set of half distances to complete the run (in the Dichotomy paradox) or catchups for the faster runner to overtake (in the Achilles paradox).

Aristotle argues his point by splitting the problem in two. First the distances being travelled, and second the time taken to travel any particular one of these distances. In terms of the Dichotomy paradox the distances being travelled are the half-way runs and the time taken refers to the time taken to run each half-way.

The argument is that the amount of time taken to complete any of the individual half-way runs in the Dichotomy paradox is a proportion of the time taken to travel the whole finite distance in the paradox. This is because the amount of time which it takes to travel half the distance will need to be half the amount of time to travel the whole distance and similarly a lesser amount of time for the smaller portions of the run[15]. Thus each successive run, which needs to be made by the runner in the Dichotomy and Achilles paradoxes, will take less and less time to complete because each successive run is over a shorter distance than those previous. Furthermore any division of an extension when added to the remaining part or parts of that extension will sum to the size of the undivided extension.

Aristotle argues this point further by showing that it is impossible for a motion over a finite distance to take an infinite time (233a30-233b15). Aristotle's argument proceeds as follows. Suppose there is some given motion from A to B. Supposed that the distance between A and B is finite. Supposed that the time taken to complete the motion is infinite. Consider a finite portion of that infinite total time. Over that finite portion of time a finite distance[16] must have been traversed. This finite distance will

15 There is an unstated assumption, in this argument, that the runner is running at a constant speed over the course of the motion

16 This argument does not take infinitesimals into account. For instance, it might be claimed that if you consider a finite period of time of this motion, which is taking an infinite time, the distance covered would not be finite but infinitesimal. In Aristotle's defence there is arguably nothing to distinguish an object which has moved an infinitesimal distance from a stationary object.

be some portion or measure of the finite distance travelled: specifically a finite number of them will fit into the total finite distance from A to B. thus to total time taken to travel from A to B must similarly be finite because there can only be a finite number of these finite periods in this finite motion. Thus it is impossible for a moving thing travelling at a constant speed to take an infinite amount of time to complete a finite motion as is apparently the case in the Dichotomy and Achilles paradoxes.

A question that remains unanswered at this point is how can an extension have an infinite number of parts to it[17]. The reason why an extension would have an infinite number of parts is because motion as the Dichotomy paradox indicates can be divided into halves (or any other portions) indefinitely.

All that Aristotle deals with in his initial solution is why a finite extension cannot be traversed in an infinite time. He does not discuss how it is possible for an extension to have an infinite number of divisions within it.

Hugget indicates that Aristotle was not completely satisfied with this first solution and returns later to give a second solution. The section which Hugget is referring to is as follows:

> Now when we first discussed the question of motion we put forward a solution of this difficulty turning on the fact that the period of time occupied in traversing the distance contains within itself an infinite number of units: there is no absurdity, we said, in supposing the traversing of infinite distances in infinite time, and the element of infinity is present in the time no less than in the distance[18]. But, although this

17 As I will later discuss, Aristotle considered this to be a problem in *On Generation and Corruption* 316a-316b

18 This passage is the summary of the argument at 233a 20-35 in the *Physics*. When Aristotle suggest that infinite distances will be traversed in infinite time he is making the claim: that if you divide up the distance travelled of a motion into infinitely many parts then there will be corresponding set of infinite divisions of the time taken to traverse the distance which when summed will equal the original period of time to complete the travel (Aristotle 233a 20-35). The important point is that when Aristotle talks about traversing the infinite distances in infinite time he is talking about an infinite set of divisions not an infinite period or extension of time.

> solution is adequate as a reply to the questioner (the question asked being whether it is possible in a finite time to traverse or reckon an infinite number of units), nevertheless as an account of the fact and explanation of its true nature it is inadequate. (Aristotle *Physics* 263a10-20)

On initial reading of Hugget's article It appears that Hugget is claiming that Aristotle revisits the Dichotomy and Achilles paradoxes specifically to solve the problem with Cauchy's method of infinite summation.

The idea is that Aristotle took a common sense view that an infinite collection of finite magnitudes would always sum to an infinite value. To use one of Hugget's examples "1+1+1+...", would equal an infinite magnitude (2010 section 2.2) [19]. Given that the partial runs in the Achilles and Dichotomy are all finite in length then common sense would indicate that the total run, in each paradox, would need to be infinite in length.

Of course a set of values like "1+1+1+..." does not describe the size of the smaller runs in either the Dichotomy or Achilles paradoxes. Aristotle appears to acknowledge this when he argues that the time taken to complete each successive run in the Dichotomy would halve each time (*Physics* 233a-233b). What Aristotle does not use in his argument however is the notion that the terms in the series approach zero and that the sum of those terms approach a particular finite value. This is the sort of reasoning which Hugget is referring to when he alludes to Cauchy's method of infinite summation.

The idea that an infinite set of values can sum to a finite value can be seen to relate to the Dichotomy paradox as follows. The lengths of the half runs in the Dichotomy paradox can be expressed as $1/2^n$ for n=1 to infinity. For instance, the sixth run which the runner makes is $1/(2^6)$ or 1/64th of the total original finite distance being run. If we were to sum up the terms in the series the total will get very close to 1 but never exceed it.

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19 This example is actually used to discuss one of Zeno's plurality paradoxes, but I suggest that it is applicable in this discussion of the Dichotomy and Achilles paradoxes.

Let us assume that the distance being run by the runner in the Dichotomy paradox is 1 unit of distance. In terms of the situation of the Dichotomy paradox, the runner runs half the distance to begin with. So the first term of our sum is ½. After this the runner then runs half of the remaining half which is ¼ of the whole run. Our sum so far is ½ + ¼. No matter how many of the half runs we include in the sum the total will never be greater than 1. For instance ½ + ¼ … + 1/1024 = 0.9990234375[20]. From this it is inferred that because the sum of *any conceivable finite set of the terms* within this series will never exceed 1, then the sum of *all of the terms* must be 1. Thus, despite the apparent infinite number of smaller motions involved in the motion, the entire motion is still very much finite. Another way to express this is to say that 1 is the *limit* of the infinite sum: $1/(2^n)$ for n=1 to infinity.

I would not suggest that Hugget is claiming that Cauchy's method of infinite summation solves Zeno's original paradox and that Aristotle's argument does not. However I would suggest that because Hugget frames his discussion of Aristotle's solution in direct comparison to Cauchy's method of infinite summation a reader would be left with the impression that Zeno's paradoxes and Aristotle's solutions are merely examples of arguments from people who lacked a method for infinite summation.

Because the original purpose of Zeno's paradoxes is unknown it is not helpful to compare modern solutions with ancient solutions in this way. It is entirely possible that Zeno's Dichotomy and Achilles paradoxes and Cauchy's method of infinite summation pertain to different problems. The reason I say this is that if the Dichotomy is treated as simply the fallacious claim that the sum of the infinite series $1/(2^n)$ for n=1 to infinity equals infinity, then the physical aspect of the paradox, the runner and the distance run, become irrelevant. Taking this kind of approach to the paradox reduces it to a simple arithmetical problem. While I would not rule out the possibility that Zeno might have been making this argument, I would suggest that his argument was more about the physical circumstance.

If it were the case that Zeno was simply making the fallacious statement that an infinite series of finite values can never sum to a finite total, then it would be fair to

20 From mathematics, the sum of n half runs is $1-1/2^n$.

say that Aristotle does not really demonstrate why that statement is wrong. However there are alternative possibilities for what Zeno might have been arguing.

Dowden in his article seems even more dismissive than Hugget of the first part of Aristotle's solution to the Dichotomy paradox. He states that:

> However, Aristotle merely asserted this [it is possible to traverse an infinite number of smaller distances in a finite period of time or that the sum of the divisions of a finite extent would always equal the original finite magnitude] and could give no detailed theory that enables the computation of the finite amount of time. So, Aristotle could not really defend his diagnosis of Zeno's error. Today the calculus is used to provide the Standard Solution with that detailed theory.
> (Dowden 2016 section 3.a.ii)

From this statement I suggest that a reader could be forgiven for thinking that Dowden is of the view that Aristotle could not really solve the Dichotomy paradox. Dowden however does assure the reader earlier in his article that he would refrain from entering into a debate about whether or not Aristotle did solve Zeno's paradoxes (2016 section 2).

I suggest however that Dowden did not even need to make this comparison between the methods employed in Calculus and Aristotle's solution to the Dichotomy. Again, this is because the purposes of Zeno's paradoxes are not known. The fact that Aristotle cannot sum infinite sets of things or, as I will discuss next, uses the concept of a potential infinity which is no longer used in modern mathematics, need not imply that he did not solve the Dichotomy paradox.

What I have not yet discussed is what Aristotle's second argument against the Dichotomy and Achilles paradox is (Aristotle *Physics* 263a-263b10). What I hope to have shown thus far however is that Hugget's and Dowden's articles give the impression that Aristotle returned to the paradoxes because he needed to provide a way of getting around the fact that he lacked a method for summing the infinite set of smaller runs which take place in the Achilles and Dichotomy paradoxes. Hugget or

Dowden might not have intended to argue this, but I suggest that the fact that they insist on comparing Aristotle's methods to modern methods and pointing out why we would not be satisfied with Aristotle's methods, does leave the reader with this impression.

Dowden discusses the second part of Aristotle's solution to the Dichotomy and Achilles paradoxes in section 4 of his article. Dowden's interpretation of Aristotle's solution to the Dichotomy and Achilles paradoxes is summarised in the following passage from his article:

> Aristotle believes it is impossible for a thing to pass over an *actually infinite* number of things in a finite time, but that it is possible for a thing to pass over a *potentially infinite* number of things in a finite time. (Dowden 2016 section 4)

Dowden notes that the term potential infinity is normally interpreted to specifically refer to a set of things which is being continually added to (Dowden 2016 section 4). An actual infinity on the other hand is normally interpreted to be a complete infinite set (Dowden 2016 section 4).

What this would imply is that Aristotle was saying that the motions which Zeno describes can be continually further divided up, but are not ever divided up the infinite number of times that the wording of the paradoxes would imply. This in turn means that there is only a finite number of distances which the runners in the paradoxes have to travel and they are all of a distinct finite size.

This avoids infinite summation by requiring that there is never an infinite number of things to sum. This does not mean that Aristotle had specifically set out to avoid performing infinite summation. The reason that I mention it is because as I previously indicated Dowden states that Aristotle was unable to defend his claim against Zeno because he lacked modern mathematical methods (Dowden 2016 section 3.a.ii). This, I suggest, would appear to imply that Zeno's paradoxes require a method for infinite summation to solve them and this, in turn, would imply that Zeno's paradoxes revolve around the idea of infinite summation.

This interpretation of potential and actual infinity assumed by Dowden is not an uncommon interpretation. For instance, it is equivalent to what Hankinson argues in the *Cambridge Companion to Aristotle* (1995 140). Hankinson argues that Aristotle was a Finitist in the sense that he rejects the idea that there are actualised infinite sets of things: specifically that he rejects that there is an actual infinite set of natural numbers[21]. Aristotle's view is instead that there are potential infinities. With regards to a set of things, a potential infinity is a set which can always have more added to it or more divisions made of it (Hankinson 1995 141). For instance, the run in the Dichotomy paradox can always be further divided up, but can never be completely divided up.

I will now briefly discuss what parts of Aristotle's arguments Hankinson is building his argument from. Consider the following passages:

> Number on the other hand is a plurality of 'ones' and a certain quantity of them. Hence number must stop at the indivisible: for 'two' and 'three' are merely derivative terms, and so with each of the other numbers. But in the direction of largeness it is always possible to think of a larger number: for the number of times a magnitude can be bisected is infinite. Hence this infinite is potential, never actual...
> (Aristotle *Physics* 207b5-10)

> ...they [mathematicians] do not need the infinite and do not use it. They postulate only that the finite straight line may be produced as far as they wish... (Aristotle *Physics* 207b30)

From these passages it would seem reasonable to say firstly that Aristotle considered infinity to be something which was continually added to rather than complete, and secondly that he did not believe mathematicians had need of infinities beyond this potential understanding of it.

21 Natural numbers are positive integers: like 1,2,3,4 and so on.

The beginning of this passage seems to indicate, as Hankinson suggests (1995 140), that Aristotle considers that there are only finite sets of numbers where the larger finite sets are derived as and when they are needed. I will return to Aristotle's understanding of infinities later, but first I will return to Dowden's discussion of Aristotle's solution to the Dichotomy and Achilles paradoxes.

With regards to the Dichotomy paradox, Dowden's interpretations of Aristotle's argument is that the number of divisions of the run which are being added together will always be finite. By virtue of this it always is possible to sum up the size of each smaller run made. For instance, consider the first three runs of the in the dichotomy plus the remainder: presume that the runner run has completed ½, ¼ and 1/8 of the run and has one more 1/8 of the run to complete. Sum up all of these elements and you will end up with the size of the whole finite run. No matter how many of the possible runs are considered, the sum of the distance run, plus the remainder, will always be finite.

Dowden does not explicitly discuss precisely what I have just described, but I believe that he is suggesting that Aristotle used this kind of argument against the Dichotomy and Achilles paradoxes (Dowden 2016 section 4).

However Dowden again adds a comparison between Aristotle's argument and modern arguments as can be seen in the following statement:

> Today's standard treatment of the Achilles paradox disagrees with Aristotle's way out of the paradox and says Zeno was correct to use the concept of a completed infinity and to imply the runner must go to an actual infinity of places in a finite time. (Dowden 2016 section 4)

Identifying that Aristotle uses methods which modern thinkers would not is fine. However, claiming that Aristotle did not solve Zeno's paradoxes as a consequence of this is unreasonable. We do not really know what Zeno originally was trying to argue. Thus modern arguments about Zeno should be treated as different arguments.

Hugget provides a slightly different way of interpreting Aristotle's argument. The claim is that size of an extension is not a result of the addition of a collection of actual divisions of that extension. Zeno's paradoxes appear to imply that in the Achilles and Dichotomy paradoxes, a runner would need to complete an infinite number of separate tasks. Hugget suggests that Aristotle argues that Zeno's description does not represent the task which a runner would actually need to undertake in order to move any given distance (Hugget 2010 section 3.1). The claim is that if it were to be actually divided up, then the extension would be intermittent or discontinuous.

Aristotle's reasoning is that if you actually divided a thing into completely separate segments then the end of one segment is separate from the start of its subsequent. What this means for the runner in the Dichotomy paradox is that the end of one of the half runs will be distinct from the start of the next, and will not strictly follow continuously from the previous. Thus, to the extent that there are divisions in a continuous extent, those divisions are are potentials which are generated as and when they are needed.

For an example of this argument, imagine that Zeno is a photographer taking photographs of Achilles racing the against the tortoise (or slower runner) as per the Achilles paradox. Presume that Zeno, being who he is, decides to take a photo at the beginning of every one of the infinite sequence of catchups described in the paradox. Unless Zeno has impossibly fast reflexes and some kind of magical camera which can function faster than the speed of light, this task is impossible[22]. However, Achilles does not need to complete a task of this nature when he runs; even if his run can be potentially divided up in this way. When Achilles runs he would not need to, for instance, carry a camera on a stick and photograph himself at each assigned moment – he would just run the finite distance required to overtake the tortoise.

Dowden also briefly refers to some similar ways of challenging Zeno's Achilles paradox. For instance: "Achilles' feet aren't obligated to stop and start again at each [of the catchup] location[s]"; Zeno is unreasonable in requiring that Achilles

22 This kind of task is known as a super task and is discussed at length by Max Black (1950)

continually aims to complete each run behind the tortoise (Dowden 2016 section 3.a.i).

Largely I agree with Hugget's interpretation of this part of Aristotle's solution. However, like Dowden, Hugget compares Aristotle's solutions with modern solutions. Hugget suggests that a modern thinker would not be satisfied with Aristotle's solution because it makes use of the concept of a potential infinite. His argument is that because we now have Set theory from Cantor, a modern thinker would appear to not require potential infinities because Set theory allows mathematicians to work directly with complete infinite sets (Hugget 2010 section 3.1).

Again, comparing Aristotle's solutions to modern solutions is an unhelpful comparison to make. It involves the assumption that Zeno was making a purely mathematical argument. Likewise it involves the assumption that Aristotle's arguments against Zeno were mathematical arguments. In contrast, I suggest that both Aristotle and Zeno were arguing about how infinities manifest in the physical world.

This problem is particularly noticeable when Hugget discusses Grünbaum's modern solutions to Zeno's paradoxes. Hugget discusses Grünbaum in relation to an argument which Aristotle makes in *On Generation and Corruption*. The reason why Hugget is discussing this argument is because it is considered to have originated from Zeno (2010 section 2.3). The arguments is as follows:

> Since, therefore, the body is divisible through and through, let it have been divided. What, then, will remain? A magnitude? No: that is impossible, since then there will be something not divided, whereas *ex hypothesi* the body was divisible *through and through*. But if it be admitted that neither a body nor a magnitude will remain, and yet division is to take place, the constituents of the body will *either* be points (i.e. without magnitudes) *or* absolutely nothing.
> (Aristotle *On Generation and Corruption* 316a 20-30)

In summary the argument being made here is that if an extended thing is infinitely divisible then one should be able to divide it up completely. The issue is that if an extension is completely divided up the resulting bits cannot be extended. If they are extended then this means that the division was not complete. Thus the end results of these divisions can only be geometric points which have no size or extension in their own right. This appears to imply that extended things are composed of non-extended things. This, according to Aristotle, does not make sense.

Aristotle uses his concept of a potential infinity to overcome this problem and I will discuss how he uses this later. First, however, I will discuss Grünbaum's argument, which Hugget is suggesting solves this problem.

Grünbaum describes the problem as being: if a line is a set of points, and points are are not extended, then how can the line have a length (Grünbaum 1967 121-125). This problem can be seen to be similar to Aristotle’s problem, differing in that Grünbaum is using a line as a particular example. Grünbaum’s description of the problem is more sophisticated and thorough than I have depicted it here, but this fairly accurately summarises the basic problem. It is, more or less, the concern that Aristotle had that an extended thing could not be composed of non-extended things (316b 5).

Grünbaum’s solution to this issue revolves around the way a *set* is defined. According to Grünbaum, extension is a property of *sets* rather than of the *points* which lie in the *sets.* Also, an extension, such as a line, is a set of *unit point sets* rather than a set of points. A *unit point set* is a set which has one point in it. Grünbaum’s argument is that a continuous extension is not “a set of points” but rather the “union of unextended unit point sets”(1967 125-126).

One of the most significant aspects of this definition is that a single “unit point set” is not extended, but a union of them is extended. Grünbaum gives the following analogy. Temperature does not apply to single molecules but does apply to a collection of molecules (Grünbaum 1967 125).

It is not immediately clear, to a novice of set theory such as myself, why a union of unit sets can become extended where a set of points cannot. I am speculating that a point is being treated like a coordinate position, and a set is being treated like the region in which the point is located. Thus the basis for the extension is the region which the points occupy, and this is the sense in which a continuous extension is a union of unit sets.

Also, for a collection of unit sets to be extended there presumably needs to be an infinite amount of them. This is because the points within them will need to be *densely ordered* for the extension to be continuous. *Densely ordered,* in this particular context refers the to a situation where between any two points there are always other points (Grünbaum 1967 37-38).

The argument which Grünbaum makes, however, does not necessarily mean that Aristotle did not adequately solve this problem of complete divisibility, which incidentally is also implied in both the Dichotomy and Achilles paradoxes. I suggest that the primary argument which Aristotle was making against Zeno was that the number of divisions which can be made of an infinitely divisible extension has no relation to the size of that extension. Another way of stating this is to say that an infinitely divisible extended thing cannot be reduced to the set of divisions which can be made of it.

I suggest that Aristotle uses the term potential infinity primarily to identify that the divisions that are made of continuous extended things are not actual elements of that continuous thing but are merely potential things which can come to be as a result of dividing that extension. I will more broadly be arguing that Aristotle was only interested in how infinities manifested in the world. I will suggest that when Aristotle denies actualised infinities he is specifically denying that any physical or temporal extensions that we encounter could either be infinite in size or completely infinitely divided. This does not, however, mean that there are no actual infinite amounts or magnitudes, because for a thing to be infinitely divisible there would need to be an actual infinity of potential ways of dividing that things up.

In terms of the Dichotomy paradox there is an infinite number of potential smaller runs which one can identify within the complete run. However the complete run is not merely this collection of conceivable smaller runs. Aristotle demonstrates this by arguing that if you completely divide an infinitely divisible extension, the results would be a collection of non-extended points. Thus Aristotle infers that the divisions of an extension, for instance the half runs in the Dichotomy paradox, must only be an accidental and potential result of the extension rather than what the extension actually or essentially is.

To begin with, I re-evaluate Aristotle arguments about infinities. Specifically, I discuss the passages of Aristotle's argument in which he uses the terms potential and actual with regard to infinities to see if there are any alternative ways of interpreting the terms from those discussed by Hugget and Dowden (and Hankinson (1995) for that matter). I suggest that Aristotle is only interested in how infinities manifest in physical and temporal extensions. There are two senses in which Aristotle calls something infinite: This is firstly with regards to how big an extension is, the second sense is with regard to how many times it can be divided (Aristotle *Physics* 233a20-30).

Firstly Aristotle denies that an extension can be infinitely large in his arguments from 206a to 208a (end of book iv). Consider the follow passages:

> What is continuous is divided *ad infinitum*, but there is no infinite in the direction of increase. For the size which it can potentially be, it can also actually be. Hence since no sensible magnitude is infinite, it is impossible to exceed every assigned magnitude; for if it were possible there would be something bigger than the heavens.(Aristotle *Physics* 207b15)

I suggest that Aristotle's argument at this point is that while extensions are infinitely divisible they cannot have an infinite size. The reason that they cannot have an infinite size is that this would imply that there is something bigger than the heavens. Whether or not the premise that there would be something bigger than the heavens leads to the conclusion which Aristotle comes to, is not of concern here.

I suggest that it is important to note that Aristotle is interested in specifically *sensible* magnitudes. I suggest that this indicates that Aristotle was talking about infinities of amounts of actual physical or temporal things rather than just numerical values.

Consider another passage, which I discussed earlier:

> Number on the other hand is a plurality of ‘ones’ and a certain quantity of them. Hence number must stop at the indivisible: for ‘two’ and ‘three’ are merely derivative terms, and so with each of the other numbers. But in the direction of largeness it always possible to think of a larger number: for the number of times a magnitude can be bisected is infinite. (Aristotle *Physics* 207b5-10)

Again, to the extent that infinity is mentioned, Aristotle is only interested in the way that infinity manifests or comes-to-be in the physical world. In this particular case, Aristotle derives an infinity from the act of dividing up a continuous extension into halves and the halves in to halves again and so forth. Thus I suggest that the term “actual” in actual infinity refers to whether or not the individual elements of the infinity are actualised in the physical word not to whether there is an actual infinite number of them.

The possibility that Aristotle was only interested in how infinities manifest in the physical world is overlooked because Aristotle’s discussion of infinities is often described in the context of how early set theorists like Cantor struggled to convince those around them that mathematicians could work with complete infinite sets[23]. Dauben summarises the kind of arguments which were being made at this time in his book *Georg Cantor: His Mathematics and Philosophy of the Infinite* (1990 122-123).

According to Dauben, the primary concern that opponents of Cantor had was that introducing an infinite term into a mathematical equation would annihilate the value of the finite mathematical terms in that equation (1990 122-123). For instance if you added infinity to the equation 1+2=3 the result would be 1+2+infinity=3+infinity

23 As I have previously mentioned, both Hugget (2010) and Dowden (2016) discuss Cantor's Set theory when they discuss Aristotle’s views on infinity.

which is equivalent to infinity=infinity. The finite constants 1, 2 and 3 become redundant when infinity is introduced. However, this kind of infinity is not what would be called an infinite set, instead it is what would be called an infinite magnitude. Instead, the kind of infinity which Cantor was interested in was more like the infinite set of points. An example of an infinite set would be the resulting bits which arise from the result of completely dividing an extension which Aristotle discusses in *On Generation and Corruption* (316a-316b).

I am not going to discuss the fundamental arguments which Cantor made about infinites, but I do suggest that Cantor was primarily interested in showing that it is meaningful to talk about sets of things which contain an infinite number of elements. I suggest that Cantor would be of the view that there is an actual or complete infinite number of points in a continuous extension. For instance, there is an infinite number of points along a continuous line.

I suggest, contrary to the standard interpretation which Hankinson discusses (1995 140), that Aristotle would agree that there is an infinite number of points in an extension. What Aristotle denies is that an extension *is* that set of points. This can be seen in the following passage:

> …a line cannot be composed of points, the line being continuous and the point indivisible. (Aristotle *Physics* 231a25)

Notice that Aristotle does not deny that there are points in a line. Instead he denies that a line is composed of those points. I appreciate that Aristotle does not mention the amount of points in the line but the amount of points plays no part in Aristotle's argument as to why a line cannot be composed of points. The reason why this cannot be the case is best summed up when Aristotle states that an extended thing cannot be composed of non-extended things (*On Generation and Corruption* 316b 5). I appreciate that these two passages come from completely different parts of Aristotle's arguments but I suggest that they are both referring to the same overall argument.

Lets return to the solution which Aristotle gives for the Dichotomy and Achilles paradoxes. Consider this passage from Aristotle's argument:

> Therefore to the question whether it is possible to pass through an infinite number of units either of time or of distance we must reply that in a sense it is and in a sense it is not. If the units are actual, it is not possible: if they are potential, it is possible. (Aristotle *Physics* 263b1-10)

As I discussed earlier in this paper, Dowden suggests that when Aristotle uses the term potential in this argument he is suggesting that the number of smaller runs that can ever be considered at any one time will always be finite and thus the size can always be determined because there will only ever be a finite number of things to be added up (Dowden 2016 section 4).

An alternative possibility is that Aristotle was suggesting that the infinite set of smaller runs (or units) are each potential in nature. What I mean by this is that the smaller runs in the Dichotomy are potential things which can come-to-be and pass-away as a result of the whole continuous motion, but that they are not what that motion actually is. I suggest that my interpretation is reflected in this passage from Aristotle's argument:

> For in the course of a continuous motion the traveller has traversed an infinite number of units in an accidental sense but not in an unqualified sense: for though it is an accidental characteristic of the distance to be an infinite number of half-distances, this is not its real and essential character. (Aristotle *Physics* 263b5-10)

Another way of putting this argument is to say that the smoothness, or infinite divisibility, of a continuous extension is *not* a result of it being an actual infinite collection of infinitely small bits. When we divide an extension we are not simply identifying a particular collection of bits within that extension. Instead, a continuous extension exhibits an actual infinity of potential divisions. These divisions come-to-be and pass-away in the extension. However, it is never the case that all of the possible ways of dividing the extension would simultaneously come-to-be. This is to say that an extension is never completely divided. If it were to be completely divided then it would be reduced to a set of non-extended points. But extension cannot be a set of non-extended points[24].

What I have just posed is one way of interpreting Aristotle's argument. I have set out to consider the possibility that modern mathematics is not required to solve the problems which Zeno was originally trying to show with the Dichotomy and Achilles Paradoxes. The reason I have avoided relying on modern methods is that Aristotle is likely to have had a better understanding of what Zeno was trying to argue.

24 And if Grünbaum's argument (1967 125) is any indication, modern mathematicians would similarly not claim that an extension can be defined merely as a collection of points. Grünbaum argued that the size of an extension was defined by the union of the unit point sets which defined the region which the points are distributed through (1967 125-126).

Chapter Three:
An Interpretation of Zeno which is Obscured by a Modern Mathematical Approach

I will now consider the question of what Zeno might have been trying to show with the Achilles and Dichotomy paradoxes. My interpretation of Zeno is inspired by an interpretation that was posed by G.E.L Owen (1957-8) and I will discuss the similarities later in the chapter. I will assume that my interpretation of Aristotle is correct and that Aristotle does satisfactorily solve the problem that Zeno was trying to show.

I will suggest that Zeno's concern was that extensions appear to have many parts and those parts likewise appeared to have many parts. The issue is that if you define an extension as simply a collection of the parts within it, then either all extensions would be infinitely large because they are composed of an infinite collection of parts which have a size, or if the parts have no size then the extensions which are composed of them would also have no size. I am suggesting that the primary assumption in this argument is that an extension is merely a collection of smaller parts.

I will now briefly summarise my interpretation of Aristotle's arguments against Zeno's Dichotomy and Achilles paradoxes, which I discussed in the previous chapter. I will then discuss what one might infer from this interpretation, about Zeno's original arguments.

In his first solution (*Physics* 233a-233b) Aristotle argued that the size of each of the runs, in each respective paradox, would be defined as proportion of the whole run. This means that the smaller runs take proportionally less time to complete such that the total time taken to complete the runs would be finite.

In his second argument (*Physics* 263a-263b10) Aristotle suggests that his initial solution was not sufficient because he did not explain how there could be an infinite number of smaller runs within the motion: all he argued was that the size of the total run would remain finite no matter how it was divided. I suggested that Aristotle argued that continuous extensions, like the runs described in the paradoxes, are never actually divided. Another way of putting this is to say that an extension is not a collection of divisions. Divisions are *potential* in the sense that they can come-to-be and pass-away within the extension but do not represent what the extension is composed of.

As for what this might imply about Zeno's paradox, Aristotle first identifies that Zeno is ignoring the concept of proportionality. Zeno does not seem to recognise that half of a run would take half the time the whole run takes to complete.

I suggest that Aristotle returns to the paradoxes because he realises that Zeno might have been treating the divisions of an extension as *actual* parts of that extension rather than *potential* parts. Based on the second argument that Aristotle makes against Zeno's Dichotomy and Achilles paradoxes, it appears that Aristotle was of the view that Zeno believed that an extension was fundamentally the collection of smaller extensions which could be found within it. The problem is that there is no identifiable set of extended parts that can define what that extension *actually* is, or in other words, that define that extension's particular size.

I argue that this the idea, that Zeno was assuming that extensions were fundamentally collections of the smaller extensions which could be found within them, is reflected in another one of Zeno's arguments. Specifically, I suggest that this is reflected by an argument which is attributed to Zeno by Simplicius:

> …if there exists [many things], each thing must have some size and solidity, and one part must stand out from the other. And the same consideration applies to what projects from this. For it will have size and a part will project from it. And it is the same to say this once and always. For there will be nothing which can serve as a final part of this nor will one part be different from another. Thus if there are many things, they must be both small and great: so small as to have no size, so great as to be unlimited. (Simplicius *Physics* 140.34-141.8)

The argument made here identifies what consequences follow if it is asserted that there are many things, specifically if extensions are considered to be pluralities. I will step through each part of this fragment in sequence to explain my interpretation of it.

The reasoning behind the claim that each, of the many, must have some "size and solidity" is discussed in a different fragment. Simplicius reports that Zeno was arguing that if a thing had no size then if it were added to something it would make no difference to that thing; furthermore if the thing which it is added to is then removed we would be left with nothing (Simplicius *Physics 1*39.5-19 B2). This would indicate that Zeno is only interested in plurality with respect to extended things or things with a size. An extended thing cannot exist if it does not have a size thus it would follow that the *many extended things that exist* all have a size[25].

The part of the passage " ...and one part must stand out from the other … " I suggest refers to the idea that an extended thing will have distinguishable parts to it which exist in their own right. For an example one end of a table is distinct from the other end of a table. This line of argument is then followed up by : "… And the same consideration applies to what project from this. For it will have size and a part will project from it …". Consider this again in terms of my table analogy. It has been established that the front of the table is distinct from the back, or that they are not the same thing. Likewise the *front* and *back* of the table, themselves each have a distinct *front* and *back* respectively.

I suggest that this passage might also indicate that Zeno is treating an extension as a collection of smaller extensions.

The following passage is less clear and more difficult to interpret: "there will be nothing which can serve as a final part of this nor will one part be different from another". I suggest that Zeno's argument was that there is no final or ultimate set of parts of a thing which a person can find. Also because this property of *having an*

25 This is similar to Aristotle's claim that an extension cannot be composed of non-extended parts (*On Generation and Corruption* 316a). As I mentioned earlier, Hugget points out (2010 section 2.3) that the arguments which Aristotle makes in this particular section of *On Generation and Corruption* (316a-316b) are considered to have originated from Zeno.

indefinite number of parts is a property of all extension then to the extent we could talk about the ultimate parts of every thing they would all be the same size. This is because if we were to consider two parts of different size, the larger part would still be further divisible in to more sub parts. Thus I would suggest that Zeno is arguing that it is impossible for there to exist an ultimate set of divisions of an extension, because there will always be more to make. However to the extent that we could talk about an ultimate set of division of an extension, they would all be the same size.

And now the final part of the argument: “Thus if there are many things they must be both small and great: so small as to have no size and so great as to be unlimited”. I suggest that the argument here is to claim that the units of this plurality of extension would be indefinitely small and there would be indefinitely many of them. The “so great as to be unlimited” is more normally interpreted to say that the size of any given extension is infinite because there are an infinite number of finite extents within every extension (Graham 2000 267).

Overall, I suggest that Zeno had assumed that if there were many parts to an extension, then it would follow that an extension was simply a collection of parts. The issue is that all extensions, including the parts of extension, have parts. Thus an extension could not be a plurality of smaller extensions because there was no ultimate, or smallest set, of extended parts. I have inferred this from the fact that I have interpreted Aristotle’s primary criticism of Zeno as being that an extension is not fundamentally the collection of the divisions which can be made of that extension.

This is a very similar, if not the same, kind of interpretation as that which G.E.L. Owen made of Zeno’s paradoxes in his article Zeno and the Mathematicians (1957-8). Owen had suggested that Zeno’s argument was that it was impossible to locate the units of a plurality (1957-8). For example in the Dichotomy paradox the runner sets out to run some distance but before they can traverse that distance they will need to traverse some part of that distance. However it is impossible to identify the first distance which needs to be travelled because for any distance considered there will always be another smaller distance which precedes it. The run therefore cannot be a plurality of smaller runs because it is impossible to determine what the individual units of that plurality would be.

Of course, there is a plurality of shorter runs in any given finite run in the same sense that "1= ¼ +¼ +¼ +¼". However the run itself is not *actually* a plurality of some set of shorter runs within it. This is because any particular one of the shorter runs, within the larger run, is also a plurality of shorter runs, and, by virtue of this, cannot be treated as a constituent part of the run. This is because an extended thing cannot be *composed* of pluralities which are themselves composed of pluralities ad infinitum. If this were the case and that extended thing was reduced to its completely divided form, it would be reduced to the infinite set of non-extended point positions which lie within the extension. By themselves, these points are not what the extension actually is. This is because an extension cannot be composed of non-extended things.

What I have discussed so far in this chapter is just an example of how one could go about using Aristotle's arguments against Zeno, independent of modern mathematical arguments and concepts, to construct an interpretation of Zeno.

Conclusion

What I hope to have shown in this dissertation is that comparing Zeno's paradoxes, and Aristotle's solutions to them, to modern solutions is not helpful for those trying to determine Zeno's original purpose. Firstly, because we are not completely sure of what Zeno was originally trying to argue, to the extent we discuss the paradoxes, I would suggest that we should refrain from trying to solve them. Instead we should primarily stick to trying to infer what he was arguing from the solutions given by other ancient philosophers. This is because what little we have of Zeno's arguments does not provide enough for one to be certain what he was trying to argue with the paradoxes.

Secondly, introducing modern solutions into the discussion of Zeno encourages people to treat Zeno's works as if they are directly aimed at modern mathematics. For instance, the size of each successive run in the Dichotomy paradox can be represented by a particular infinite series. This particular infinite series happens to sum to a finite magnitude. However, if the paradox is explained in these terms, then the physical aspect of the circumstance which Zeno discusses becomes irrelevant and the problem simply becomes one of adding up abstract magnitudes. I hope to have shown in the first chapter that Aristotle appears to first consider the Dichotomy and Achilles paradoxes in a similar way but then later decides that he might have missed Zeno's point. Aristotle initially argued that no matter how many times you divide an extension, because the size of each division is defined as a proportion of the whole, the sum of the size of all of those divisions would always equal the original size of the whole. While Aristotle does not use the idea of infinite summation, his initial solution is about the more abstract concept of proportionality and specifically that Zeno appears to be ignoring that concept.

The fact that Aristotle returns to the problem, claims that his initial solution was not sufficient, and then proceeds to argue that an extension is not actually divided or is not composed of actual divisions, would indicate that he believed that Zeno might have been making an argument about physical and temporal extension.

From this I suggest that to the extent that Zeno talked about addition of magnitudes, he would have been referring to the act of summing up extended things. I suggest that Zeno's main concern is that because physical and temporal extensions appear to be infinitely divisible, it is hard to see what the parts of these extensions could be. This involves the assumption that an extension is a collection of smaller extensions and that the size of an extension arises when the size of all of its parts are summed up. If this is the case then several problems appear to follow. Firstly, the smallest parts cannot be extended, because this would imply that there are smaller parts yet to be identified. An extension cannot be a collection of non-extended things. Even if it is allowed that the smallest parts have a size, there would be an infinite number of them. This would mean that the size of any given extension would be infinite because it is a collection of infinitely many extended bits.

I have inferred this interpretation of Zeno from my interpretation of Aristotle's solution. Specifically I argue that Aristotle had set out to show that an extension is neither the set of divisions which can be made of it, nor the set of points which can be found within it. I argue that Aristotle is of the view that the divisions which can be made of a continuous extension and the points at which those divisions are made do not represent what the extension actually is. Instead the divisions and points are things which can potentially be by virtue of there being a continuous extension. For instance a division can only exist as a division *of an extension* and a point can only exist as *a position within an extension*. In Aristotle's terminology there is an infinity of potential divisions which can be made of a continuous extension. More generally I argue that Aristotle is of the view that there cannot be infinities of *actual things* or *actual things* which are infinitely large.

This interpretation differs from the standard interpretation of Aristotle's solution. Usually Aristotle is seen to have denied that there are complete infinities. When Aristotle uses the term potential to describe an infinity it is more usually seen as indicating that an infinity is something which is finite but can always be added to, increased, or alternatively further sub divided. This is not an unreasonable interpretation, given that Aristotle also suggests that mathematicians have no need for complete or actual infinite sets.

The reason why I suggest my alternative interpretation is because on the occasions where Aristotle talks about infinites he discusses them in terms of physical or temporal things. Again my argument is that what Aristotle denies is that an extension is an infinite number of actual parts and also that there can be actual extensions of infinite size.

When Hugget and Dowden discuss Aristotle's solutions to Zeno's paradoxes they insist on pointing out that modern mathematicians would not use the idea of potentiality when discussing infinities. When they discuss this they are assuming the more standard interpretation of Aristotle where he is assumed to be arguing that a potential infinity is a finite set which can always be added to. Hugget and Dowden would have no reason to consider my interpretation given that they appear to be discussing Zeno and Aristotle primarily in mathematical terms and modern mathematics has a particular way of imagining infinitely small divisions. My suggestion that Aristotle was of the view that the elements of an infinity must always be potential in nature is not relevant to mathematics. My suggestion would indicate that Aristotle was posing a physics argument rather than a mathematical argument. My argument is that Aristotle was not arguing about numbers or amounts, but about the constitution of extended things.

Thus if it is possible that Aristotle's argument against Zeno was an argument about the constitution of extension, then it is also possible that Zeno was originally trying to pose an argument about the constitution of extension. If I am right in my assertion that Aristotle was claiming that an extension is *not* a collection of smaller extensions, then it is possible that Zeno had assumed that an extension was a collection of smaller extensions.

The possibility of interpreting Zeno and Aristotle in this way is obscured by the ways scholars like Hugget and Dowden insist on comparing Zeno and Aristotle's arguments to modern mathematical arguments. This is not to say that the modern mathematical arguments, which are based on, or inspired by, Zeno's paradoxes, should not be discussed. Rather, these discussions should be clearly separated from discussions about what Zeno might have originally intended to show with his paradoxes.

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