Investigations of the Temkin-Poet Model for Electron-Hydrogen Iso-Electronic Series Scattering

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to the Degree of Doctor of Philosophy
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by

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Declaration

I declare that this thesis is my account of my research and contains as its main content work which has not previously been submitted for a degree at any tertiary educational institution.

Tony Shackleton

Crave wisdom of God, the sense to understand,

Else meddle not herewith, nor take it in hand.

For it will cost thee much wordly wealth;

But trust not to other, but do it thyself.

Learn, therefore, first to cleanse, purify and sublime,

To dissolve, congeal, distill and sometime

To conjoin and separate, and how to do all,

That when you think to rise, thou do not fall,

Trust to thyself and not to another;

I can say no more to thee if thou were my brother.

– Simon Forman, 1597

To Mum, Dad, Patricia, Michael, Stephen,

Terry, Robert and Candice

Acknowledgments

Jules Henri Poincaré (1854-1912) once made the remark, "The scientist does not study nature because it is useful to do so. He studies it because he takes pleasure in it, and he takes pleasure in it because it is beautiful". For me this is only half the story as I know I have gained something that cannot be measured by the yardstick of science. Not only have I learned much in my chosen discipline, I have met some wonderful people along the way who have made my life all the richer for the experience. It is to these people and others whom I owe my sincere gratitude.

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Abstract

An important model in the study of electron-atom collisions was developed by Temkin and Poet. Although the model has been used to test many theories and approximation methods, the novel method of solution used by Poet has not been developed further to any great extent. The Temkin-Poet model of electron scattering simplifies the three-body scattering problem by suppressing all angular dependence of the wavefunction. In this thesis we return to Poet's method and apply it to a range of calculations for the hydrogen-isoelectronic series. Firstly it is demonstrated that the method provides high-precision solutions for elastic and inelastic scattering. These solutions will then be used to investigate various aspects of electron scattering including the ionisation of hydrogen in the near-threshold region, resonant states in helium, and the scaling of elastic cross sections for a number of hydrogen-like ions.

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