Dynamic Sliding Window Width Selection Strategies for Rate-Distortion Optimal Vertex-Based Shape Coding Algorithms

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ABSTRACT

Vertex-based operational-rate-distortion (ORD) shape coding algorithms frequently use a sliding window (SW) to both avoid trivial solutions and improve computational efficiency, with the choice of the SW-width enabling the encoder to trade between bit-rate and computational complexity. This paper presents some new strategies for dynamically determining the SW-width adaptive within the rate-distortion constraints. From a constrained bit-rate perspective, this is achieved by estimating the maximum number of significant points feasible for a prescribed bit-rate, while for an admissible distortion, an additional *rate trade-off* parameter selects the SW-width, with the optimal width determined for an encoder constrained by both rate and distortion. An efficient shape-adaptive technique is also presented that exploits the curvature of a shape to compute the SW-width. Experimental results confirm that the SW-width determined using the proposed strategies can be seamlessly and efficiently embedded into the ORD shape coding framework.

1. INTRODUCTION

While facilitating increasingly effective retrieval, manipulation and interactive editing functionality for both natural and synthetic video sequences, object-oriented coding using shape information remains a challenging research topic [1]-[3]. Inherent bandwidth limitations and computational speed requirements of the existing communication technologies mean that applications such as video-on-demand, video streaming over the Internet and mobile video transmission for handheld devices will benefit significantly from faster and efficient shape coding strategies.

Operational-rate-distortion (ORD) shape coding algorithms such as [1]-[3] determine a set of *significant points* (SP) for a particular shape contour from a set of vertices contained in the *admissible control-point band* (ACB) around the boundary inclusive. The SPs are then encoded instead of all boundary points. In the ORD shape coding framework, distortion is measured by either the *shortest absolute distance* (SAD) or the *distortion band* (DB) approach. Since these ORD algorithms consider all possible combinations of control points to form the *directed acyclic graph* while searching for SPs, they are computationally inefficient. In addition, both ACB and DB can generate trivial solution, so Katsaggelos et al. [1] introduced the use of *sliding window* (SW) to ensure the encoder followed the boundary so avoiding trivial solutions, as well as gaining computational speed by limiting the search space for SPs. In fact, the SW limits the search space for the next SP to only within the SW-width, so this provides an inherent lower bound upon the number of SP, and thereby the requisite number of bits to encode a particular shape. The bit-rate overhead becomes higher as the SW width is decreased and vice versa. Conversely, a smaller SW ensures computationally faster encoding.

While Katsaggelos et al. [1] introduced the concept of use SW, no formal method exists for determining the appropriate SW-width for different shapes within the ORD shape coding framework. This paper investigates this issue by developing mathematical models to determine the appropriate width within the *rate-distortion* (RD) constraints. It also presents an efficient shape adaptive SW-width technique based on the curvature at the boundary points. The experimental results confirm that the SW-width derived from these proposed methods can be seamlessly embedded into existing ORD shape coding algorithms to also enhance their performance.

The remainder of this paper is organised as follows: Section 2 provides a short overview of the ORD shape coding framework indicating the importance of SW-width, while Section 3 presents the proposed mathematical foundations for determining the window width within the RD constraints. Section 4 presents experimental results endorsing the effectiveness of the proposed techniques, with some concluding remarks given in Section 5.

2. ORD OPTIMAL SHAPE CODING ALGORITHMS

ORD shape coding algorithms seek to determine and then encode a set of SP to represent a particular shape. Let the boundary $B=\{b_1,b_2,\cdots,b_{N_B}\}$ be an ordered set of shape points, where N_B is the total number of shape points and $b_1=b_{N_B}$ for a closed boundary. $P=\{p_1,p_2,\cdots,p_{N_P}\}$ is an ordered set of SP used to approximate B, where N_P is the total number of SP and $P\subseteq A$, where A is the set of vertices in ACB. The ORD shape coding approach in Algorithm 1 [1] (except Step 1) then determines the optimal P for a boundary B within the constraints of the rate-distortion criteria.

Algorithm 1: The basic ($A=B$) ORD-optimal coding algorithm.

Inputs: Boundary $B=\{b_1,b_2,\cdots,b_{N_B}\}$, admissible distortion $D_{\max}$.
Variables: $MinRate(b_k)$ is the current minimum bit rate to encode up to vertex b_k from b_1; while $pred(b_k)$ stores the preceding SP of b_k.
Output: $pred$ is the set of ordered SP approximating shape B.

1. *Determine the width of the sliding window*;
2. Initialise $MinRate(b_1)$ with the total bits required to encode the first shape point b_1; Set $MinRate(b_k), 1<k\le N_B$ as infinity;
3. FOR each vertex $b_i,\ 1\le i<b_{N_B}$;
4. FOR each vertex pair $(b_i,b_j)\ i<j\le N_B$;
5. Check the edge distortion $d(b_i,b_j)$;
6. Find bit requirement $r(b_i,b_j)$ and weight $w(b_i,b_j)$;
7. IF $((MinRate(b_i)+w(b_i,b_j))<MinRate(b_j))$;
8. $MinRate(b_j)=MinRate(b_i)+w(b_i,b_j)$; $pred(b_j)=b_i$;

The computational complexity of Algorithm 1 is $O(N_B^3)$ as the nested loops in Steps 3-4 take $O(N_B^2)$ and within these loops, Step 5 incurs $O(N_B)$ computation cost by using either the edge distortion measure [4] or SAD. A SW of width L however, sets the range of

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j in Step 4 to $i < j \le \min\{(i+L), N_B\}$ and makes the computational complexity $O(N_B L^2)$, since for a SW-width L the loop at Step 4 is executed L rather than N_B times in the worst case and Step 5 will also incur $O(L)$ time complexity. Again, DB requires $O(N_B^2)$ for Step 5, so the overall complexity of Algorithm 1 using DB is $O(N_B^4)$ which becomes $O(N_B L^3)$ when a SW is applied. The computational cost is improved if $L < N_B$, however as the SW forces the encoder to select the next SP from only the vertices associated with the next L boundary points, for a small SW the number of SP generated is larger, with a commensurately higher bit-rate being incurred.

3. DYNAMIC SLIDING-WINDOW WIDTH CALCULATION STRATEGIES

Various RD constraint scenarios are now individually examined.

3.1. Constrained by admissible bit-rate

Typically, the admissible bit-rate directly determines the level of distortion in the encoding process, however as a SW controls the number of SP and hence the bit-rate, its influence on the distortion whenever a SW is used is lower. Therefore, in an encoder with rate constraints, an appropriately small SW-width within the available bit budget should firstly be estimated and then be used in the conventional RD optimisation process proposed in [1].

Since bit-rate is the total number of bits required to encode all SP, the encoding technique used has a direct impact on the number of SP and hence the SW-width L. If c is the average number of bits required to encode a SP then:-

$$L \ge \frac{N_B \cdot c}{R_{\max} - r} \tag{1}$$

where r bits are required to encode the starting vertex and $R_{\max}$ is the admissible bit rate. However, this L holds true for a strictly maintained c. If $p(i)$ is the probability that the next SP will be selected at distance i in the SW, the effective width of SW of width L becomes $E = \sum_{1 \le i \le L} i \cdot p(i)$ and the normalised effective ratio to the actual width is $e = E/L$. Therefore:-

$$L \ge \frac{N_B \cdot c}{e \cdot (R_{\max} - r)} \tag{2}$$

The encoding methods proposed in [2] use a combination of run-length and chain code. The direction of the next SP is coded by a 3-bit chain, and the distance is considered the run-length and can be encoded by either a *fixed length code* (FLC) or a *variable length code* (VLC). It is noteworthy to mention that the 2-bit improved orientation encoding scheme for direction coding is unsuitable for both SW and VLC table [2], since for zero distortion, it fails to encode a straight line of length longer than the SW or VLC code.

With fixed length codes: For a SW-width L, FLC of $\lceil \lg L \rceil$ bits are required to encode the run-length as the next SP has to be within L points, so $c = 3 + \lceil \lg L \rceil$ and (1) becomes:-

$$L \ge \frac{N_B \cdot (3 + \lceil \lg L \rceil)}{R_{\max} - r} \tag{3}$$

If $p(i), 1 \le i \le L$ are equally probable, $E = \frac{(L+1)}{2}$ and $e = \frac{L+1}{2L}$. Thus,

$$L \ge \frac{2L \cdot (3 + \lceil \lg L \rceil)}{(L+1) \cdot (R_{\max} - r)} \cdot N_B \tag{4}$$

However, in the rate minimisation process the farther points in the SW are more likely to be selected as the next SP. If the probabilities $p(i)$ follow an arithmetic progression within the SW, $p(i) = \frac{2 \cdot i}{L(L+1)}$ and $e = \frac{2L+1}{3L}$, so from (2):-

$$L \ge \frac{3L \cdot (3 + \lceil \lg L \rceil)}{(2L+1) \cdot (R_{\max} - r)} \cdot N_B \tag{5}$$

With variable length codes: This paper also uses the 15 codeword based logarithmic coding [2], so a SP i pel apart from the previous SP can be coded with $c = 3 + 2 + \lfloor \lg i \rfloor$ bits. With equi-probability and a SW of L, $c = 5 + \frac{1}{L} \sum_{i=1}^{L} \lfloor \lg i \rfloor = \frac{1}{L}\left(L \cdot m + m - 2^{m+1} + 2\right)$ bits, where $m = \lfloor \lg L \rfloor$, so (1) and (2) respectively become:-

$$L \ge \frac{5 + \frac{1}{L}\left(L \cdot m + m - 2^{m+1} + 2\right)}{R_{\max} - r} \cdot N_B \tag{6}$$

$$L \ge \frac{2L \cdot \left(5 + \frac{1}{L}\left(L \cdot m + m - 2^{m+1} + 2\right)\right)}{(L+1) \cdot (R_{\max} - r)} \cdot N_B \tag{7}$$

Similarly, for an arithmetic progression probability $c = 5 + \frac{2}{L(L+1)} \sum_{i=1}^{L} i \lfloor \lg i \rfloor$, so if $m = \lfloor \lg L \rfloor$ then:-

$$L \ge \frac{3L \cdot \left(5 + \frac{1}{L(L+1)}\left(L(L \cdot m + m) + \frac{2}{3}\left(4^m - 1\right) + 2^{m+1}\left(1 - 2^m\right)\right)\right)}{2 \cdot (2L+1) \cdot (R_{\max} - r)} \cdot N_B \tag{8}$$

The value of L can be found by using numerical methods and used in all the RD optimisation algorithms [1]-[3].

3.2. Constrained by admissible distortion

As rate is a non-increasing function of distortion, without loss of generality this is also upheld when using a fixed SW, so if the full admissible distortion is utilised, the encoder has more bits available to ensure a narrower SW, and thus more efficient coding. Again, for a given admissible distortion, bit-rate is a non-increasing function of SW-width, though there is no direct relationship between window-width and distortion. Whenever an encoder is constrained by only admissible distortion, it is in principle always possible to use an arbitrary SW-width, with the pyrrhic cost of high bit-rate or computational overhead. A better strategy is to explore reducing the SW-width using a *rate trade-off* parameter, which is now discussed.

For a given distortion, the bit rate ($R_{L_{\max}}$) using the maximum SW-width ($L_{\max}$) gives the lowest bit-rate. To have an implicit constraint, a *rate trade-off* parameter α is introduced, which allows the encoder to use an additional $\alpha \cdot R_{L_{\max}}$ bits to $R_{L_{\max}}$ to trade between bit-rate and computational cost. For a fixed distortion therefore, the SW-width can be determined by using Lagrangian multiplier optimisation method constrained only by the admissible bit-rate keeping the distortion constant as the admissible distortion. The unconstrained problem can be formulated as:-

$$\min_{1 \le i \le L_{\max}} \left(L(i) + \lambda \cdot R(i)\right) \text{ subject to } D = D_{\max} \tag{9}$$

and as follows from [2], an optimal solution to (9) with respect to $R(i)$ is also an optimal solution to the constrained problem:-

$$\min_{1 \le i \le L_{\max}} L(i), \text{ subject to } R(i) \le (1 + \alpha) \cdot R_{L_{\max}} \text{ and } D = D_{\max} \tag{10}$$

$L(i)$ will be the optimal width of the SW for the encoder at a prescribed admissible distortion and the parameter α. For VLC, (10) can be solved straightforwardly using an iterative search method; however for FLC a 2-stage search is necessary. Firstly, the optimal range $2^k < i \le 2^{k+1}, k > 0$ is determined and then the optimal solution is sought within this range. $L_{\max}$ is determined from the SP coding strategy used so for FLC $L_{\max} = 2^n$, where n is the code length in bits. Since for VLC, $L_{\max} = 15$ is used throughout this paper, $n = 4$ is selected to ensure comparable results.

3.3. Optimal SW constrained by both rate and distortion

For an encoder constrained by both bit-rate and distortion, the optimal width $L(i)$ of the SW is also determined using the Lagrangian multiplier method (10). However, in this particular case $\alpha=0$ and the admissible bit-rate $R_{\max}$ replaces $R_{L_{\max}}$.

3.4. Shape-adaptive sliding window

As the *cornerity* of shape points do not change abruptly, but rather gradually follow the trend to and from any extreme value, it is used to determine the optimal SW-width. By monitoring the relative cornerity of the shape points, an adaptive strategy can be developed so that boundary points with a larger cornerity induce a smaller SW and vice versa. This will ensure more SP for the boundary-segment with sharp changes and corners and fewer SP in segments where the rate of change in shape is small (long window width). For this purpose, a corner detection technique [5] is used since this accurately reflects the results perceived by a human [6]. However, instead of using the original form of calculating the following modified cornerity is used:-

$$K[j]=\left(\sum_{s=s_1}^{s_2}\left[\ln(t_1+1)\cdot\ln(t_2+1)\cdot\sum_{i=j}^{j+s}d[i]\right]\right)/(s_2-s_1) \qquad (11)$$

where $K[j]$ is the cornerity of the $j-th$ boundary point, t_1 and t_2 are respectively the backward and forward links, d is the local curvature, s is width of lookup window ranging from s_1 and s_2; with fuller details given in [5]. The modification in (11) occurs inside the logarithms, where t_1 and t_2 are incremented to ensure the existence of local cornerity values, even when either is 1.

The maximum and minimum SW-widths are respectively assigned to the boundary points with the lowest and highest cornerity values. A mapping is performed between the cornerity value of each boundary point and the SW-width, so the SW-width for the $j-th$ boundary point is given by:-

$$L[j]=L_{\min}+\frac{L_{\max}-L_{\min}}{\exp(K_{\max})-\exp(K_{\min})}\cdot\left(\exp(K_{\max})-\exp(K[j])\right) \qquad (12)$$

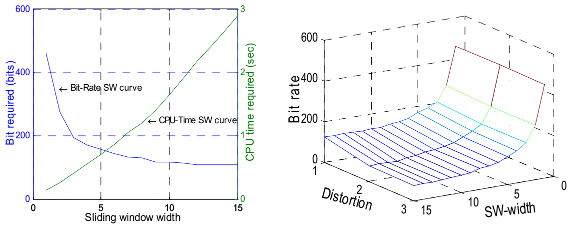
where $K_{\max}$ and $K_{\min}$ are the maximum and minimum cornerity of the vertices on a shape boundary respectively.

For VLC, $L_{\max}$=15 pel [2], if the bit-rate is constrained, $L_{\min}$ is estimated using (6), while (3) is used for FLC. If the distortion is constrained then in both cases $L_{\min}=\lceil D_{\max}\rceil$ as otherwise the encoder will not utilise the full admissible distortion. For FLC, $L_{\max}$ is 2^n, where n is the code length in bits..

4. RESULTS AND ANALYSIS

The performance of the proposed SW-width calculation model has been analysed for many different natural and synthetic object shapes, though in this section the analysis are provided for the *neck* shape of the 30th frame of the *Miss America* video sequence. This is because it possesses both sharp and gradual parts and is used extensively in the literature [1], [2].

The effect of the SW-width upon both the computational time (CPU time required to run the Matlab 6.1 program on a PC with 2.8GHz Pentium-IV processor, 512MB RAM under Windows XP operating system) and bit-requirements were firstly analysed. Figure 1(a) plots the bit-rate and CPU-time requirements for different SW-widths (including SW-width calculation) for the *neck* shape with $D_{\max}=1$ pel and using VLC. It is visually noticeable that a smaller SW requires less computational time and higher bit-rate. Moreover, there is a diminishing rate of decrease in the bit-rate as the window width increases. For instance, when SW-width is increased from 1 to 2 pel, the bit rate reduction is much steeper whereas, the curve for SW-width greater than 10 pel is almost flat. Conversely, the computational time increases commensurately with SW-width, e.g., the computational time for SW=1 pel and 15 pel are 180ms and 3s respectively. Since the SW-width directly impacts on the computational time, in the remainder of this section, SW-width is mainly considered instead of computational time.



(a) (b)

Figure 1: a) Effect of SW-width on bit-rate and CPU-time, b) rate–distortion and SW-width dynamics using VLC.

Table 1: Estimated SW-width with constrained bit-rate obtained distortion and utilisation of bit-rate (distortion is in *pel*).

	FLC (4)			FLC (16 pel SW)		VLC (8)			VLC (15 pel SW	
$R_{\max}$	SW (pel)	Disto-rtion	Bit used	Disto-rtion	Bit used	SW (pel)	Disto-rtion	Bit used	Disto-rtion	Bit used
75	16	1.42	70	1.42	70	14	3	74	3	74
90	15	1	86	1	86	12	2	78	2	77
120	11	0.71	120	0.71	120	8	1.42	118	1.42	93
150	8	0.71	118	0.71	120	6	1	149	1	124
180	6	0.5	160	0.5	170	5	1	166	1	124

To prove the effectiveness of the proposed model, Table 1 summarises the numerical results for a range of admissible bit-rates for FLC with equi-probability (4), VLC with arithmetic progression probability (8), and VLC and FLC with respective maximum SW-widths of 15 and 16 pel. Table 1 shows that for $R_{\max}=120$ bits with (4) the estimated SW-width was 11 pel and utilised the entire 120 bits with a corresponding distortion of 0.71 pel, while with FLC using a fixed SW of 16 pel the distortion and bit utilisation respectively were also 0.71 pel and 120 bits. While both cases produced similar RD results, the proposed method (4) used a smaller SW and is thus more efficient. For the VLC (8) with the same $R_{\max}$, the estimated SW is 8 pel and produced distortion of 1.42 pel utilising 118 bits while using a maximum 15 pel SW it also produced peak distortion of 1.42 pel but used only 93 bits of the 120 bits available. Clearly, the proposed SW-width estimation methods better utilise admissible bits and provide more efficient encoding without compromising RD performance. The results also indicate that increasing the admissible bit-rate results in a smaller SW-width and vice versa. It should be noted however that in the FLC, the utilised bit-rate with SW=11 pel is greater than that of 8 pel because with the former the code length for each run is 4 bits, but is only 3 bits for the latter. As a result, though the number of SP has increased with a smaller SW, the bit-rate is lower.

Figure 1(b) presents the rate–distortion–SW-width dynamics from which it is clear that for a given distortion, the bit-rate decreases as the SW-width increases and vice versa. The graph also reveals that for a particular SW-width there is a little change in bit-rate with distortion variation. This is reinforced by the results in Table 2 which presents the optimal SW-width together with utilised bits for both RD constraints using VLC. For example, with an admissible bit-rate of 150 bits for distortion within the range 1.42 pel to 3 pel, the SW-width is 5 pel, which means that though the distortion has increased, the window width has not and that it is linked much more to the admissible bit-rate. In addition, an encoder constrained by $R_{\max}=90$ bit and $D\max=2$ pel requires an optimal SW-width of 11 pel, which means that use of other SW-widths will compromise the maintenance of the RD constraints.

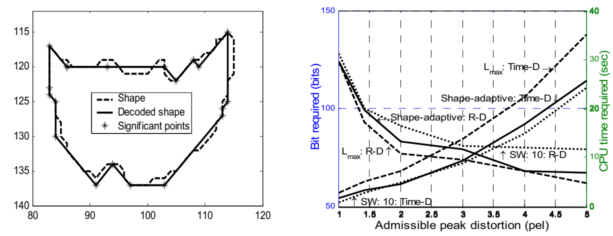
Table 2: Optimised rate–SW-width results for a given distortion and admissible bit-rate using VLC ('X' = unachievable).

Peak	Bit-rate = 90		Bit-rate = 120		Bit-rate = 150	
Dist (pel)	SW(pel)	Bit used	SW(pel)	Bit used	SW(pel)	Bit used
1	X	X	X	X	6	149
1.42	X	X	8	118	5	146
2	11	84	6	120	5	138
2.24	10	89	6	119	5	134
3	9	89	6	117	5	133

Table 3: Optimal SW-width and bit requirement considering *rate trade-off* parameter α for given distortions (VLC, *neck* region).

Admissible distortion	α =0.1		α =0.2		α =0.3		α =0.4	
	SW	Bits	SW	Bits	SW	Bits	SW	Bits
1	8	136	6	148	6	148	5	166
1.42	10	100	9	107	8	118	6	130
2	11	84	10	91	9	99	8	101

Table 3 presents the optimised SW-width and bits used for admissible peak distortion and *rate trade-off* parameter using VLC. By increasing α, the encoder uses more bits and thus provides a smaller SW. For example, for $D\max=1$ pel, the SW and bit-rate pairs for α =0.1 and α =0.4 were respectively 8 pel and 136 bits, and 5 pel and 166 bits.



(a) (b)

Figure 2: a) Shape coding upon the *neck* using shape-adaptive SW and VLC for $D\max=1$ pel, b) RD and time-distortion comparisons among shape-adaptive SW, SW=15 pel and SW=10 pel.

Figure 2 summarises the performance of the shape adaptive SW-width. Figure 2(a) shows the decoded shape using a shape-adaptive SW-width of $D\max=1$ pel, where it is clear that the shape has retained all its sharp corners by using more SP in these regions, while concomitantly using fewer SP in the flatter regions, so vindicating the use of shape cornerity at the boundary points in the adaptation. Figure 2(b) compares RD and time–distortion characteristics of shape-adaptive SW, SW=15 ($L\max$) and SW=10 (SW_{10}) pel for VLC. It is observed that both shape-adaptive (SA) and $L\max$ give almost similar bit-rates while SW_{10} always incurs a higher bit-rate, so that for $D\max=1$ and $D\max=4$ pel, both SA and $L\max$ required the same bit-rate, and the maximum difference in bit-rates occurred when $D\max=3$ and was only 8%. Conversely, the computational time requirements for $L\max$ are always much higher than those for the adaptive SW while the SW_{10} is closely followed by the adaptive SW. The minimum time-requirement difference was 28% for $D\max=3$ pel and the absolute time requirements were higher for higher distortion using $L\max$ than the adaptive SW. Hence, as well as preserving the sharp features of a shape, shape-adaptive SW is an efficient algorithm which retains both bit-rate and computational cost at a quiescent level.

Estimation of the SW-width for bit-rate constraint purposes is computationally efficient because it is only performed once outside the main loops and the complexity is that of the numerical method applied to determine L. In the optimal SW-width cases for the RD constraints, computational complexity is higher due to searching for optimal solution. In practice however, there always has to be search operations to find the width if an encoder is constrained by both rate and distortion. The adaptive SW is efficient and also performed outside the main loop and the computational complexity is $O(N_B)$ to calculate the cornerity and determine SW-width for all boundary points.

In [1], a rigorous review of the shape coding techniques has been presented where it is established that the algorithm proposed in [1] has outperformed the others. Though the proposed SW-width selection is designed for algorithms [1], for completeness some comparative results with the baseline-based method [7], which was shown efficient (intra mode) in [1], are presented in Figure 3. From the results in Figure 3, it is obvious that the proposed adaptive SW strategy provided better results than the baseline-based method.

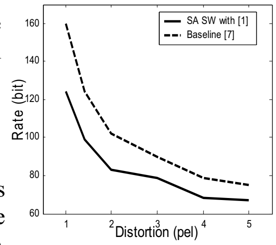


Figure 3: RD results for baseline-based method and [1] with the proposed shape adaptive SW for the *neck*.

5. CONCLUSIONS

The sliding window (SW) is an established mechanism in vertex-based, operational-rate-distortion (ORD) shape coding algorithms to both reduce the computational cost and avoid trivial solutions. This paper has investigated how to determine the most appropriate SW-width, in the context that the choice directly impacts upon both the bit-rate and computational overhead. The paper has presented a formal mathematical framework for determining the SW-width from different RD constraint aspects and can be seamlessly embedded into existing ORD algorithms. A shape-adaptive SW technique has also been developed that varies the width depending upon the cornerity of a shape point, so in flat shape regions there is a longer SW with fewer SP, while at sharp edges, the SW is much smaller so the sharp features are retained during encoding.

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