A note on self-maximal numbers

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Given an infinite word $\boldsymbol{x}$ over an alphabet $\mathcal{A}$, a letter b occurring in $\boldsymbol{x}$, and a total order $\leq$ on $\mathcal{A}$, the smallest word (resp. greatest word) with respect to $\leq$ starting with b in the shift orbit closure of $\boldsymbol{x}$ is called a *minimal word* (resp. *maximal word*) of $\boldsymbol{x}$. By a result of Parry (1960), it is known that an infinite word $\boldsymbol{s}$ over the alphabet $\{0, 1, \ldots, \lfloor\beta\rfloor\}$ forms the digits of a greedy β-expansion of 1 for some (non-integer) real number $\beta > 1$ if and only if $\boldsymbol{s}$ is a maximal word (with respect to the natural order on its alphabet) beginning with $\lfloor\beta\rfloor$ (the maximal letter in its alphabet), in which case β is unique. We call such numbers β *self-maximal*. Using combinatorial properties of the maximal words of certain classes of infinite words, we consider the problem of determining whether the corresponding self-maximal numbers are transcendental.

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