Estimating Transformer Parameters for Partial Discharge Location

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***Abstract*—Partial discharge (PD) location in power transformers using electrical methods require transformer parameters to estimate the PD location. Previous research using a lumped parameter model of a transformer consisting of inductance (L), series capacitance (K) and shunt capacitance (C) has shown an algorithm for PD location. This algorithm does not require L, K and C values for the transformer in their explicit form. Rather, the products LC and LK are required. This paper presents three methods of estimating LC and LK values for a power transformer, which could then be used for PD location. The paper shows that all three methods give identical results confirming that either of these methods could be used for estimating LC and LK values. Results based on impedance measurements from two transformer windings are also presented.**

Index Terms--power transformer; partial discharge location; parameter estimation; impedance measurements; frequency response analysis

I. Introduction

In high power, high voltage transformers, one of the main causes of insulation failure is partial discharge (PD) [1]. Partial discharges [2] in power transformers can gradually deteriorate the insulation leading to total breakdown. If partial discharges are identified at an early stage of their development, the location becomes important for preventive maintenance. PD location in transformers can be performed with either the acoustic signals generated by the PD or using electrical methods. Location of PD due to acoustic methods can be difficult due to attenuation, reflections and refractions due to non-uniform structure within a transformer.

Most electrical methods developed to locate partial discharges in transformers either assumes that the transformer construction data are readily available for computer based simulations [3] - [7], or measurements requiring access to the interior of the transformer is possible [8] - [11]. However, most utilities operate power transformers which have reached their designed lifetime, and do not seem to have their construction details. There is also no easy way of getting access to the interior of a power transformer without a major overhaul, which requires considerable amount of time and financial commitment.

A measurements-based PD location algorithm [12] can overcome these shortcomings to some extent. According to this algorithm only a few measurements are required at the line-end of a power transformer to estimate the PD location. This paper gives an overview of the PD location algorithm [12], and methods to estimate the transformer parameters, which are required for estimating the PD location.

The transformer ladder network used in the analyses has only three parameters: series inductance (L), series capacitance (K) and shunt or parallel capacitance (C). With the PD location algorithm [12] requiring the transformer parameters in their product form (LC and LK), the methods suggested here are easier to implement. Explicit forms of calculating transformer parameters, which make use of the transformer impedance – frequency characteristic can be found in other literature [14] - [17].

II. Partial Discharge Location – Analyses

A. An Overview of the Location Algorithm

In deriving the location algorithm [12], the transformer winding is modelled using an LKC ladder network. Fig. 1 shows the ladder network with C_b representing the capacitance of a bushing connected at the line end; the neutral end is solidly earthed. In Fig. 1, i_{PD} is a current due to a PD at a distance x_0 from the line-end; i_l and i_n are the line-end and the neutral-end currents due to the PD. The total length of the winding from the line to neutral-end is taken as unity for simplicity.

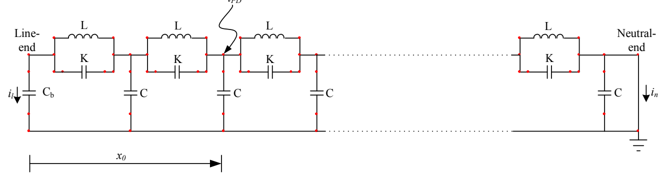

Fig. 1. Ladder network of a transformer at high frequencies.

With no PD ($i_{PD} = 0$), if a current impulse is applied at the line-end, the voltage (v) and current (i) distributions along the winding will be given by (1) and (2) [13], where the distance x is measured from the line-end.

$$v(x, j\omega) = A\cosh(rx) + B\sinh(rx) \tag{1}$$

$$i(x, j\omega) = \frac{1}{Z}[A\sinh(rx) + B\cosh(rx)] \tag{2}$$

In (1) and (2), r is given by,

$$r^2 = \frac{-LC\omega^2}{1 - LK\omega^2} \tag{3}$$

and,

$$Z = \sqrt{\frac{L}{C(1 - LK\omega^2)}} \tag{4}$$

A and B in (1) and (2) are constants determined by the boundary conditions of the winding, and ω is the fundamental angular frequency of the current impulse. If a PD occurs at a distance x_0 from the line-end, it can be shown [3], [4], that the current at the line-end (i_l) due to the PD current (i_{PD}) is given by,

$$i_l = \left(\frac{\frac{C_b}{C} r\sinh(r(1 - x_0))}{-\cosh(r) + \frac{C_b}{C} r\sinh(r)} \right) i_{PD} \tag{5}$$

It can be seen that the PD location x_0 is embedded in (5). To find x_0, the roots of $i_l = 0$ have to be found. This corresponds to finding the series resonant frequencies of i_l, which make $i_l = 0$. This leads to finding the roots of

$$r\sinh r(1 - x_0) = 0 \tag{6}$$

Since, $r \neq 0$, except when $\omega = 0$ (which is of no interest), this gives,

$$\sinh r(1 - x_0) = 0 \tag{7}$$

The solution for (7) is given by,

$$(1 - x_0)^2 = -\left(\frac{n\pi}{r}\right)^2 \tag{8}$$

where n is an integer. Equation (8) can be re-arranged to find x_0 as,

$$x_0 = 1 - \frac{n}{2f}\sqrt{\frac{1 - 4\pi^2 f^2 LK}{LC}} \tag{9}$$

where, n is a positive integer. Equation (9) shows that the PD location, x_0, can be found, when the transformer parameters LC and LK are known; the integer n and frequency f are known. It has to be emphasized here that the integer n and the frequency f are co-related. For each integer value, there is a corresponding frequency, and accurate PD location depends on identifying the correct frequency for each integer value.

Having found an expression for the PD location in (9), the next step involves finding the LC and LK values. The following three sections show the three different methods proposed in this paper to find LC and LK values.

B. *Use of a PD Calibration Signal to Estimate LC and LK*

A PD calibration signal injected at the bushing tap of the transformer corresponds to a PD signal at the line-end, with $x_0 = 0$ in Fig. 1. The corresponding line-end current i_l is given by (5) with $x_0 = 0$, leading to (10).

$$i_l = \left(\frac{\frac{C_b}{C} r\sinh(r)}{-\cosh(r) + \frac{C_b}{C} r\sinh(r)} \right) i_{PD} \tag{10}$$

The frequencies which make $i_l = 0$, called the series resonant frequencies, are given by equating the numerator of (10) to zero. That is,

$$\frac{C_b}{C} r\sinh(r) = 0 \tag{11}$$

Simplifying (11) shows that $r = 0$ solution is of no interest, and the roots of interest are given by,

$$\sinh(r) = 0 \tag{12}$$

The solution for (12) can be re-arranged into,

$$LC + n^2\pi^2 LK = \left(\frac{n}{2f}\right)^2 \tag{13}$$

where, n again is an integer. Equation (13) can be used to find LC and LK values, using two resonant frequencies. Assume that the two resonant frequencies f_1 and f_2 are found corresponding to integers $n = n_1$ and $n = n_2$. LC and LK values are then given by,

$$LC = \frac{(n_1 n_2)^2}{4({n_2}^2 - {n_1}^2)}\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) \tag{14}$$

$$LK = \frac{1}{4\pi^2({n_2}^2 - {n_1}^2)}\left[\left(\frac{n_2}{f_2}\right)^2 - \left(\frac{n_1}{f_1}\right)^2\right] \tag{15}$$

C. *Use of Winding Impedance to Estimate LC and LK*

Fig. 2 shows the ladder network shown in Fig. 1, with the line-end open and the neutral-end at earth potential. The voltage (v) and current (i) distributions in the winding described by (1) and (2) can be used to find the measured impedance at the line-end ($x = 0$). Assume i_{PD} to be equal to 1 and to be injected at the line-end in Fig. 2.

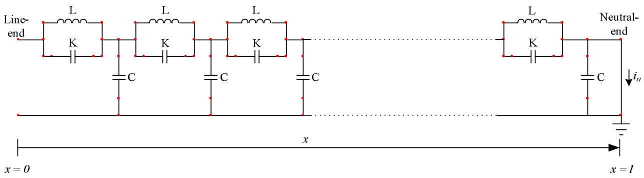

Fig. 2. Impedance measurement in transformer ladder network.

At x = 0, from (2),

$$i(0, j\omega) = \frac{1}{Z}\left(A\sinh(0) + B\cosh(0)\right) = 1 \tag{16}$$

Giving,

$$B = Z \tag{17}$$

At the earthed neutral-end, x = 1, $v(1, j\omega) = 0.$ From (1),

$$v(1, j\omega) = A\cosh(r) + B\sinh(r) = 0 \tag{18}$$

Giving,

$$A = -\frac{B\sinh(r)}{\cosh(r)} \tag{19}$$

The impedance of the winding seen from the line-end is given by,

$$Z_w = \frac{v(0, j\omega)}{i(0, j\omega)} \tag{20}$$

This gives,

$$Z_w = Z\left(\frac{A}{B}\right) \tag{21}$$

Substituting for A/B from (19) gives, the impedance of the winding, Z_w,

$$Z_w = -Z\left(\frac{\sinh(r)}{\cosh(r)}\right) \tag{22}$$

The line-end terminal of the winding is brought out of the transformer tank through the bushing, which is connected to the earthed transformer tank. The total impedance, Z_{imp}, is therefore, the parallel combination of Z_w with the impedance of the bushing, which is modelled by a capacitor, C_b.

$$Z_{imp} = \frac{Z_w \times \frac{1}{j\omega C_b}}{Z_w + \frac{1}{j\omega C_b}} \tag{23}$$

Substituting for Z_w from (22) and simplifying (23) using (3) and (4) gives,

$$Z_{imp} = \frac{Z\sinh(r)}{-\cosh(r) + \frac{C_b}{C} r\sinh(r)} \tag{24}$$

The series resonant frequencies of the winding impedance with the bushing capacitance, Z_{imp}, are given by equating the numerator of (24) to zero. That is,

$$Z\sinh(r) = 0 \tag{25}$$

Inspecting (25), it is clear that Z cannot be equal to zero for finite values of frequency. Hence, the only solution is given by

$$\sinh(r) = 0 \tag{26}$$

which is same as (12), leading to the same solutions (14) and (15).

D. Use of Frequency Response Analysis to Estimate LC and LK

In frequency response analysis (FRA), the voltage ratio between the neutral-end voltage (V_2) and the line-end voltage (V_1) is calculated as a transfer function. Fig. 3 shows the ladder network of the transformer winding with the neutral-end earthed through a resistor R. In FRA measurements, the transfer function is given by,

$$FRA = \frac{V_2}{V_1} \tag{27}$$

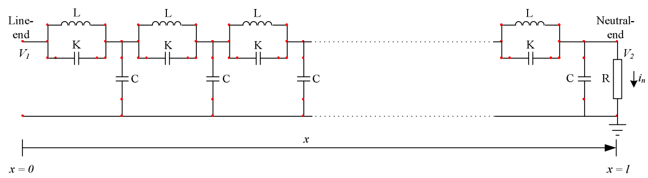

Fig. 3. Frequency response analysis (FRA) using transformer ladder network.

The voltage distribution along the winding given by (1) can be used to find V_1 and V_2. At the line-end, $x = 0$,

$$V_1 = v(0, j\omega) = A \tag{28}$$

At the neutral-end, x = 1,

$$V_2 = v(1, j\omega) = A\cosh(r) + B\sinh(r) \tag{29}$$

Substituting (28) and (29) into (27) gives,

$$FRA = \cosh(r) + \frac{B}{A}\sinh(r) \tag{30}$$

Applying Ohm's Law to the resistor connected at the neutral-end gives,

$$V_2 = i_n R \tag{31}$$

Substituting for V_2 and i_n in (31) gives,

$$A\cosh(r) + B\sinh(r) = \frac{1}{Z}(A\sinh(r) + B\cosh(r))R \tag{32}$$

This gives,

$$\frac{B}{A}=\frac{\cosh(r)-\frac{R}{Z}\sinh(r)}{\frac{R}{Z}\cosh(r)-\sinh(r)} \tag{33}$$

Substituting for $\frac{B}{A}$ in (30) from (33), and simplifying gives,

$$FRA=\frac{R}{R\cosh(r)-Z\sinh(r)} \tag{34}$$

The denominator in (34) can be re-arranged, and the FRA can be re-written as,

$$FRA=\frac{R}{-\sqrt{Z^2-R^2}\,\sinh(r-\theta)} \tag{35}$$

Where, θ is given by,

$$\tan\theta=\frac{R}{Z} \tag{36}$$

If R is selected such that, $R << Z$, (35) can be approximated into,

$$FRA\approx-\frac{R}{Z\sinh(r)} \tag{37}$$

The poles of the FRA transfer function (or the parallel resonant frequencies) can be obtained by equating the denominator of (37) to zero. That is,

$$Z\sinh(r)=0 \tag{38}$$

This leads to,

$$\sinh(r)=0 \tag{39}$$

giving the same solutions as in (14) and (15).

It is therefore, clear that for estimating LC and LK values, either the series resonant frequencies of the line-end signal during PD calibration, or the series resonant frequencies of the measured impedance, or the parallel resonant frequencies of the FRA can be used.

III. Measurements

Impedance measurements available from two transformer windings were used to calculate LC and LK values. The two transformer windings used in the measurements were rated 6.6kV and 11kV, and they were both plain (continuous) disc type windings.

A. 6.6kV Transformer Winding

The 6.6kV winding had an air core with 22 plain discs. The impedance of the 6.6kV winding was measured using a spectrum analyser, with and without a bushing capacitance, C_b = 220pF, and the result is shown in Fig. 4. Fig. 4 shows that the series resonant frequencies (troughs in the top plot in Fig. 4), are independent of the bushing capacitance value (at least for the first resonance), confirming the validity of (26). Table I gives these series resonant frequency values (without C_b).

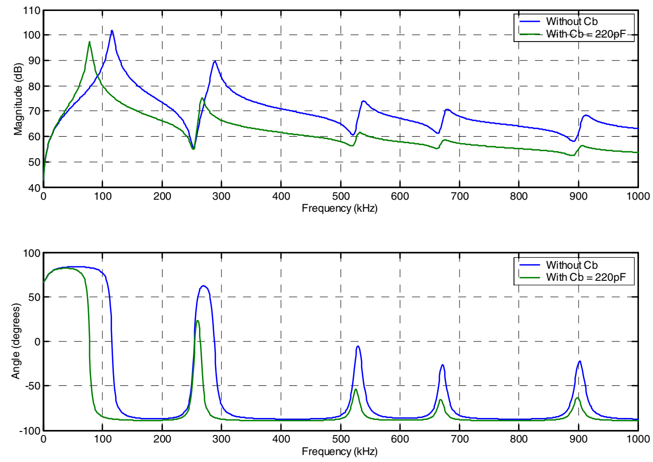

Fig. 4. Measured impedance of the 6.6kV winding with and without 220pF bushing capacitance C_b.

TABLE I. Measured Series Resonant Frequencies (kHz) of 6.6kV Winding

z_1	z_2	z_3	z_4
253	521	663	892

B. 11kV Transformer Winding

The 11kV transformer winding had an iron core with 72 plain discs. The impedance measurements of the 11kV winding using the spectrum analyser is shown in Fig. 5, with and without a bushing capacitance, C_b = 220pF. Table II gives the series resonant frequency values (without C_b) for the winding.

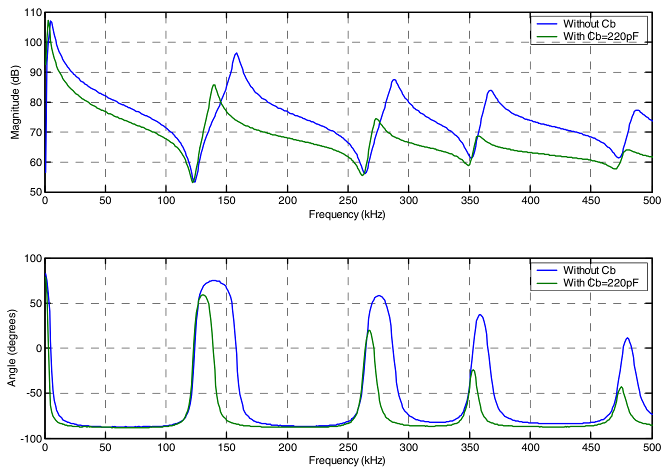

Fig. 5. Measured impedance of the 11kV winding with and without 220pF bushing capacitance C_b.

TABLE II. MEASURED SERIES RESONANT FREQUENCIES (KHZ) OF 11KV WINDING

z_1	z_2	z_3	z_4
122	262	349	470

IV. CALCULATIONS AND RESULTS

LC and LK values calculated according to (14) and (15) are given in Table III and Table IV for the two transformer windings. Each table shows three sets of LC and LK values. The first set of values were calculated using the first two series resonant frequencies (z_1 and z_2), with $n_1 = 1$ and $n_2 = 2$ in (14) and (15), and $f_1 = z_1$ and $f_2 = z_2$. The second set of values in each table was calculated using the second and third series resonant frequencies. That is with $n_1 = 2$ and $n_2 = 3$ in (14) and (15), and with $f_1 = z_2$ and $f_2 = z_3$. The third set of values were calculated using a similar approach with $n_1 = 3$ and $n_2 = 4$, and $f_1 = z_3$ and $f_2 = z_4$.

TABLE III. LC AND LK VALUES FOR 6.6KV TRANSFORMER WINDING

	z_1	z_2	z_3	z_4
	253	521	663	892
LC	3.9796e-12			
LK	-7.4862e-15			
LC		2.5364e-12		
LK		2.9071e-14		
LC			5.2362e-12	
LK			-1.3230e-15	

TABLE IV. LC AND LK VALUES FOR 11KV TRANSFORMER WINDING

	z_1	z_2	z_3	z_4
	122	262	349	470
LC	1.7539e-11			
LK	-7.5270e-14			
LC		1.1444e-11		
LK		7.9129e-14		
LC			1.8942e-11	
LK			-5.2834e-15	

V. DISCUSSION

It can be seen from Table III and Table IV that LC values are always positive whereas LK values can either be positive or negative. LC values are always positive, since the series resonant frequencies were selected such that $f_2 > f_1$, and the integers $n_2 > n_1$, always. However, LK values will only be positive if $n_2/f_2 > n_1/f_1$, which does not always happen, as the results in Table III and Table IV show. This could be due to the simplicity of the model used in the analysis, which does not consider the mutual inductances, resistances and the dielectric losses in the winding.

With each transformer winding producing three sets of values for LC and LK, which set of values to use in PD location becomes an interesting question. An obvious choice would be to use the positive set of values. However, research has shown [12] that any one of the three sets of values could be used, as long as the corresponding frequency is used in PD location using (9). This means, if z_1 and z_2 are used in calculating LC and LK, then the first series resonant frequency of the detected PD signal has to be used. If z_2 and z_3 are used in calculating LC and LK, then the second series resonant frequency of the detected PD signal has to be used, and finally, if z_3 and z_4 are used in calculating LC and LK, then the third series resonant frequency of the detected PD signal has to be used. Research has shown that even with this simple model the PD location can be accurately estimated with an error margin of less than 10% of the winding length.

VI. CONCLUSIONS

A simplified lumped parameter model of a transformer winding is used in the parameter estimation. The simplified model used has only three elements: series inductance (L), series capacitance (K) and the parallel capacitance (C) per each unit. The aim of the parameter estimation was PD location. The equation used in calculating the PD location does not require explicit values for L, C and K; rather the products LC and LK. The paper shows three methods to estimate LC and LK values. One method is based on measurement of line-end terminal signal's series resonances during PD calibration. The second method presented uses the impedance – frequency characteristic of the winding measured using a spectrum analyser. The series resonant frequencies of the winding impedance can be used to estimate the parameters. The third method presented uses parallel resonant frequencies of the frequency response analysis. Either of these methods gives the same LC and LK values.

The paper also presents LC and LK values calculated for two plain disc type transformer windings using their winding impedance – frequency characteristics. Whilst LC values are always positive, LK values can either be positive or negative. This could be due to the simplicity of the winding model used with no mutual inductances, resistances or dielectric losses used in the model.

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