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A NEW RANGE-BASED IMAGE TEXTURE MEASURE WITH APPLICATION TO MAMMOGRAM IMAGE ANALYSIS

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Abstract: This paper describes a new, spatially isotropic, neighbourhood-based, image texture measure, and illustrates its use for mammographic image analysis. It is based on the extended Russ operator, first introduced by J.C. Russ, and founded on the work of H. E. Hurst. An octagonal neighbourhood is defined, centred on each pixel in an image, and the difference between the maximum and minimum pixel values in each set of pixels at a given Euclidean distance d from the centre pixel is computed to be the range d. log d is plotted against log d, and a straight line fitted to the data. The square of the correlation coefficient, $q^2$, is associated with the position of the centre pixel. Preliminary experiments suggest that $q^2$ is a promising texture measure that may be used to detect the edge of the pectoral muscle, define the boundary of the mammographic parenchyma, and in conjunction with other features, possibly detect circumscribed lesions.

Keywords: extended Russ operator, texture measure, mammogram image analysis, parenchyma, pectoral muscle, circumscribed lesion detection

INTRODUCTION

Image texture is easily perceived and intuitively understood. It is, however, difficult to characterise mathematically with sufficient generality to encapsulate the whole gamut of perceived textures. Multi-pronged approaches to texture description and analysis have therefore been developed [1, 2]. One of these is to classify textures as strongly ordered, weakly ordered and disordered, with the observation that these textures could best be described using structural, oriented-flow, and statistical/fractal descriptors respectively [3].

We present here a new texture measure, extending from the work of J. C. Russ [4], and apply it to mammogram texture analysis. It is based on a statistical quantity: the range of greyscale values in successive, equidistant annuli, around a centre pixel. It is unclear whether the new measure captures fractal as well as statistical behaviour. While further investigation of its theoretical basis is required, the new measure appears well-suited to characterising mammogram texture, which lies somewhere in the texture spectrum between a weakly ordered and a disordered texture.

THE EXTENDED RUSS OPERATOR

In 1990, J.C. Russ [4] introduced a "local Hurst operator" to perform texture discrimination. He claimed that this operator implemented in two dimensions the method H.E. Hurst [5] had introduced in studying time-series related to river flow data. While we have shown that Russ’s implementation differs from Hurst’s at least five counts [6, chapter 9], the operator introduced by Russ is nevertheless very interesting and useful in its own right.

Mandelbrot and co-workers [7, 8, 9] introduced the fractional Brownian motion (fBm) model to account for the behaviour extracted by Hurst’s method, although it is not known at present whether this is the only applicable model. Moreover, it is unclear whether fBm is a valid model for explaining what the Russ operator does, given its departures from Hurst’s method.

The Russ operator [4] is most easily understood with an example. An octagonal neighbourhood of given “radius” is first defined for any particular pixel in an image. For example, the neighbourhood shown in Figure 1 is defined for a “radius” of 4 or a “diameter” of 9 pixels; the centre pixel is labelled 0. This neighbourhood is an approximation to a circle, which is the equidistant locus in the Euclidean norm. The distances associated with each labelled pixel in Figure 1 are shown in Table 1. Russ defined his “local Hurst operator” so:

1. For each set of pixels lying within the neighbourhood, at the same Euclidean distance d from the centre pixel, find the maximum and the minimum pixel values. Their difference is the range, d.

2. Plot log d against log d and fit a straight line to the data to minimize the square of the error.

3. The slope of the plot, $m$, is a measure of “local roughness (in the sense of the Hurst coefficient)” [4, p 250] at the centre pixel. It could be used to plot a transformed image, scaled for display, that could later be segmented on the basis of the $m$ values.

As an example, Figure 2 shows a sample plot for
FIGURE 1: Octagonal neighbourhood of "radius" \( p = 4 \) and "diameter" 9. Pixels at the same Euclidean distance \( d_k \) from the central pixel, numbered 0 above, are labelled with the same index, \( k \). See Table 1 for the relevant distances.

<table>
<thead>
<tr>
<th>Pixel Label</th>
<th>No. of Pixels</th>
<th>Euclidean Distance ( d_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>( \sqrt{2} )</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>( \sqrt{5} )</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>( 2\sqrt{2} )</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>( \sqrt{10} )</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>( \sqrt{13} )</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>( \sqrt{17} )</td>
</tr>
</tbody>
</table>

Table 1: This table shows the Euclidean distances \( d_k \) corresponding to the indices \( k \) shown on the octagonal neighbourhood of radius \( p = 4 \) in Figure 1.

RESULTS WITH MAMMOGRAMS

Results from preliminary experiments on applying the extended Russ operator to mammograms are presented in this section, with emphasis on the \( \eta^2 \)-component.

In Figure 3(a), we show an original mammogram, mdb00911, from the MIAS database [10, 11]. Figures 3(b) to (d) show the three transformed images resulting from our definition of the extended Russ operator. The \( c \)-image clearly emphasises edges as we have shown elsewhere [12]. The \( m \)- and \( \eta^2 \)-images exhibit similarities as well as differences and seem to be sensitive to intensity and texture.

The intensity histograms of the original, \( m \)-, \( c \)- and \( \eta^2 \)-images are shown in Figure 3(e) to (h) re-
FIGURE 3: Images from the extended Russ operator with $p = 3$. (a) The original image is MIAS mammogram mdb0098. (b) This is the $m$-image, or "image of slope values", scaled for display. It results from the original definition of Russ [4], with $p = 3$. (c) This is the $c$-image, or "image of y-axis intercept values", again scaled for display. Note the edge sensitivity of the $c$-component which has been described elsewhere [12]. (d) This is the $\eta^2$-image, or image of "the square of the coefficients of correlation", scaled for display. Note its similarities and differences with the image in (b). Because $\eta^2$ has a fixed range of values (from 0 to 1), it may be better suited to extract calibrated texture information. (e) Intensity histogram of original mammogram at 400 $\mu$m per pixel. (f) to (h) Intensity histograms of the $m$, $c$- and $\eta^2$-images from the extended Russ operator. Note that all three histograms are different from each other and the original, confirming that the three components are indeed extracting independent information from the image.
respectively. They appear markedly different.

The original and $\eta^2$ images from another MIAS mammogram, mdb00511, are shown in Figure 4. The original mammogram has two circularly shaped circumscribed lesions close to each other, appearing like a figure-of-eight at the lower middle of the image. The $\eta^2$ image shows a whitish elliptical region with a blacker core at the corresponding site.

DISCUSSION

Qualitative observations

The question may arise as to whether the $c$- and $\eta^2$-components of the extended Russ operator are not really capturing the same information as the $m$-component of the original Russ operator. The fact that the intensity histograms in Figures 3(f) to (h) represent different pixel intensity distributions, and are not scaled versions of each other, confirms that we are indeed extracting independent information from the two new components, $c$ and $\eta^2$, of the extended Russ operator. It is also reasonable to infer that $c$ is strongly indicative of edges whereas $m$ and $\eta^2$ seem to be correlated with local intensity and visual texture.

In our opinion, $\eta^2$ as a local texture measure has the advantage of spanning a pre-defined range of values, lying between zero and one, and may therefore be used quantitatively as a calibrated measure.

When the $\eta^2$ component is used to analyse mammograms, we have observed the following:

1. it can clearly outline the breast border;
2. the dark, predominantly fatty region close to the breast border on the original mammogram appears as a predominantly whitish band on the $\eta^2$-image;
3. the fibroglandular mammographic parenchyma appear as a predominantly blackish region which may have patches of whiter regions on it;
4. clearly discernible lesions show up as whitish regions (sometimes having a blackish core) in the midst of the parenchyma;
5. the edge of the pectoral muscle shows up as a characteristically whitish band adjoining the darker parenchymal region;
6. the mottled appearance of the mammogram background in the $\eta^2$-image indicates that it is highly non-uniform. The "patches" on the $\eta^2$-image, when magnified, may be identified with the shape of the octagonal neighbourhoods used in the operator.

Further experiments with a large number of mammograms are needed to confirm these behaviours generally and to establish the use of the $\eta^2$-component as a reliable texture measure for analysing mammograms. Additional experiments are also required to identify the theoretical basis of the operator, determine what post-processing is needed for texture analysis and segmentation, compare its performance with other texture operators, and explore scale dependence.

Theoretical ramifications

The straight line fit performed by the extended Russ operator imposes the relationship

$$\log d = m \log d + c$$

(2)

which in turn implies the power law relationship

$$R = (10^c)d^m$$

(3)

i.e., the range at successive equidistant annuli from the centre pixel increases as the $m$th power of distance from the centre pixel. While this statement describes the observed behaviour, it does not yield insight into the class of underlying pixel distributions that could lead to this behaviour.

If one assumes that this power law behaviour is characteristic of fractals, then, the degree to which the straight line fits the data may indicate the "degree of fractality" in the neighbourhood of any pixel.

The square of the coefficient of correlation, $\eta^2$, by definition, lies between 0 and 1. It may therefore be used as a calibrated, absolute measure of the degree to which the power law behaviour, being tested for by the extended Russ operator, is indeed obeyed. Values close to 1 signify power law behaviour; values close to zero do not.

We may then conjecture that by setting a threshold for $\eta^2$ close to 1, and thresholding the image, we may be segmenting those portions of the image that exhibit fractal characteristics from those that do not.

CONCLUSIONS

We have defined the extended the Russ operator as one that maps an octagonal neighbourhood of a pixel to three components, $m$, $c$, and $\eta^2$, each of which may be scaled and plotted as images. The $\eta^2$-component always takes values between zero and one and seems to discriminate textures. Preliminary experiments have revealed that it may be used to define the mammographic parenchyma and the pectoral muscle edge. We conjecture that it may also be capable of highlighting circumscribed masses, very likely in conjunction with other features.

References

FIGURE 4: Images from the extended Russ operator with $p = 3$. (a) The original mammogram is MIAS image mdb00511. There are two closely spaced circular lesions of higher intensity, arranged like an oblique figure-of-eight, in the lower middle of the mammogram. The pectoral muscle appears as a higher intensity triangle at the top left of the image and has a somewhat diffuse border. (b) $\eta^p$-image of the mammogram, scaled for display. A white elliptical region with a black core marks the lesion site on the $\eta^p$-image. The pectoral muscle edge is predominantly whiter than either the parenchyma to the right or the rest of the muscle to the left.


