Abstract submitted for 54th Annual AustMS Meeting

Title: Rich, Sturmian, and Trapezoidal Words
Author(s): Amy Glen
Session: Combinatorics

The *factor complexity function* $C_w(n)$ of a finite or infinite word w associates to each integer $n \geq 0$ the number of distinct *factors* (i.e., blocks of consecutive letters) in w of length n. A finite word w of length $|w|$ is said to be *trapezoidal* if the graph of its factor complexity $C_w(n)$ as a function of n (for $0 \leq n \leq |w|$) is that of a regular trapezoid: $C_w(n)$ increases by 1 with each n on some interval of length r, then $C_w(n)$ is constant on some interval of length s, and finally $C_w(n)$ decreases by 1 with each n on an interval of length r. Necessarily $C_w(1) = 2$ (since there is one factor of length 0, namely the *empty word*), so any trapezoidal word is on a binary alphabet. Trapezoidal words were first introduced by A. de Luca (1999) when studying the behaviour of the factor complexity of *finite Sturmian words*, i.e., factors of infinite "cutting sequences", obtained by coding the sequence of cuts in an integer lattice over the positive quadrant of $\mathbb{R}^2$ made by a line of irrational slope. Every finite Sturmian word is trapezoidal, but not conversely. However, both families of words (trapezoidal and Sturmian) are special classes of so-called *rich words* – a new (wider) class of finite and infinite words characterised by containing the maximal number of palindromes – recently introduced by J. Justin, S. Widmer, L.Q. Zamboni, and myself.

In this talk, I will discuss various interconnections between rich words, Sturmian words, and trapezoidal words. I will also briefly outline current research and open problems concerning rich words.

This talk is based on joint work with A. de Luca (Università degli Studi di Napoli Federico II, Italy) and L.Q. Zamboni (Université Lyon 1, France).