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Multi-model Robust Control for Nonlinear Chemical Processes: A Passivity Based Approach

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Abstract

In this paper, the robust control problem for nonlinear multi-unit chemical processes is addressed. The processes discussed admit a set of plant decompositions in an operating region which corresponds to the variation of the operating point caused by disturbances. A passivity based approach is proposed to design decentralized controllers for these processes. Namely, the proposed method determines the best local plant decompositions based on evaluation of a passivity index, and assesses the closed-loop stability and performance using the best local plant decompositions. Local decentralized controllers can be designed based on the best local plant decomposition to achieve desired closed-loop stability and performance.

1 Introduction

Multi-unit nonlinear chemical processes are generally complicated high-dimensional systems. Centralized control structures for such systems are often too complicated or costly to be implemented in practice, and the central control structure is also less fault tolerant. Therefore, it is often preferred to introduce a decentralized control structure in controlling multi-unit nonlinear chemical processes.

In general, decentralized design for a multi-unit nonlinear chemical process involves two parts, namely, linearizing the nonlinear process model at an operating point and then decomposing the linear model into some subsystems such that lower-dimensional local controllers can be designed for these subsystems independently. Process decomposition plays an important role as several possible decompositions may exist for the local linear model [9, 7]. An ill-defined decentralized structure may lead to complete failure of the design or poor closed-loop performance when the control structure is applied to the true process. Therefore, it is necessary to analyse interactions between subsystems and find a best decomposition for the process at the specific operating point [9, 7].

A multi-unit nonlinear chemical process may not always operate at a fixed setpoint due to the inherent nonlinearity and perturbations. Instead, a multi-unit process may operating within a certain operating space produced by the joint effect of manipulated variables and perturbations. Unfortunately, the best decomposition structure is not unique across the whole operating space. Therefore, multiple models are required in this case to tackle stability and performance issues.

A multi-model approach was proposed in [7] to control nonlinear processes operating in an operating space. It was shown that an operating space for a process can be divided into several sub-regions with each subregion admitting the same best process decomposition structure. Therefore, a unified local decentralized controller can be designed for every sub-region to guarantee closed-loop stability and better performance over the whole operating space. The approach introduced in [7] is based on gap metric methodology.

Recent work on robust control design has employed the concept of passivity [1]. Often robust stability of a system can be determined by evaluating passivity, or energy dissipation, of a subsystem. Many uncertain systems can be converted into equivalent interconnected feedback systems which consist of a linear block and possibly a nonlinear time-varying block. By studying the passivity of the interconnected systems, sufficient stability conditions can be derived for the original uncertain systems. Specifically, if the linear block is strictly passive and the nonlinear block is passive, then the original uncertain system is robustly stable. Equivalent loop transforms can be applied to render the transformed nonlinear block passive, if it is not already passive.

The passivity concept can also be applied to decentralized control design for multi-unit nonlinear processes. Specifically, a decomposition of the local linear model at a specific operating point can be treated as the linear block in the interconnected feedback system, and the difference between the decomposition and the full local model can be viewed as the nonlinear block. Therefore, the design objective for a decentralized controller is to render the linear block strictly passive.

It is observed that passivity design can lead to less conservative results for robust control of some uncertain systems. It is possible to consider the phase properties of a system in a passivity based approach. Furthermore, using a passivity concept may result in a simpler decentralized control design.

This paper proposes a multi-model robust control design based on the passivity concept. A methodology for robust stability criteria and controller synthesis is pro-

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posed for chemical processes operating in an operating space.

2 Preliminaries

In the following, we briefly introduce preliminaries of the passivity approach.

Consider the following linear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t) \\ z(t) &= Cx(t) + Dw(t) \end{aligned} \quad (1)$$

System (1) is said to be dissipative if there exists a nonnegative real storage function V such that

$$V(x) \leq V(x_0) + \int_0^t s(t)dt, \quad \forall t \geq 0, \quad (2)$$

where $s(t) \in \mathbb{R}$ is the supply rate.

System (1) is said to be passive if it is dissipative with a supply rate $s(t) = 2z^T(t)w(t)$ and the storage function $V(x)$ satisfies $V(0) = 0$. System (1) is called strictly passive if there exists a positive definite function $\tilde{S}(x(t))$ such that

$$V(x) = V(x_0) + \int_0^t s(t)dt - \int_0^t \tilde{S}(x(t))dt, \quad \forall t \geq 0.$$

For linear systems, passivity is equivalent to positive realness [3, 6].

It can be observed from (2) that $\dot{V}(x) \leq s(t)$, therefore, the supply rate represents instantaneous energy flow into the system. If there are two supply rates $s_1(t)$ and $s_2(t)$ with $s_1(t) \leq s_2(t)$, then if the system (1) is dissipative with supply rate $s_1(t)$, it is also dissipative with supply rate $s_2(t)$. Hence the dissipativeness with supply rate $s_1(t)$ guarantees larger dissipated energy in terms of supply rate $s_2(t)$ [8].

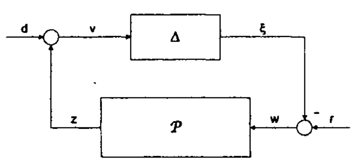

Figure 1: Interconnected Feedback System

Consider the feedback system depicted in Figure 1, where $\mathcal{P}$ is a linear operator and Δ can be a nonlinear/time-varying operator. The following lemma reveals a sufficient condition for the stability of this interconnected system:

Lemma 1 *[3] Suppose $\mathcal{P}$ is linear and strictly passive and Δ is passive. Then, the feedback system is stable.*

In the following, we call the Δ block near passive if it is not passive, but is close to being passive. Similar, we call Δ over passive if it remains passive even under certain additive perturbations. It is obvious that if Δ is strictly passive, then it is over passive.

Lemma 1 is not applicable when Δ is near passive. On the other hand, directly applying Lemma 1 to the interconnected systems may lead to conservative results if Δ is over passive. Fortunately, transforms can be applied on a given interconnected system. We will adopt one of the transforms only in this paper. The transformed feedback system is depicted in Figure 2. Other transforms can be found in [3] and their applications to robust passivity analysis can be found in [10].

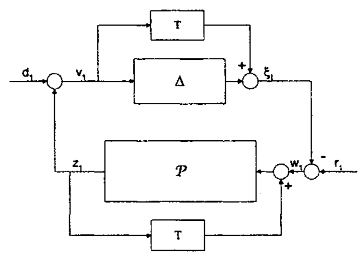

Figure 2: Transformed feedback System

Lemma 2 *[3] Given the feedback system in Figure 1, consider the transformed version in Figure 2. Suppose T is linear and stable. Then, the stability of the systems in Figure 1 and Figure 2 are equivalent.*

In general, there is no unique way to find the transform T. If Δ is linear time-invariant and near passive, then the easiest way to find T is to use the passivity index as defined below:

Definition 1 *[1] For a given linear system $G(s)$, the passivity index is defined as*

$$v(\Delta) = -\inf_{\omega \in \mathbb{R}} \{\lambda_{min}(\frac{1}{2}[\Delta(j\omega) + \Delta^*(j\omega)])\} \quad (3)$$

In this case, $T = v(\Delta)I$ and $T + \Delta$ is passive. If Δ is over passive, then it is always possible to set $T = \beta I$ where β can be any scalar such that $T + \Delta$ is still passive. We will use this simple form only in the development of our methodology. A general form for T when Δ is over passive can be found in [10].

If Δ is near-passive, T can be used to render the transformed Δ block passive, and controllers can be synthesized to guarantee that the transformed $\mathcal{P}$ block is strictly passive. If the Δ block is over-passive, it is only sufficient to render the $\mathcal{P}$ block strictly passive. Since passivity theory requires that the Δ block be passive only, this design will inevitably lead to conservative results. In order to take advantage of the over passive Δ block, T can also be used. The purpose of T for the over-passive case is to render the transformed $\mathcal{P}$ block close to strictly passive while maintaining passiveness of the transformed Δ block. Thus a controller can be easily synthesized to render the transformed $\mathcal{P}$ block strictly passive, and the closed-loop system may achieve better robust stability or robust performance.

3 Methodology

A nonlinear chemical process can be represented by the diagram in Figure 3, where Δ is generally used to represent the neglected dynamics of the process, $\mathcal{P}_p$ is the linearized model for the process at a specific operating point, and K is a controller. Further constraints can be manually imposed on Δ to cope with the influence of nonlinearity and perturbations [1].

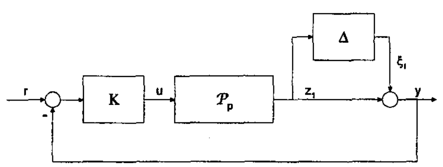

Figure 3: Diagram of a typical chemical process

It is easy to check that Figure 3 is equivalent to Figure 1 with

$$\mathcal{P} = -\mathcal{P}_p K(I + \mathcal{P}_p K)^{-1}.$$

If Δ is passive, a controller K can be synthesized to render $\mathcal{P}$ strictly passive. By Lemma 1, the same controller will guarantee that the closed-loop system in Figure 3 is robustly stable.

If Δ is near passive or over passive, then by Lemma 2, a transform T can be applied, as shown in Figure 4. T is set to $T = \beta I$ for both near passive and over passive Δ in this paper, and $\beta \geq v(\Delta)$ given in (3) for the near passive case.

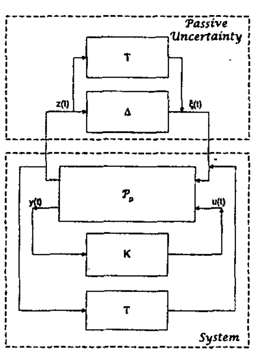

Figure 4: Transformed interconnect feedback system

The transformed feedback system can be viewed as an interconnection of two transformed blocks, namely, 'Passive Uncertainty' block with input $z(t)$ and output $\xi(t)$ and 'System' block with input $\xi(t)$ and output $z(t)$, as shown in Figure 4. By Lemma 1, K only needs to render the 'System' block strictly passive.

The 'System' block can be described in the following state-space form:

$$\begin{aligned} \dot{x}(t) &= A(\beta)x(t) + B(\beta)u(t) + H(\beta)\xi(t) \\ y(t) &= Cx(t) \\ z(t) &= E_1 x(t) + E_2 u(t) + E_3 \xi(t) \end{aligned} \tag{4}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^m$ is the measured output of the process, $u(t) \in \mathbb{R}^r$ is the manipulated input to the process. $\xi(t)$ and $z(t)$ can be viewed as the input and output of the 'System' block. β is a preselected constant scalar based on the passivity index for Δ.

Based on Lemma 1, Lemma 2, we have the following result:

Theorem 1 *If there exists a controller $u = Ky$ which renders system (4) strictly passive, then the same controller guarantees that the closed-loop system shown in Figure 3 is robustly stable.*

Note that passivity index measures how far a system $P(s)$ is from being passive. It is strictly negative if $P(s)$ is strictly passive. The passivity index may also be viewed as an indication of disturbance tolerance. Suppose that $P(s)$ is perturbed to be $\tilde{P}(s) = P(s) + \Delta_1(s)$ where $\Delta_1(s)$ can be either passive or non-passive. By the Weyl inequality, it can be easily verified that:

$$v(\tilde{P}) \leq v(P) + v(\Delta_1).$$

Obviously, $\tilde{P}(s)$ will remain strictly passive if $v(P) + |v(\Delta_1)| < 0$. Therefore, the passivity index as defined in the form (3) for the 'System' block in Figure 4 may be used as an index for best process decomposition selection when decentralized design is concerned.

Unfortunately, the passivity index for the 'System' block in Figure 4 is only available when a controller is designed and applied to the transformed $\mathcal{P}_p$ block. Therefore, it is necessary to introduce am alternative index which can be accessed using open loop information.

To this end, we will use γ-passivity to address the 'System' block instead.

Definition 2 *[8] Let $0 \leq \gamma < 1$. The closed-loop system for (4) is called γ-passive if it is dissipative with a supply rate of $s(\xi, z) = \frac{1}{2}(\gamma^2\|\xi + z\|^2 - \|\xi - z\|^2)$ and the storage function $V(x)$ satisfies $V(0) = 0$. Let $P(s)$ represent the closed-loop transfer function matrix for system (4). $P(s)$ is γ-passive if it is analytic in* Re $s \geq 0$ *and satisfies*

$$(\gamma^2-1)P^*(s)P(s)+(\gamma^2+1)(P^*(s)+P(s))+(\gamma^2-1)I \geq 0. \tag{5}$$

It can be verified that if a system is γ-passive, then it is strictly passive. Conversely, if a system is strictly passive, then it is also γ-passive for some $0 < \gamma < 1$.

Furthermore, the passivity index of a γ-passive $P(s)$ satisfies the following inequality:

$$v(P) \leq \frac{\gamma^2 - 1}{\gamma^2 + 1} \inf_{\omega \in \mathbf{R}} \lambda_{min}[P^*(j\omega)P(j\omega) + I] \quad (6)$$

From the definition of γ-passivity, a smaller γ reduces the supple rate $s(\xi, z)$, and hence guarantees larger dissipated energy for the system.

If the transformed 'Passive Uncertainty' block in Figure 4 is such that its passivity index is a fixed real value, then a smaller $\gamma < 1$ for the transformed 'System' block means that the feedback system in Figure 4 is robustly stable with larger energy dissipation, hence the feedback system should have better performance. (6) further implies that s smaller γ indicates that the feedback system has better perturbation tolerance.

It is interesting to notice that there is a connection between a γ-passivity problem and a H_∞ problem:

Theorem 2 *[8] Let $\gamma < 1$ and suppose $I + E_3$ is nonsingular. A controller K renders system (4) γ-passive if and only if K solves the following H_∞ problem with L_2 gain γ:*

$$\begin{aligned} \dot{x}(t) &= (A(\beta) - H(\beta)(I + E_3)^{-1}E_1)x(t) \\ &\quad + (B(\beta) - H(\beta)(I + E_3)^{-1}E_2)u(t) \\ &\quad + \sqrt{2}H(\beta)(I + E_3)^{-1}\xi(t) \\ y(t) &= Cx(t) \\ z(t) &= \sqrt{2}(I + E_3)^{-1}E_1x(t) + \sqrt{2}(I + E_3)^{-1} \times \\ &\quad \times E_2u(t) - (I - E_3)(I + E_3)^{-1}\xi(t) \end{aligned} \quad (7)$$

The H_∞ solution for system (7) is readily available in the literature. The convex optimization method can be adopted to find the optimal solution in Matlab [2, 5]. For example, an iterative procedure based on the work by Gahinet and Apkarian [4] can be applied to find the minimal γ and an explicit formula for the controller K.

Moreover, solvability of the H_∞ problem for the system (7) and the achievable minimal γ are only related to the open loop system (4). This inspires us to use the γ as an index for best decomposition selection in multi-model approach. Therefore, the following procedure is proposed for multi-model robust control:

Design procedure for multi-model robust control

1. Operating space justification: For a nonlinear chemical process, the possible disturbance sources and their corresponding extents can be identified. The combined effect of manipulated variables and the disturbances leads to an operating space for the nonlinear process;
2. Linearization: Once the operating space is obtained, a grid method can be used to analyse the operating space. Linearization of the nonlinear process is performed on every grid point which is also an operating point. This results in a class of linearized local models;
3. Process decomposition: At each grid point, the linearized local model is used for analysis. Since a decentralized control structure is often desired in chemical process control, decompositions can be obtained for the local model to represent various decentralized structures (see [9] and [7] for details about decompositions);
4. Best decomposition selection: All decompositions at a grid point are screened through by passivity analysis. Let the local open loop linear model be $G(s)$ and a particular decomposition be $G_d(s)$, then $\mathcal{P}_p(s) = G_d(s)$ and $\Delta(s) = [G(s) - G_d(s)]G_d^{-1}(s)$ in Figure 3. Next, a transform $T = \beta I$ in Figure 4 is adopted such that $\beta + v(\Delta) = -\epsilon$ where $\epsilon \geq 0$ is a pre-defined scalar. Finally, the γ-passivity analysis is performed on the 'System' block (4) in Figure 4 and a corresponding minimal γ is obtained using theorem 2. The best local decomposition at the specific grid point is selected as the one which achieves the smallest γ;
5. Local decentralized controller design: Both (4) and (7) have decentralized structures. The best local decentralized controller is obtained by applying theorem 2 on the best local decomposition;
6. Operating space evaluation: Steps 3-5 are repeated throughout all grid points of the operating space. This results in a class of best local decompositions;
7. Operating space partition: Neighbouring grid points may admit the same best decomposition structure. All of these neighbouring "similar" grid points can be joined together to form a subregion. As a result, the whole operating space is separated into several sub-regions with each subregion admits the same best decomposition structure as well as the same decentralized controller structure. Instead of designing a decentralized controller on every grid point, it is now possible to use one or several decentralized controllers to cover the whole sub-region. This will simplify the final multi-model control structure;
8. Closed-loop verification: Once the decentralized controllers are designed, the passivity index for $\mathcal{P}$ is re-evaluated through all of the grid points. Necessary modifications can be performed to get better sub-region partitions.

4 An Illustrative Example

The reactor/separation process shown in Figure 1 is adopted for the illustrative example. The process has five controlled variables and five manipulated variables.

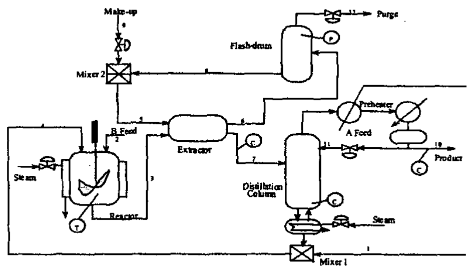

Figure 5: A reactor/separation process

The controlled variables are the reactor temperature T_R, the raffinate composition x_{G7}, the product composition x_D, the bottom composition x_B and the flash-drum pressure P. The five manipulated variables are the steam flowrate S, the make-up flowrate Mk, the reflux flowrate L, the boilup rate V and the purge flowrate F_{12}. The process is adopted from Lee *et al.* [7]. It is assumed that the disturbance sources have been identified. The disturbance variables are the A feed flowrate F_1, A feed temperature T_{F0}, B feed temperature T_2, steam temperature T_{J0}, the bottom flowrate B, the bottom temperature T_B and the recycle-flash flowrate F_8. For simplicity and clarity, it is further assumed that only feed A flowrate F_1 and the bottom flowrate B change from 10 kmol/hr to 20 kmol/hr. The rest of the disturbance variables remain constant.

Nine major decompositions can be generated for this reactor/separation process, using the physical decomposition and the decomposition across units. A summary of these nine decompositions can be found in Lee *et al.* [7], therefore, the details are omitted for brevity.

Applying the design procedure, it was found that the candidates for the best decompositions were decomposition DAU7 and decomposition DAU8.

Figure 6 shows the γ plane of DAU7 over the whole operating region. Figure 7 shows the γ plane of DAU8 over the whole operating region. The zero crossing curve of the γ different plane is shown in Figure 8. Obviously, γ of DAU7 is greater than γ of DAU8 in sub-region 1, while the situation is reversed in sub-region 2. According to step 4 of the design procedure, it can be declared that DAU8 is the best decomposition for sub-region 1 while DAU7 is the best decomposition for sub-region 2.

Once open loop subregion partition is obtained, it is possible to construct a decentralized control scheme for each subregion and then apply the decentralized controllers to form closed-loop systems over the whole operating space. Then, closed-loop stability and performance can be verified by evaluating the closed-loop passivity index over the operating space. Figure 9 shows the passivity index plane for the closed-loop system using a DAU7-based decentralized controller, Figure 10 shows the passivity index plane for the closed-

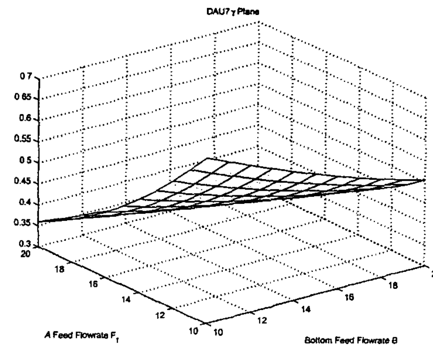

Figure 6: γ plane for DAU7

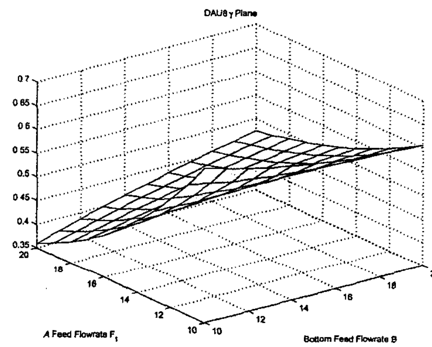

Figure 7: γ plane for DAU8

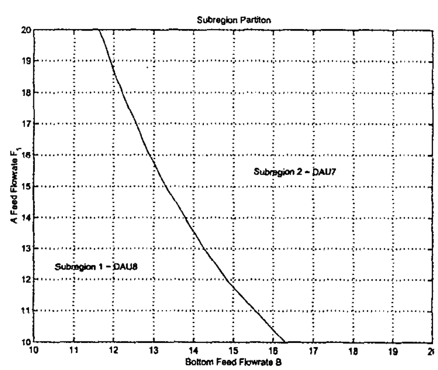

Figure 8: Open loop subregion partition

loop system using a DAU8-based decentralized controller. It can be seen from the zero crossing curve of the closed-loop passivity index different plane in Figure 11 that the operating space is partitioned into two closed-loop subregions. Although the closed-loop region shapes are different from the open loop ones, it is clear that two decentralized schemes have to be deployed in different operating regions in order to reach the best overall stability and performance over the whole operating space.

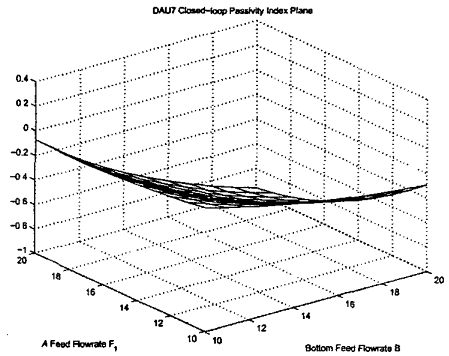

Figure 9: Closed-loop passivity index plane for DAU7

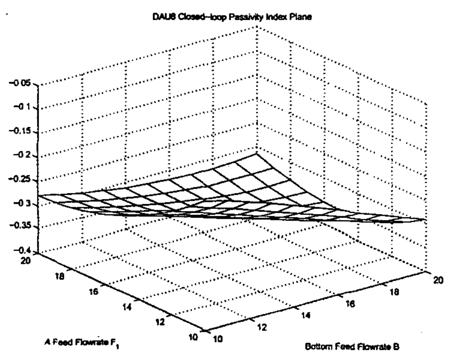

Figure 10: Closed-loop passivity index plane for DAU8

5 Conclusion

This paper has proposed a passivity based approach to decentralized control design for nonlinear multi-unit chemical processes. The proposed method involves evaluating a passivity index for a set of alternative plant decompositions in an operating region. Based on the values of the index, the operating region is partitioned into several subregions with each sub-region admitting the same best local plant decomposition. Thus local decentralized controllers can be designed using the best local plant. Closed-loop studies are further applied to verify that decentralized control design based

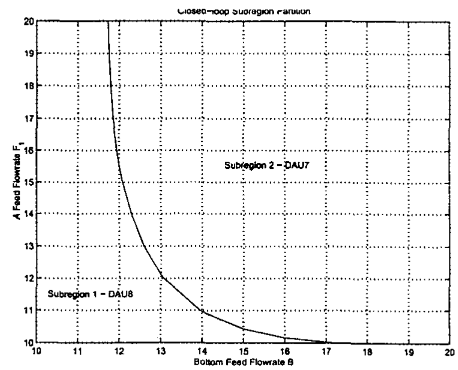

Figure 11: Closed-loop subregion partition verification

on the open loop sub-regions satisfies adequate closed-loop stability and performance requirements.

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