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Simple Arithmetic Processing: The Question of Automaticity.

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Abstract

In adult simple arithmetic performance, it is commonly held that retrieval of solutions occurs automatically from a network of stored facts in memory. However, such an account of performance necessarily predicts a uniform reaction time for solution retrieval and is therefore not consistent with the robust finding that reaction time increases with problem size and difficulty. Additionally, past research into arithmetic performance has relied on tasks that may have actually induced and measured attentional processing, thereby possibly confounding previous results and conclusions pertaining to automaticity. The present study therefore, attempted to more reliably assess the influence of automatic processing in arithmetic performance by utilizing a variant of the well-established semantic word-priming procedure with a target-naming task. The overall results revealed significant facilitation in naming times at SOAs of 240 and 1500 ms for congruent targets i.e., targets that represented the correct solutions to problems presented as primes (e.g., 6 + 8 and 14). Significant inhibition in comparison to a neutral condition (0 + 0 and 17) was also observed at 120 and 240 ms SOAs in naming incongruent targets (e.g., 6 + 8 and 17). Furthermore, response times were found to vary as a function of both arithmetic fluency and problem size. Differences in performance to addition and multiplication operations and implications for cognitive research and education are considered.

PsycINFO classification: 2343; 2346*Key Words:* Arithmetic, Fluency, Automaticity, Priming, Naming

1. Introduction

How is simple arithmetic knowledge organised in and accessed from the adult human brain? Over the past three decades, most models of adult arithmetic processing have converged on the notion that adults solve single-digit addition and multiplication problems solely through automatic fact retrieval from memory (Ashcraft, 1992; LeFevre et al., 1996b). Foremost amongst these models has been Ashcraft's (1992) Associative Network Retrieval model, which posits that arithmetic facts exist in a network of stored associations that are based on the operands and their related nodes. Retrieval of facts is thought to occur via automatic spreading activation, a process that is considered to be fast, accurate, obligatory, and requiring minimal cognitive load (LeFevre & Kulak, 1994).

Support for the associative network model and in particular the notion of obligatory (i.e., unintentional) activation of arithmetic knowledge derives from the presence of cross-operation confusion effects in the performance of production and verification tasks (LeFevre & Kulak, 1994). For example, in production tasks, cross-operation errors occur where the incorrect solution that is produced represents the correct solution to an alternative operation involving the same operands e.g., $2 + 3 = 6$ (Ashcraft, 1992; Campbell, 1987; Cipolotti & Butterworth, 1995). Likewise, in verification tasks, it takes longer to determine that a cross-operation equation is false than it does to determine that an equation with an unrelated solution (e.g., $2 + 3 = 11$) is false (Ashcraft, 1992; LeFevre & Kulak, 1994; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Thus, as opposed to being obtained through procedures and rules, arithmetic solutions appear to be directly retrieved from a highly organised and associated network in long-term memory (Ashcraft, 1992; LeFevre & Kulak, 1994).

Further support for this model and the notion of obligatory activation stems from a series of investigations undertaken by LeFevre and her colleagues that employed a number-matching task. In this task, participants were first presented with a pair of numbers (e.g., 3 + 4) and then following a given inter-stimulus interval, were simply required to decide whether a target number (e.g., 7) was one of the two numbers originally presented. The results of investigations involving both the addition (LeFevre, Bisanz & Mrkonjic, 1988; LeFevre & Kulak, 1994) and multiplication (Thibodeau, LeFevre & Bisanz, 1996) operations showed that in contrast to other unrelated numbers (e.g., 3 + 4 and 9), the presentation of the correct sum or product (respectively) led to lengthier decision times. Moreover, this interference effect occurred quickly, at very short SOAs, and was found regardless of the exclusion of the arithmetic operator. Importantly, consistent with the notion of automatic spreading activation, the solution was produced irrespective of the intentions of participants to simply match numerical symbols (LeFevre et al., 1988; LeFevre and Kulak, 1994; LeFevre, et al., 1996a, 1996b; Thibodeau et al., 1996).

In a similar study, Galfano, Rusconi and Umilta (2003) employed a number-matching task to determine whether multiplicatively related facts (e.g., 16) that either precede or follow the correct product (i.e., 24) could be automatically activated following the presentation of two numbers (i.e., 8 and 3). The results showed that decision times to target numbers that were adjacent to the correct product in the table (related to either of the operands within the prime) were increased in comparison to other unrelated targets. The authors again, concluded that multiplication facts are represented in a highly associated network, with automatic activation also spreading from the correct product to adjacent nodes.

Thus, it would seem that the evidence in support of the associative network model and obligatory activation is rather convincing. However, a disadvantage of this explanation of arithmetic processing arises from the fact that, in principal, it can not account for the problem size (or difficulty) effect. This refers to the apparently robust finding that it becomes more difficult and takes longer to process problems as they become larger in size (Ashcraft, 1992; Brysbaert, 1995). Such an effect is different to the uniform reaction time pattern that is predicted by automatic retrieval models.

Nonetheless, in keeping with the associative network model, explanations for the problem size effect have centred on structural rationales relating reaction times to numerical indices (such as the distance to be traversed within a network) or to the frequency of exposure to particular problems in early education (Ashcraft, 1992; Ashcraft, Donley, Halas & Vakali, 1992; LeFevre et al., 1996a). In the latter case, data taken from elementary textbooks, shows that smaller problems appear earlier in instruction and more frequently than do larger problems (Hamman & Ashcraft, 1986). Smaller numbers also appear more frequently than larger numbers in naturally occurring settings (Ashcraft, 1992). Accordingly, it is not inconceivable that greater exposure to, and more practice of, smaller problems will result in fact retrieval being increasingly automated in comparison to larger, more difficult problems (Ashcraft, 1992; Ashcraft et al., 1992; Koshmider & Ashcraft, 1991; Siegler, 1988; Siegler & Jenkins, 1989).

The notion that there may be differences in the way that particular problems are solved is not new to the cognitive arithmetic literature. In fact, this very idea serves as the basic premise underlying Siegler and Jenkins' (1989) Distributions of Associations model, which posits that knowledge representations of particular problems develop a set of associated solutions and a set of methods for their accurate retrieval. Furthermore, depending on the strength of associations between problems and their correct solutions (a factor influenced by frequency of exposure), retrieval of arithmetic facts may occur using either automatic or strategic processing mechanisms. Importantly, the inclusion of both mechanisms in performance allows for the prediction of reliable differences in reaction time and consequently a possible explanation of the problem size effect (Ashcraft et al., 1992; Koshmider & Ashcraft, 1991).

Empirical support for a difference in processing between problems of varying size and difficulty derives from a priming study conducted by Ashcraft et al. (1992). Simple multiplication problems and their solutions were first divided into three problem difficulty groups (i.e., low, medium and high). All problems were then presented twice, once neutrally primed by a line of two dashes (e.g., -- primed 6 x 5 = 30) and once primed by either the correct solution, a related solution or an unrelated solution (e.g., 30, 25, or 21 primed 6 x 5 = 30, respectively). Participants were simply required to decide whether the presented problem, was true or false. The results showed that correct primes had a positive effect on reaction time, although, for the high difficulty problems, this occurred only at a long SOA. Furthermore, related and unrelated (irrelevant) primes were found to yield negative effects, especially for the more difficult problems. This finding was deemed consistent with the notion that the incorrect problem led to confusion and that, in contrast to low and medium difficulty problems, the more difficult problems were solved using conscious processing.

Two main difficulties arose with the methodology employed in the Ashcraft et al. (1992) study. The first of these occurred in that the order of stimulus presentation may have led to an overestimation of the levels of priming and inhibition that occur in normal arithmetic processing. For instance, in true trials, exposure to the correct solution before exposure to the problem would have led to prior activation of this number in memory, and consequently faster responses than would normally occur following simple exposure to an arithmetic problem. Similarly, in false trials, pre-exposure to an incorrect solution possibly created greater levels of confusion than would normally be encountered in arithmetic tasks.

The second difficulty with the Ashcraft et al. (1992) study occurred in relation to their choice of verification procedure. In such procedures, access to arithmetic processing may be confounded by responses to problems that are carried out using some sort of familiarity judgement, possibly involving comparison of the equation as a whole to information in memory (LeFevre et al., 1996). Additionally, Campbell (1987) argues that participants may rely on plausibility judgements in terms of approximate magnitude or on the odd-even status of the presented answer in relation to the problem's operands. Moreover, for incorrect trials, previous research has shown that when the difference in magnitude between an incorrect and correct solution is large it is verified more quickly than if this difference is only small (commonly referred to as the 'split effect'; Ashcraft, 1992; Campbell, 1987). Such an effect, whilst not distorting reaction times to correct trials, may confound other conditions and consequently influence the final outcomes of the study (Campbell, 1987). Finally, similar arguments to those levelled at verification procedures in the single-word semantic priming literature (i.e., lexical decision tasks) can be made with regard to those employed in studying simple arithmetic. Specifically, it has been suggested that 'attentional' decision processes, that occur after the simple matching of a stimulus with its lexical representation, may confound the overall reaction time measured in the lexical decision task (Balota & Lorch, 1986; Friedrich, Henik & Tzelgov, 1991; Lorch, Balota & Stamm, 1986; Neely, 1991; Sereno, 1991; Slowiaczek, 1994; Smith, Besner & Myoshi, 1994). In the case of the aforementioned studies then, it could be argued that the requirement to actively make a binary decision as to the relationship between the prime and the target might interfere with the automatic processes, essentially thought to occur without intention or awareness, that they purport to measure.

Having acknowledged the difficulties inherent in verification tasks, Campbell (1987, 1991) resolved to employ a production task in the examination of differences in processing between multiplication problems of varying difficulty. In two studies (employing different SOAs of 300 and 200 ms, respectively), problems were first divided into easy and difficult categories, based on normative production error rates. Participants were then presented with one of four prime types: the correct product, a neutral prime (##), a related false prime (frequently occurring as an error for a given problem) and an unrelated false prime (occurring with low frequency as an error response to the problem). Following this, they were presented with a problem and required to produce the correct solution. In both studies, facilitation to more difficult problems was greater than to easy problems. According to Campbell, this showed that priming using the correct answer improved retrieval of less accessible (i.e., more difficult) answers in comparison to more automatic answers that had already reached a ceiling such that no appreciable effects on performance could be realised.

Interestingly, in Campbell's (1987, 1991) studies, inhibition was found when a related but incorrect prime preceded each problem. As noted by Campbell, the presence of inhibitory effects suggests the use of attentional, conscious processes in performance and, in the context of these studies, may have reflected a deliberate attempt by the participant to ignore interference caused by the prime. Furthermore, with the subject always intending to accurately perform arithmetic calculations, it could again be argued that the line between unintentional, automatic processing and conscious processing was blurred. Finally, the use of only short SOAs did not allow for the analysis of changes in facilitatory and inhibitory effects over time (Koshmider & Ashcraft, 1991).

More recently, LeFevre and colleagues addressed the issue of differences in processing in both addition and multiplication procedures using self-report measures. In two studies, samples of undergraduate students were first required to provide solutions to given problems and then to describe how they obtained them. In the addition study, the results indicated that an amazing 25% of all solutions from a 'relatively skilled' university sample were achieved through a strategic transformation (6 + 5 = 6 + 4 + 1) or counting (3 + 2 = 3, 4, 5) procedure (LeFevre et al., 1996a). This figure was again reflected in the multiplication study, with the use of such conscious retrieval methods as rules (0 x n = 0), repeated addition (2 x 3 = 3 + 3), number series (3 x 3 = 3, 6, 9) and derived facts (3 x 4 = [3 x 3] + 3) reported on 20% of all trials (Lefevre et al., 1996b). In addition, examinations into individual differences revealed significant correlations between arithmetic fluency and the percentage use of retrieval in both operations. Thus, the authors concluded that learning and experience had a continuing influence on adult arithmetic performance and that solely automatic fact retrieval explanations of performance did not provide a complete account of adult processing.

Unfortunately, as noted by Lefevre and Colleagues (1996a, 1996b), the use of self-report as a valid and reliable measure of performance was critical to the interpretation of their data. The self-report methodology has nonetheless been criticised on the grounds that, when asked to describe mental processing, people may change or be unable to accurately describe their behaviours (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003). Additionally, individual differences and instructional demands may bias verbal reports and the solution procedures that are reported (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003). Indeed, a recent study by Smith-Chant and LeFevre (2003) showed that low skill participants responded more slowly and accurately when asked to describe their solution procedures for large and very large problems. Furthermore, low skill participants exhibited greater variation in procedures and were more likely to alter their selection of retrieval method, with changes in instructional emphasis between speed and accuracy. Thus, the possibility of reactivity in the LeFevre et al (1996a, 1996b) study could not be ruled out, leading these authors to the call for the use of alternative, more reliable methods in the investigation of the role of automaticity in arithmetic performance.

In an attempt to address this, the present study borrowed from the well-established single word semantic priming paradigm and employed a procedure similar to that used in the previous Campbell (1987, 1991) studies. In contrast to the earlier research, however, the present study involved the presentation of a problem as the prime and a solution as the target, in the order that they would appear in a naturally occurring setting. Additionally, a naming task that simply required the subject to state the target number as it appeared and not to perform any verification, calculations, or relationship matching based on the prime, was utilised. This served to both minimise the possibility of decision-induced attentional processing and to reduce the influence of errors in production on subsequent trials (Campbell, 1991). Furthermore, as recommended in Koshmider and Ashcraft (1991), SOAs representing both automatic and conscious processing conditions were employed. Problems were then randomly assigned to appear in all conditions and equally divided into problems containing both small and large numbers, and a mix of the two. This allowed for a comparison of processing between problem sizes (and difficulty) over time. Finally, to allow for an investigation into the influence of skill on arithmetic performance, the participants arithmetic fluency was measured using the arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) (Form C; 1984).

2. Method

2.1 Participants

Thirty-nine psychology students, including 16 males and 23 females, from Murdoch University participated in the present study. The participants' ages ranged from 16 to 53 years, with a mean age of 27.

2.2 Design and stimulus materials

Three independent variables were examined in the present study. The first of these determined the arithmetic operation i.e., addition or multiplication. The second variable incorporated three prime-target relationships, including congruent (e.g., 2 + 4 = 6), incongruent (2 + 4 = 9) and neutral (0 + 0 = 6) conditions. The final independent variable was SOA with three levels: 120 ms, 240 ms and 1500 ms.

Four sets of primes were constructed for each of the addition and multiplication operations (see Appendix A). The first set for each operation consisted of 18 simple arithmetic facts selected from the 2s through 9s matrices (e.g., 2 + 3). The second set comprised the reverse operand placement equivalents of the first set (3 + 2). The third set for each operation contained a mix of problems taken from the first and second sets, such that no two problems represented the same arithmetic fact (i.e., if 2 + 3 was already chosen in the third set, then 3 + 2 was not also selected). The final set consisted of the reverse operand placement equivalents of the third set.

Arithmetic facts resulting from ties (e.g., 3 + 3 and 3 x 3) were excluded from use as primes, because previous research indicates that these problems are solved more quickly than others (LeFevre et al., 1988). Additionally, to balance each prime set, half of the arithmetic facts were produced so that the smaller of the two operands in each problem was placed on the left-hand side and half with the smaller operands on the right hand side. Finally, to enable testing for the presence of the problem size effect, each stimulus set consisted of six smaller problems (i.e., with both operands of a magnitude less than or equal to five; 2 + 3), six larger problems (operands greater than or equal to six; 8 + 9), and six of mixed magnitude (2 + 9).

Target sets constructed for each of the congruent, incongruent and neutral conditions consisted solely of the correct solutions corresponding to the simple arithmetic facts investigated in this study. For the incongruent condition, these targets were simply paired with an alternative problem, making them mathematically erroneous. To guard against split effects in the multiplication condition, incongruent solutions were paired with problems so that they differed by at least 16 from the correct solutions to these problems. Similarly, for the addition condition, incongruent solutions differed by at least three from the correct solutions. Further constraints on the incongruent target sets addressed possible confounding relationships between the prime and the target. For example, incongruent targets were not permitted to be one of the operands or the numbers plus or minus one from the operands, used in the prime. Additionally, where possible, multiples or factors of the operands and number series relations were excluded. Finally, incongruent targets were such that they could not be the correct solution to the prime using a different operation, a double-digit number containing the operand, or a number containing the correct solution (i.e., if the correct solution was 7, then numbers such as 17 and 70 were also excluded).

Neutral conditions have not been widely used in the study of arithmetic but have been useful in assessing facilitation and inhibition and hence distinguishing automatic from conscious processing in word priming research (Neely, 1991). As such, in the present study, the neutral stimuli (i.e., 0 + 0 for the addition condition and 0 x 0 for the multiplication condition) were developed in accordance with three main criteria that were outlined in a review of the word priming literature by Neely (1991). The first of these was that neutral primes should be equated with other primes in relation to their value as a warning signal that a target will soon appear. In the present study, the numerical prime 0 + 0 can be likened perceptually to the other primes such as 2 + 3, with both consisting of two numerical operands separated by an arithmetic operator. Secondly, according to Neely, neutral primes should be unassociated to the target so that they are a neutral baseline by which to assess spreading activation between related stimuli. The 0 + 0 and 0 x 0 stimuli were unrelated to the targets that were employed in the present study both in terms of arithmetic relatedness and distance along the number line. Lastly, Neely suggested that neutral primes should not offer any information as to the semantic nature of the target to follow in order to provide a baseline by which to compare expectancy effects. In the case of this last criterion, as with the use of neutral primes such as *ready, neutral* or *blank* in the word priming literature, it may be argued that the 0 + 0 and 0 x 0 stimuli are not necessarily semantically neutral (possibly leading to the expectation that 0 will be presented as the target). Importantly however, their repeated presentation ensures that semantic satiation is rapidly attained and that less processing capacity is consumed, thereby enabling them to serve as an effective neutral baseline.

2.3 Psychometric testing

The arithmetic section of the ACER SCT incorporated 60 arithmetic problems that variously included the addition, subtraction, division and multiplication of single, two and three digit numbers (ACER, 1984). Participants were given five-minutes in which to accurately complete as many problems as they could. They were instructed to start from the first problem and to work through each in turn, without omitting any problems (ACER, 1984). Rough working out could be undertaken anywhere on the page and participants were informed that if they completed the first column, they should immediately go onto the second one (ACER, 1984).

The total number of problems solved correctly served as the participant's fluency score. One participant did not return for this test. The remaining participants' scores ranged between 10 and 47. A median split procedure was then used to allocate 19 participants who scored less than or equal to 17 to the low skilled group, and 18 participants scoring greater than 17 to the high skilled group. According to the ACER SCT manual, a score of 17 corresponds to a percentile rank of 2% in a normed sample of 124 candidates who had completed a three to four year degree or diploma in a tertiary institution. The mean correct score for the low skilled group was 14, which was lower than any score obtained by the normed sample. In contrast, the mean correct score for the high skilled group corresponded to a percentile rank of 14%, with the highest score in this group corresponding to a percentile rank of 92% in the normed sample.

2.4 Procedure

Participants were individually tested in a well-lit cubicle room containing an Amiga 1200 microcomputer with 1084S monitor, that controlled stimulus presentation, trial sequencing, timing and data collection. An additional monitor outside of the cubicle displayed reaction times and target stimuli so that accuracy could be monitored. All stimuli were centrally presented, white against an amber background. Individual operands within each problem did not exceed dimensions of 5 x 15 mm on the computer screen. Arithmetic operators (i.e., x and +) did not exceed 5 x 10 mm and a 5 mm gap separated operands from the operator within each problem. A chin rest was used to stabilise the participant's head at a viewing distance of 60cm from the screen.

Each testing session began with 20 unique practice trials and thereafter comprised six blocks of 54 experimental trials (i.e., three for each of the addition and multiplication operations corresponding to each of the three SOA conditions). Addition and multiplication trials were separately blocked so as not to produce cross operation or relatedness errors. Half of the participants started with the addition block first and half started with the multiplication block first. Additionally, half of the participants were exposed to the first set in the 120 ms SOA condition, and half to the second set. The set not assigned to the 120 ms condition was then presented in the 240 ms condition. Participants were exposed to both sets in order to reduce repetition of the priming stimuli at the short SOA's, whilst keeping the target stimuli the same. Half of the participants were then presented with the third set and half with the fourth set in the 1500 ms condition. Repetition of the first and second set trials at the longer SOA allowed for a level of familiarity with the stimuli, drawing attention to the prime-target relationship. Finally, the computer randomly generated the order of presentation of the individual congruent, incongruent and neutral trials within each block and exposure to all stimuli was counterbalanced across participants.

Prior to testing, participants were instructed on the need to respond both quickly and accurately. At the start of each trial participants were required to focus their gaze on a 1 x 1 mm blue central fixation dot, exposed for 600 ms. The screen then went blank for a period of 150 ms before the prime was presented for 100 ms. Following the given SOA, the target appeared and remained exposed until the participant identified the given number. An interval of two seconds separated the participant's response and the onset of the next trial. A microphone connected to a headset, with padded ear guards preventing external noise intrusions, was used to detect participant vocal response sounds. The microphone amplifier triggered an electronic relay interfaced to the computer, which determined the time of relay closure using a hardware timer. The value of the timer, accurate to 1 millisecond, measured the participant's vocal reaction time from the onset of the target.

On finishing the computer task participants completed the Arithmetic section of the ACER SCT. They were then debriefed, with the session having taken approximately 40 minutes to complete.

3. Results

3.1 Overall analysis

The correct mean response latencies were initially screened for outliers using a criterion of +/- 2.5 z-scores and replaced using mean substitution. This led to adjustment of less than 0.60% of all scores. The resulting reaction time data are presented in Table 1.

Table 1.
Mean Reaction Times (ms) and Standard Deviations (in parentheses) for all Prime-Target Relationships as a Function of SOA and Operation.

	SOA		
	120 ms	*240 ms*	*1500 ms*
Addition			
Congruent	443 (52.6)	419 (50.0)	431 (52.1)
Incongruent	446 (51.6)	433 (53.5)	451 (50.7)
Neutral	435 (45.4)	430 (48.4)	447 (47.7)
Multiplication			
Congruent	466 (49.0)	440 (49.0)	445 (61.8)
Incongruent	470 (48.2)	456 (53.7)	470 (52.6)
Neutral	464 (49.4)	447 (44.7)	465 (55.4)

An overall repeated measures analysis of variance, including operation, SOA and prime-target relationship as within group variables, was performed on these data. Significant main effects were found for all three variables. Firstly, reaction times to addition-related targets were 21 ms faster overall than to multiplication-related targets ($F(1, 37) = 46.1$, $MSe = 1{,}617.4$, $p < 0.001$). This difference in performance is best explained by differences in target magnitude. For example, in the present study, addition-related targets only ranged from 5 through 17 as compared to multiplication-related targets, which ranged from 6 through 72. Previous research has indicated that it takes longer to perform number naming tasks when numbers are large than when they are small (Ashcraft, 1992; Brysbaert, 1995). This finding was again supported in the problem size analysis below. Given this fundamental difference in processing, following the first analysis, the addition and multiplication operations were analysed separately.

Secondly, a significant main effect of SOA was found ($F(1.4, 52.7) = 10.7$; $MSe = 2{,}335.1$, $p = 0.001$. Violations of the assumption of compound symmetry were corrected throughout the present analyses by adjusting the degrees of freedom using Huynh-Feldt epsilons). Reaction times to the 120 ms condition were 16 ms slower than to the 240 ms condition but were no different from those obtained to the 1500 ms condition. One possible explanation for the lengthier response times at the 120 ms SOA is that the short interval between the onset of the prime and the presentation of the target interfered with the effectiveness of the prime as a warning signal for the target (Posner, Klein, Summers & Buggie, 1973). Posner and colleagues showed that 200 ms is the optimal period for a warning stimulus to precede a target in a simple spatial choice reaction time task, with shorter or longer inter-stimulus intervals leading to progressively longer overall reaction times. Additionally, the advantage in response times at the 240 ms SOA may have in part reflected a speed accuracy trade off. For example, whilst very few errors in number naming were found (i.e., less than 0.50% of all trials), 49% of errors occurred at the 240 ms SOA in comparison to only 28% at the 120 ms SOA and 23 % at the 1500 ms SOA.

Thirdly, in the overall analysis, a significant main effect of prime-target relationship was found ($F(2, 74) = 27.9$; $MSe = 366.8$, $p < 0.001$). Congruent trials had a 13 ms advantage over incongruent trials, and a 7 ms advantage over neutral trials. This overall pattern of performance was then further qualified by a significant interaction between SOA and prime-target relationship ($F(4, 148) = 9.2$, $MSe = 352.1$, $p < 0.001$). The facilitatory (neutral – congruent) and inhibitory (incongruent – neutral) differences describing this interaction are presented in Fig. 1. All 95% confidence intervals were calculated using the *MSe* terms for individual one-factor repeated measures ANOVAs involving the difference scores representing each of the facilitation and inhibition functions (Loftus & Masson, 1994; Masson & Loftus, 2003).

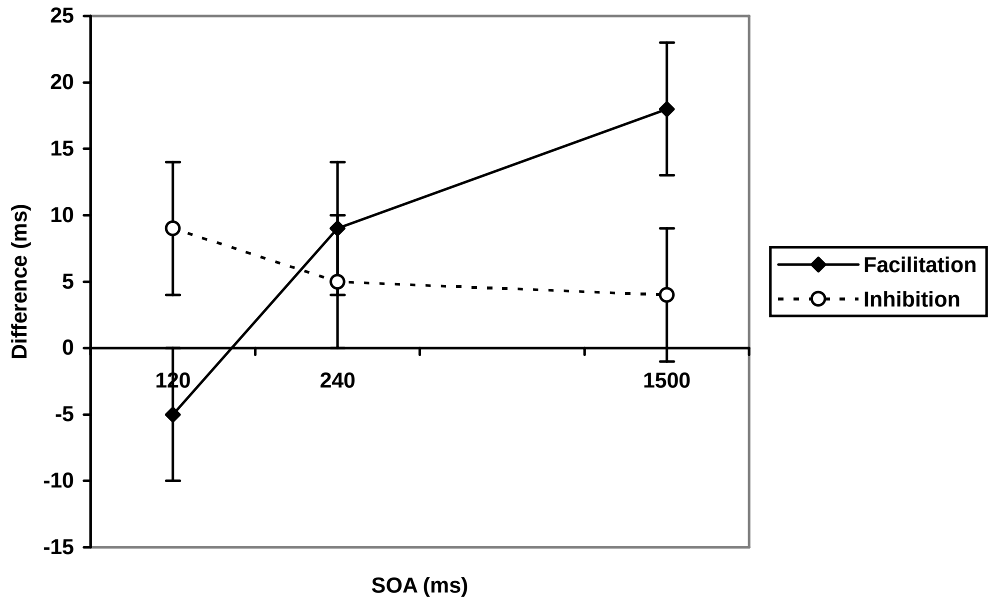

Fig. 1 Showing facilitation and inhibition as a function of SOA. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects.

Repeated measures t-test comparisons showed that the facilitation was significant at both the medium (t = 4.2, df = 37, p < 0.001) and long SOAs (t = 5.1, df = 37, p < 0.001). Furthermore, a one-way repeated measures ANOVA showed a significant increase in facilitation over time (F(2, 74) = 17.7, MSe = 277.9, p < 0.001), with increments at both the 240 (t = 4.3, df = 37, p < 0.001) and 1500 ms SOAs (t = 2.3, df = 37, p = 0.027) reaching significance. In contrast, t-test comparisons showed that inhibition to incongruent targets reached significance only at the short (t = 3.4, df = 37, p = 0.001) and medium SOAs (t = 2.2, df = 37, p = 0.034) and a repeated measures ANOVA showed that it generally remained constant over time (F(1.7, 61.6), = 0.9, MSe = 301.5, p = 0.402). The results of the present study, utilizing a naming task, are therefore generally consistent with those employing production and verification procedures in demonstrating positive effects of congruent primes on reaction times (Ashcraft et al., 1992; Campbell, 1987, 1991). Additionally, the pattern of performance, with significant inhibition found to incongruent targets at short SOAs, is consistent with that previously found in number-matching and verification procedures (LeFevre et al., 1988; Zbrodoff & Logan, 1986).

Finally, in relation to the overall analysis, t-test comparisons revealed equivalent levels of facilitation and inhibition at the medium SOA ($t = 0.99$, $df = 37$, $p = 0.324$) and facilitation dominance at the long SOA ($t = 2.7$, $df = 37$, $p = 0.010$). The findings of the present study, employing the priming paradigm and arithmetic stimuli, are thus similar to those described in studies investigating the time course of facilitation and inhibition in the investigation of associatively related word primes and targets (Neely, 1991).

3.2 Arithmetic Fluency

A separate split plot analysis of variance for each of the addition and multiplication operations was used to explore the influence of the between group variable arithmetic fluency. For the multiplication operation, as in the overall analysis, significant main effects of SOA ($F(1.7, 59.0) = 7.2$, $MSe = 1{,}664.3$, $p = 0.003$) and prime target relationship ($F(2, 70) = 28.3$, $MSe = 210.1$, $p < 0.001$) and an interaction between SOA and prime-target relationship ($F(4, 140) = 6.1$, $MSe = 221.9$, $p < 0.001$) were again found. Furthermore, a significant two-way interaction between fluency and prime-target relationship ($F(2, 70) = 5.6$, $MSe = 210.1$, $p = 0.006$) and a three-way interaction between fluency, SOA and prime-target relationship ($F(2, 70) = 6.9$, $p = 0.002$) were found. Facilitation and inhibition differences underlying this interaction are presented in Fig. 2.

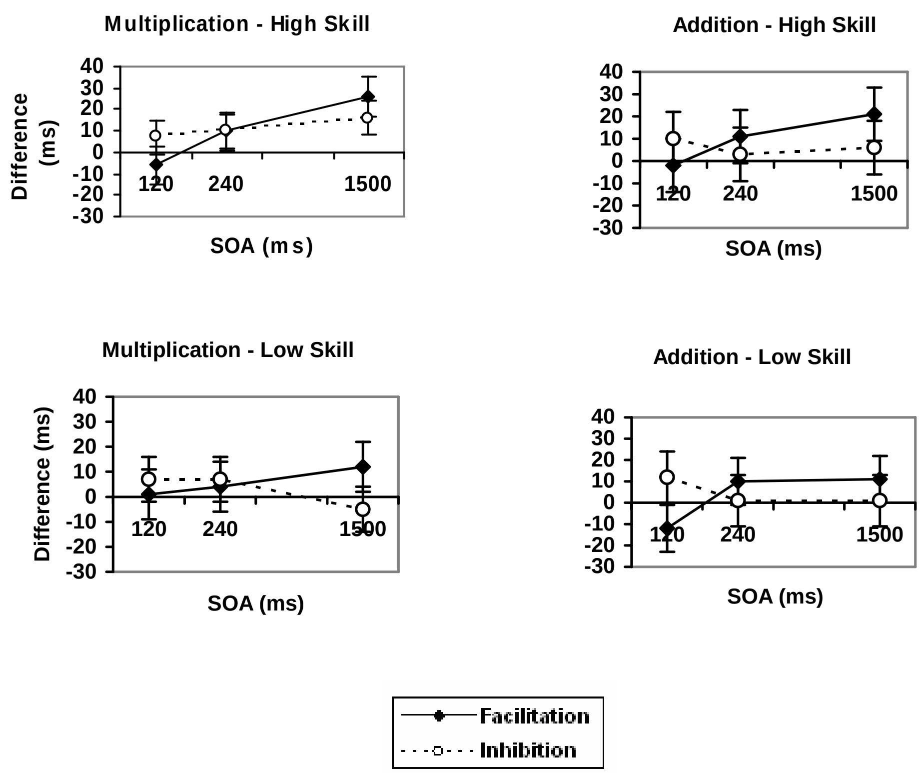

Fig. 2 Showing facilitation and inhibition for the multiplication and addition operations as a function of SOA and arithmetic fluency. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects.

For the high skilled group, a one-way repeated measures ANOVA showed that the facilitation to congruent targets increased significantly over time, across all SOAs ($F(2, 34) = 12.1$, $MSe = 372.6$, $p < 0.001$). Additionally, paired sample *t*-test comparisons showed that the level of facilitation at the medium SOA approached but did not quite reach significance ($t = 2$, $df = 17$, $p = 0.060$), whilst at the longest SOA it was highly significant ($t = 5.7$, $df = 17$, $p < 0.001$). The finding of such a strong advantage for the congruent condition, given a lengthy interval between the onset of the prime and the presentation of the target, possibly reflects the use of an expectancy strategy. Described in Neely (1991), this occurs where the participant deliberately generates a set of related targets that could be expected to follow a given prime. Consequently, the processing of expected targets is facilitated, whilst the processing of unrelated targets is inhibited. In accord with this interpretation, examination of Fig. 2 suggests that the cost in response to incongruent targets was greatest at the longest SOA for this group and was significantly greater than that observed for the low skilled group ($F(1, 36) = 13.5$, $p = 0.001$). However, this finding was not supported statistically, with a repeated measures ANOVA showing that the level of inhibition for the group remained constant over time ($F(2, 34) = 1.3$, $MSe = 282.9$, $p = 0.276$). Within group *t*-test analyses revealed significant levels of inhibition at each of the 120 ($t = 3.2$, $df = 17$, $p = 0.006$), 240 ($t = 2.2$, $df = 17$, $p = 0.040$) and 1500 ms ($t = 3.9$, $df = 17$, $p < 0.001$) SOAs for this group.

In contrast, for the low skilled group, no significant increase in facilitation was observed over time ($F(2, 36) = 1.4$, $MSe = 453.1$, $p = 0.249$). Moreover, the only observable effects in the data were a significant level of facilitation reached at the long SOA ($t = 2.1$, $df = 18$, $p = 0.049$) and a decrease in the level of inhibition observed between the 120 and 1500 ms SOAs ($t = 2.2$, $df = 18$, $p = 0.039$). Finally, in relation to the multiplication analyses, a between groups ANOVA showed that the difference between high and low skilled facilitation levels at the long SOA approached significance ($F(1, 36) = 3.7$, $p = 0.063$).

For the addition operation, as in the previous analysis, significant main effects of SOA ($F(1.8, 61.4) = 7.3$, $MSe = 1178.4$, $p = 0.002$) and prime-target relationship ($F(2, 70) = 12.2$, $MSe = 348.5$, $p < 0.001$), and a significant interaction between SOA and prime-target relationship were found ($F(4, 140) = 5.0$, $MSe = 290.5$, $p = 0.001$). No main effect of arithmetic fluency was present and, in contrast to the multiplication operation, it was not involved in any interactions. Nevertheless, in line with the particular interests of the present study in the effects of arithmetic fluency on processing, facilitatory and inhibitory differences were considered. For the high skilled group, a significant increase in facilitation over time was again observed ($F(1.5, 25.8) = 4.0$, $MSe = 818.0$, $p = 0.042$), occurring between the 120 and 240 ms SOAs ($t = 2.7$, $df = 17$, $p = 0.016$). Paired sample t-test comparisons showed that the level of facilitation was significant at both the 240 ($t = 4.2$, $df = 17$, $p = .001$) and 1500 ms ($t = 2.4$, $df = 17$, $p = 0.030$) SOAs. In contrast, the level of inhibition was significant only at the short SOA ($t = 2.4$, $df = 17$, $p = 0.026$) and, as in the multiplication condition, it remained constant over time. An ANOVA involving operation, SOA and prime-target relationship as within group variables, showed that although there was a main effect of operation (with responses to addition-related targets found to be 16 ms faster overall than to multiplication related targets; $F(1, 17) = 12.5$, $MSe = 1728.5$, $p = 0.003$), it was not involved in any interactions. High skilled performance in the addition and multiplication operations is therefore, generally the same.

As with the high skilled group performance, the low skilled results for the addition condition revealed a significant increase in facilitation over time ($F(2,36) = 6.0$, $MSe = 565.4$, $p = 0.006$), occurring between the 120 and 240 ms SOAs ($t = 2.6$, $df = 18$, $p = 0.018$). Furthermore, the level of facilitation approached significance at the 240 ms SOA ($t = 2.09$, $df = 18$, $p = 0.051$) and reached significance at the 1500 ms SOA ($t = 2.6$, $df = 18$, $p = 0.017$). No inhibitory differences reached significance in the data and they did not change significantly over time. Finally, no significant differences in the levels of facilitation or inhibition were observed between high and low skilled groups at any of the SOAs in the addition data.

In summary, the patterns of performance observed for both the high and low skilled groups in the addition condition were very similar, with increasing facilitation over time and advantages in performance evident for both groups at the long SOA. At the 240 ms SOA, automaticity in processing typified high skilled performance and most likely also characterised low skilled performance, given that the level of facilitation so narrowly missed significance. Performance by high skilled participants did not vary statistically between operations, with the facilitation at the 240 ms SOA again approaching significance in the multiplication condition. However, unlike the addition condition, at the long SOA, high skilled participants appeared able to apply their multiplication fact knowledge strategically to advantage performance in the naming task. This was in direct contrast to the low skilled performance, with the facilitation at the long SOA barely reaching significance and no advantage evident at the 240 ms SOA.

3.3 Problem size analysis

In order to determine the influence of problem size on arithmetic processing a subset of the data that included only the reaction times to small (consisting of operands ≤ 5) and large problem sizes (operands > 5) was selected. These data were initially screened for outliers using a cut off score of +/- 2.5 z-scores, leading to 1.24% of all scores being replaced using mean substitution.

In contrast to the earlier analyses in which each solution was presented in every condition, the use of only a subset of the data created a mis-match between the solutions in the congruent and incongruent conditions, and between problems and solutions of differing magnitudes. For example, in the multiplication condition congruent targets for small problems ranged between 6 and 20, whilst in the incongruent condition, except for the target 6, all other targets ranged between 30 and 63. Similarly, congruent targets for large problems ranged between 42 and 72, with incongruent targets mostly ranging between 15 and 24 (with the exception of targets 56 and 72). This led to difficulties in making direct comparisons within and between problem sizes because previous research indicates that as number magnitude increases, reaction time increases (Brysbaert, 1995). The raw data for all problems within the original stimulus set were thus entered into correlation and regression analyses to first ascertain any effect of target magnitude and then to account for this variable in the obtained reaction times. Pearson correlation coefficients and the best fitting model between the mean overall reaction time and number magnitude are presented in Table 2.

Table 2.
Pearson Correlation Coefficients (*r*) and Models of Best Fit between Reaction Time and Number Magnitude.

	SOA		
	120 ms	*240 ms*	*1500 ms*
Addition			
Congruent	.30	.41	.66*
Incongruent	.06	.04	.09
Neutral	.30	.37	.16
Model of Best Fit	Reaction Time = 0.79(Number Magnitude) + 429**		
Multiplication			
Congruent	.79**	.82**	.80**
Incongruent	.61**	.64**	.50*
Neutral	.48*	.64**	.70**
Model of Best Fit	Reaction Time = 0.48**(Number Magnitude) + 443**		

Note. $*p < 0.05$, two-tailed. $**p < 0.01$, two-tailed.

The results again supported the previous findings of an increase in reaction time with number magnitude. Strong positive correlations were present across all prime-target relationships, over the extensive range of magnitudes covered in the multiplication condition. Furthermore, number magnitude was shown to be a significant predictor of reaction time for this condition. For addition however, the association between number magnitude and reaction time was only evident at the longest SOA and number magnitude failed to reliably predict reaction time. The addition condition nonetheless, covered a much smaller range of magnitudes than that covered by the multiplication condition. Both models were thus employed to compute predicted reaction times scores for their respective operations. Following this, residual reaction time scores were calculated by subtracting the predicted reaction times from the observed ones. Mean residual reaction time scores were then produced for each of the small and large problem sizes, for all participants, by averaging the residual reaction times for the six smallest and the six largest problems, respectively. Analyses were then performed independently on both the raw and residual reaction time subsets. Both sets of data generally produced the same effects and so, only the residual analysis is reported here. The data for this analysis is presented in Table. 3.

Table 3.
Mean Residual Scores (ms) and Standard Deviations (in parentheses) for all Prime-Target Relationships as a Function of SOA, Operation and Problem Size.

	SOA		
	120 ms	*240 ms*	*1500 ms*
Addition			
Small Congruent	0 (54.1)	-23 (56.6)	-11 (47.4)
Small Incongruent	12 (47.4)	0 (57.8)	13 (53.1)
Small Neutral	-7 (41.5)	-15 (36.0)	12 (45.6)
Large Congruent	4 (51.0)	-24 (42.7)	1 (57.5)
Large Incongruent	5 (51.1)	-7 (53.9)	10 (54.5)
Large Neutral	1 (50.1)	-3 (55.8)	11 (54.5)
Multiplication			
Small Congruent	5 (49.1)	-19 (57.6)	-24(58.6)
Small Incongruent	16 (51.9)	-2 (57.9)	11(53.9)
Small Neutral	4 (43.2)	-9 (47.4)	1(54.1)
Large Congruent	7 (53.7)	-15 (56.2)	-11 (67.5)
Large Incongruent	2 (45.0)	-12 (48.3)	7 (53.6)
Large Neutral	1 (56.6)	-14 (52.5)	6 (63.6)

A repeated measures ANOVA, including SOA, size and prime-target relationship was undertaken using the residual reaction time subset for each of the addition and multiplication operations. For the multiplication operation, significant main effects of SOA ($F(1.6, 60.9) = 8.2$, $MSe = 2598.2$, $p = 0.001$) and prime-target relationship ($F(2, 74) = 14.5$, $MSe = 780.6$, $p < 0.001$) and a significant interaction between SOA and prime-target relationship ($F(4, 148) = 5.4$, $MSe = 680.8$, $p < 0.001$) were again evident. Additionally, a significant interaction between size and prime-target relationship was found ($F(1.6, 58.6) = 5.2$, $p = 0.014$). Figure 3, showing the facilitation and inhibition evident in the residual reaction time subset for small and large problems, illustrates this interaction.

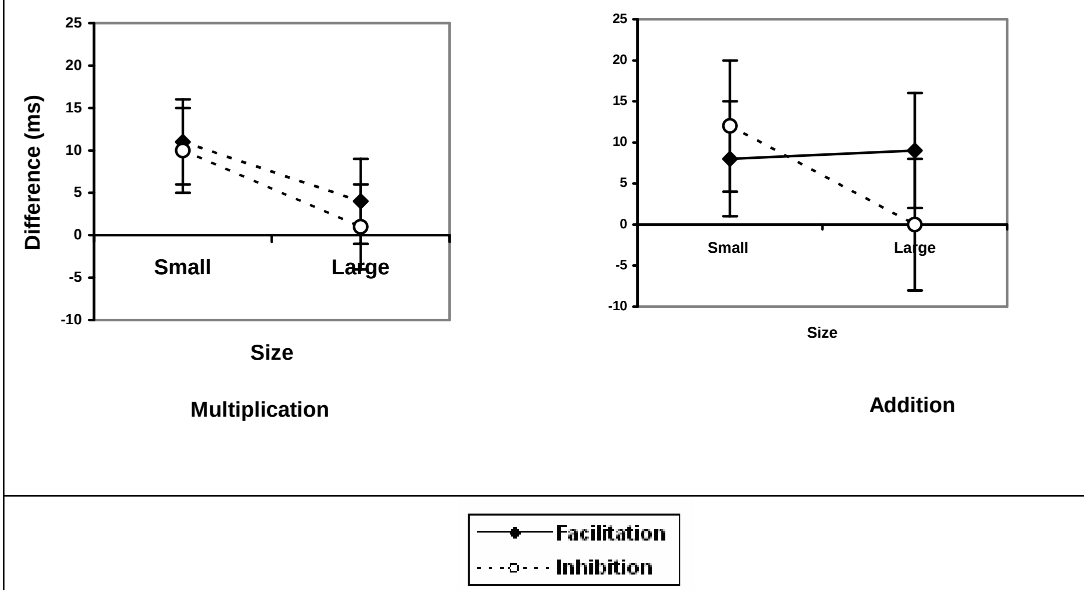

Fig. 3 Showing the facilitation and inhibition evident in residual reaction time scores (ms) for small and large problems i.e., after adjustment for target magnitude. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects.

For small problems, the facilitation to congruent targets (t = 3.7, df = 37, p = 0.001) and inhibition to incongruent targets (t = 3.0, df = 37, p = 0.005) both reached significance. Additionally, responses to small incongruent targets were significantly inhibited in comparison to large incongruent targets (t = 2.5, df = 37, p = 0.016). No significant facilitation or inhibition was observed in response to large problems.

Similarly, in the addition condition, significant main effects of SOA (F(1.8, 65.3) = 11.0, MSe = 2085.4, p < 0.001) and prime-target relationship (F(2, 74) = 14.9, MSe = 819.5, p < 0.001) and a significant interaction between SOA and prime-target relationship (F(3.7, 137.7) = 4.7, MSe = 671.6, p = 0.002) were again found. The interaction between size and prime target relationship approached but did not quite reach significance (F(2, 74) = 3.0, MSe = 852.6, p = 0.058). Planned comparisons showed that for small problems, the facilitation to congruent targets approached significance (t = 1.9, df = 37, p = 0.061), whilst the inhibition to incongruent targets was highly significant (t = 3.1, df = 37, p = 0.004). In contrast, for large problems, the facilitation reached significance (t = 3.0, df = 37, p = 0.005), whilst no significant inhibition was observed in the data. No significant differences in the levels of facilitation and inhibition for small and large problems were observed between operations.

No interactions between SOA, size and prime-target relationship were found for either operation. Nevertheless, in view of the interest in the present study in facilitation and inhibition changes over time, planned comparisons were undertaken. With such large numbers of comparisons to be made (i.e., 6 comparisons at each problem size for each operation), a Bonferroni adjustment was used to reduce the alpha level to a more conservative level of 0.004 (i.e., 0.05/12 = 0.004). In the multiplication condition, at the long SOA, significant facilitation of 25 ms was observed to small congruent targets (t = 4.6, df = 37, $p < 0.001$) and 16 ms to large congruent targets (t = 3.1, df = 37, p = 0.004). Inhibition of 13 ms to small incongruent targets approached significance at the short SOA (t = 2.4, df = 37, p = 0.022). In the addition condition, at the long SOA, significant facilitation of 23 ms to small congruent targets was observed (t = 3.8, df = 37, $p < 0.001$). Significant facilitation of 21 ms to large congruent targets occurred only at the medium SOA (t = 3.1, df = 37, p = 0.003). Responses to small incongruent targets were significantly inhibited by 19 ms at the short SOA (t = 5.0, df = 37, $p < 0.001$). Inhibition in responding to these targets of 16 ms approached significance at the medium SOA (t = 2.2, df = 37, p = 0.031). No other significant effects were observed in the data although the earlier findings of constant inhibition and increasing facilitation over time were again supported.

In summary, when collapsed across SOA, responses to the small congruent multiplication condition were markedly facilitated in comparison to the small neutral condition. This facilitation was accompanied by significant interference to small incongruent targets, suggesting that in comparison to large problems, simple exposure to small problems led to the obligatory activation of their correct solution in memory. The results for the addition operation were similar, although less convincing. For example, although facilitation to small congruent targets approached significance, significant facilitation was observed for large incongruent targets only. As in the multiplication condition, the inhibition to small incongruent targets was highly significant and no inhibition was observed to large incongruent targets. Finally, inclusion of the SOA variable in analysis showed that the observed facilitatory effects occurred largely at the longest SOA for both problem sizes, whilst the inhibitory effects occurred only for the small problems at the shortest SOAs. Caution is advised in the interpretation of this data however, as the preceding adjustment for the effects of magnitude depended on the assumption that target magnitude is statistically additive with priming effects and SOA. Additionally, as noted previously, the present analysis involved only a subset of the data, possibly making it less reliable than the overall analyses reported earlier.

4. Discussion

The aim of the present study was to assess the influence of automatic processing on simple arithmetic performance. The overall results of a priming procedure employing a naming task revealed significant facilitation and inhibition effects consistent with those found in previous arithmetic and word-priming research.

That is, significant facilitation emerged at the 240 ms SOA and increased significantly over time, whilst inhibition was found at the two shortest SOAs and remained constant over time. Additionally, individual differences in multiplication performance were found, with highly skilled arithmeticians demonstrating stronger and more reliable advantages in naming at the 240 and 1500 ms SOAs. Increased facilitation and a significant inhibitory effect at the long SOA for this group indicated that they were able to apply their fact knowledge strategically to speed processing in the naming task. In contrast, advantages in performance were indicated for both groups at the 240 and 1500 ms SOAs in the addition condition. Finally, significant facilitation to small congruent targets in the multiplication condition was accompanied by significant costs in performance to small incongruent targets. Similar results were found in the addition condition, with responses to small incongruent targets found to be significantly inhibited and the level of facilitation to small congruent targets approaching significance. Facilitation to large congruent targets was evident only in the addition condition and no significant inhibition was observed for large incongruent targets in either operation.

A number of observations stem from the overall results. Firstly, the finding of significant facilitation to congruent targets at the 240 ms SOA is consistent with the notion that exposure to simple arithmetic problems results in the automatic activation of their correct solution in memory. Three main factors support this conclusion. Firstly, at the 240 ms SOA, the time period between the onset of the prime and the presentation of the target is too brief to permit conscious awareness and strategic processing of the prime before exposure to the target (Velmans, 1999). Secondly, if the benefit to congruent targets had resulted from processing that occurred after exposure to the target, then facilitation should also have been observed at the shortest SOA. Finally, the use of a target naming procedure ensured that intentional processing of the prime and calculation were irrelevant to performance of the task. The function describing the time course of facilitation is thus indicative of the operation of an automatic spreading activation mechanism that arises at the 240 ms SOA and leads to marked facilitation at the long SOA, consistent with the use of expectancy (Neely, 1991). The results of the present study, therefore, improve on the findings of earlier arithmetic studies demonstrating obligatory activation and in doing so, highlight the utility of the word-priming procedure in the investigation of automatic processing in arithmetic research.

The second observation stemming from the present findings is that the facilitatory and inhibitory effects obtained in the naming task result from the operation of two independent mechanisms. Differences in the time course of the facilitation and inhibition functions support this notion. For instance, in contrast to the path described by the facilitation function, the inhibition function emerged at the shortest SOA and, regardless of changes in facilitation, remained constant over time. As noted earlier, SOAs of 120 and 240 ms are too short to allow for strategic processing to either speed or inhibit performance to the target. This, coupled with the finding of no increase in the level of inhibition at the long SOA, suggests that it must have resulted from processing that occurred after presentation of the target.

What mechanism is responsible for this inhibition? One possible explanation is that it arises due to a process of selective inhibition. Outlined in Tipper, Weaver and Houghton (1994), such a process in the context of the present study begins with exposure to a problem (i.e., the prime), which elicits a verbal response code for its correct solution. This response code then directly competes with the verbal naming response required to the incongruent target, thereby leading to inhibition. Support for the operation of such a mechanism arises from the finding of significant inhibition in high skilled performance across all SOAs in the multiplication condition i.e., an operation that conventionally relies on verbal rote learning. However, on the basis of such an explanation, a comparable advantage in response time to the congruent condition might also be expected, a prediction that was not supported at the 120 ms SOA in the present study.

An alternative explanation is that the participants performed a self-regulatory validity check on their responses before vocalisation (see Siegler, 1988, for a discussion of similar mechanisms). That is, after exposure to the target and shortly before responding, the participants may have quickly compared the target to the correct solution evoked from memory. In the incongruent condition, in which the two did not match, this in turn would have led to hesitation in responding. Again, the finding of significant inhibition at all SOAs in the multiplication condition and at the shortest SOA in the addition condition for the high skilled group (who could be expected to show a greater tendency to engage in such a process) supports this conclusion. Additionally, such an explanation fits well with the 120 ms SOA findings in that it does not predict an advantage in response times to congruent targets. Furthermore, the notion of an obligatory self-regulatory mechanism operating toward exactness in arithmetic performance is compatible with the importance that is placed on accuracy in computation both in learning environments and in every day life (Smith-Chant & LeFevre, 2003). Certainly, the workings of such a mechanism, even at a voluntary level, could be seen to complement explanations of arithmetic performance such as Siegler and Jenkins (1989) Distribution of Associations model.

A third observation stemming from the overall analyses, relates to the clear parallels that can be drawn between performance in the present study and those investigating performance to associatively related word primes and targets (e.g., *rake* and *leaf*; Neely, 1991). This finding adds support to the notion that number knowledge may be represented in a similar form to word knowledge and accessed through similar mechanisms (Ashcraft, 1992; Dehaene, 1992; LeFevre et al., 1988). An assumption of this kind appears reasonable when one considers the mutual reliance of the two knowledge domains on the use of visual, written symbols in education, the fundamental dependence of numerical knowledge on language, and the use of verbal, mechanical word repetition in learning simple multiplication facts (Ashcraft, 1992). Yet, unlike word knowledge, arithmetic problems have only one correct solution and dozens of other relationships with differing degrees of association, built upon 10 basic symbols (Anderson, 1983). As such, the knowledge gained from the study of simple arithmetic performance may provide a valuable benchmark by which to compare information on word knowledge (Campbell & Clark, 1989).

Finally, in relation to the overall analysis, it is noteworthy that the average facilitation and inhibition effects observed in the present study are smaller than those evidenced in the previous arithmetic research. For example, the average levels of facilitation and inhibition observed in Ashcraft et al's (1992) study were 30 ms and 75 ms, respectively. This finding possibly reflects differences in procedural and task requirements between the two studies, with the effects observed in Ashcraft et al's verification study possibly enhanced due to prior exposure to, and activation of solutions, in memory. Additionally, it is noteworthy that a common finding in the word priming literature is that naming tasks produce smaller facilitatory effects than lexical decision tasks (Neely, 1991). Nonetheless, the use of a naming task ensured that such factors as decision-induced attentional processing, retrieval of arithmetic solutions, and calculation were irrelevant to the task, thereby strengthening the present conclusions in relation to automaticity.

The findings of the arithmetic fluency analysis however, suggest that these conclusions need to be further qualified. For example, performance in the addition condition indicated automaticity in processing, regardless of fluency (although, the level of facilitation did not quite reach significance for the low skilled group at the 240 ms SOA; $p = 0.051$). In the multiplication condition high skilled performance followed a similar pattern to that observed in the addition condition although again, it just failed to reach significance at the 240 ms SOA ($p = 0.060$). In contrast, low skilled facilitation at this SOA did not even approach significance, revealing no advantage in exposure to congruent targets whatsoever. In fact, only at a lengthy SOA did any evidence of an advantage in processing begin to emerge for this group and even then it was not strong. This in turn, contrasted with the performance of the high skilled group, who appeared able to use this initial advantage and apply their knowledge to speed processing in the congruent condition at the long SOA.

The results of the present study therefore revealed between group differences in multiplication performance and similarities in addition performance. In the former case, the findings possibly reflect a greater sensitivity of the word priming methodology to individual differences in multiplication performance. As noted earlier, in contrast to the addition operation, the multiplication operation is usually rote learnt and hinges on the development of verbal associations between words (Ashcraft, 1992; Butterworth, 1999; Dehaene, 1992). Consequently, high skilled participants who develop strong associations between problems and their correct solutions may be more likely to stand apart from their counterparts in the present naming task. In the latter case, the findings are at odds with the results of a study by LeFevre and Kulak (1994) who found a significant between group difference in addition performance, with only skilled subjects demonstrating significant effects of obligatory activation in a matching task. In light of this, a replication of the present study employing a larger sample size and more distinctive groups than is accomplished via median split may be useful.

Alternatively, it may be the case that between group similarities are in fact, fundamental to addition performance. For instance, in most Western cultures, even before schooling, children begin to develop an understanding of simple addition through the use of finger counting (Butterworth, 1999). Once at school, they are formally taught addition through counting procedures and the use of concrete visual representations of numerosity involving small numbers (e.g., ☐☐ + ☐ two squares added to one square equals three squares) (Butterworth, 1999). Only after this, is an understanding of the multiplication operation developed, again based on the notion of repeated addition (Butterworth, 1999; Swan, 1990). Multiplication facts, dealing with much greater quantities that are not easily visually or mentally portrayed, are then gradually rote learnt up to the age of approximately nine years (Butterworth, 1999). Thus, for the addition operation, earlier and greater exposure to simpler and more meaningful constructs may enable even the least fluent individual a comparable level of automaticity to other more skilled arithmeticians.

Tied in with the above explanation of between group similarities in addition performance is the notion that a central factor in both learning and performance is problem size. This notion is supported in the present study by differences in the patterns of facilitation and inhibition observed for each problem size between the addition and multiplication operations. For example, in the multiplication condition, significant facilitation was found to small congruent targets (i.e., problems with small operands and their correct solutions) only. However, in the addition condition, facilitation to small congruent targets approached significance and facilitation to large congruent targets (i.e., problems consisting of large operands yet still involving much smaller target magnitudes than those in the multiplication condition) also reached significance. Moreover, a significant level of inhibition was found to small incongruent targets only in both operations. Thus, small correct solutions appear to be accessed from memory more quickly than large correct solutions and small incorrect solutions lead to greater levels of interference in naming. The results of the present study are therefore generally consistent with the notion that problems of differing size are processed differently (Ashcraft et al., 1992; Campbell, 1987, 1991; Koshmider & Ashcraft, 1991; LeFevre et al., 1996a, 1996b).

Finally, in relation to the problem size analyses, it is worth noting that the advantages to small congruent targets were not evidenced in the automatic processing conditions, instead occurring only at the long SOA in both operations. Furthermore, significant inhibition was observed to small incongruent targets in the short addition condition and inhibition to these targets approached significance in both the 240 ms addition condition and the 120 ms multiplication condition. These findings again suggest that two different mechanisms are responsible for the facilitation and inhibition observed in the data and provide support to the notion of the operation of an obligatory self-regulatory mechanism. However, the finding of an advantage to large congruent targets at the 240 ms SOA in the addition condition that did not persist to the 1500 ms SOA condition and that occurred in the absence of any comparable facilitation to small congruent targets, is difficult to explain. One possibility is that this finding somehow reflects underlying differences in performance between fluency groups. Unfortunately, the data were too inconsistent in analysis at this level, suggesting again the use of greater numbers and better defined groups in testing.

A number of interesting educational implications stem from the present results. The first of these occurs in relation to the mathematical reform that has occurred over the last three decades in most major industrialised countries throughout the world. This reform, encouraged by rapid technological advance (i.e., with the inception of calculators) and shifting theoretical paradigms (such as the notion that children should think before they 'fact;' Lochhead, 1991, pp. 77) has seen the traditional role of rote learning be greatly undermined (Willis, 1990; Willoughby, 2000; Woodward & Montague, 2002). Such a situation is disturbing in light of the finding that greater access to multiplication facts and the ability to then apply this knowledge distinguishes performance between groups. Advantages of this kind possibly serve to free cognitive space and extend the number of functions that can be performed at once (Campbell, 1987; Koshmider & Ashcraft, 1991; Reed, 1988; Willoughby, 2000). This is important when one considers that students and programs, especially at the primary level, are still examined using standardised tests that are often speeded and rely on pencil and paper skills (Tsuruda, 1998). Even at the high school level, the ability to quickly approximate a solution and be confident that an answer obtained on a calculator is accurate can be seen to be an advantage in an exam situation (Meissner, 1980).

Secondly, the finding that in the addition condition, regardless of fluency, there was facilitation to the correct condition over both the 240 and 1500 ms SOA's, raises serious doubts about the utility of verification tasks in the assessment of addition competence (Campbell, 1987). As Campbell (1987) notes, with the presented solution to a given problem already priming the correct answer in memory, the probability of subsequently retrieving an error may be reduced. Consequently, the individual's performance in a verification task may be an overestimate of their ability in a normal production task.

In summary, the present study demonstrated the utility of the word-priming paradigm (with naming task) in accessing facilitatory and inhibitory mechanisms associated with simple arithmetic performance. More specifically, it showed that brief exposure to simple addition problems leads to the automatic activation of correct solutions in memory in high skilled individuals and, most likely, in individuals of low skill also. Exposure to multiplication problems however, revealed individual differences in performance, with facilitatory and inhibitory effects at the longer SOA's indicating that only the high skilled arithmeticians applied their multiplication fact knowledge toward superior performance in the naming task. Furthermore, the results indicated significant advantages in performance to small problems that are more frequently encountered in educational and natural settings than larger ones. The results of the present study therefore, demonstrate the need for further elaboration and revision of network retrieval models to account for differences between individuals, between problem sizes and between operations (LeFevre & Kulak, 1994; Lefevre et al., 1996). Furthermore, they highlight the importance of the continued use of the more traditional rote learning of simple arithmetic facts in mathematical education.

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Appendix A

Prime Sets and Correct and Incorrect Targets for Multiplication Operation					
Set 1	Set 2	Set 3	Set 4	Correct	Incorrect
2 x 4	4 x 2	2 x 4	4 x 2	8	30
3 x 5	5 x 3	5 x 3	3 x 5	15	42
3 x 7	7 x 3	7 x 3	3 x 7	21	48
4 x 5	5 x 4	4 x 5	5 x 4	20	63
5 x 6	6 x 5	6 x 5	5 x 6	30	10
5 x 9	9 x 5	5 x 9	9 x 5	45	27
6 x 8	8 x 6	8 x 6	6 x 8	48	15
7 x 9	9 x 7	7 x 9	9 x 7	63	56
8 x 9	9 x 8	9 x 8	8 x 9	72	24
3 x 2	2 x 3	3 x 2	2 x 3	6	54
4 x 3	3 x 4	3 x 4	4 x 3	12	6
5 x 2	2 x 5	5 x 2	2 x 5	10	40
6 x 4	4 x 6	6 x 4	4 x 6	24	8
7 x 6	6 x 7	6 x 7	7 x 6	42	21
8 x 7	7 x 8	8 x 7	7 x 8	56	20
8 x 5	5 x 8	5 x 8	8 x 5	40	12
9 x 3	3 x 9	9 x 3	3 x 9	27	45
9 x 6	6 x 9	6 x 9	9 x 6	54	72

Prime Sets and Correct and Incorrect Targets for Addition Operation					
Set 1	Set 2	Set 3	Set 4	Correct	Incorrect
2 + 4	4 + 2	2 + 4	4 + 2	6	13
3 + 5	5 + 3	5 + 3	3 + 5	8	16
3 + 7	7 + 3	7 + 3	3 + 7	10	15
4 + 5	5 + 4	4 + 5	5 + 4	9	13
5 + 6	6 + 5	6 + 5	5 + 6	11	8
5 + 9	9 + 5	5 + 9	9 + 5	14	7
6 + 8	8 + 6	8 + 6	6 + 8	14	17
7 + 9	9 + 7	7 + 9	9 + 7	16	5
8 + 9	9 + 8	9 + 8	8 + 9	17	6
3 + 2	2 + 3	3 + 2	2 + 3	5	14
4 + 3	3 + 4	3 + 4	4 + 3	7	10
5 + 2	2 + 5	5 + 2	2 + 5	7	14
6 + 4	4 + 6	6 + 4	4 + 6	10	15
7 + 6	6 + 7	6 + 7	7 + 6	13	9
8 + 7	7 + 8	8 + 7	7 + 8	15	12
8 + 5	5 + 8	5 + 8	8 + 5	13	10
9 + 3	3 + 9	9 + 3	3 + 9	12	7
9 + 6	6 + 9	6 + 9	9 + 6	15	11