Simple arithmetic processing: individual differences in automaticity

Author	Jackson, N., and Coney, J.
Year	2007
Source	*European Journal of Cognitive Psychology, 19*(1), 141-160.
Official URL	http://dx.doi.org/10.1080/09541440600612712

Simple Arithmetic Processing: Individual Differences in Automaticity

Natalie Jackson and Jeffrey Coney

Murdoch University

Running Head: Individual Differences in Automaticity

Address for Correspondence: School of Psychology

Murdoch University

Murdoch, Western Australia, 6150

Australia

Email: N.Jackson@Murdoch.edu.au

Phone: +61 (08) 9360 2387, Fax: +61 (08) 9360 6492

Abstract

This study investigated individual differences in the ability to automatically access simple addition and multiplication facts from memory. It employed a target-naming task and a priming procedure similar to that utilized in the single word semantic-priming paradigm. In each trial, participants were first presented with a single digit arithmetic problem (e.g., 6 + 8) and were then presented with a target that was either congruent (e.g., 14) or incongruent (e.g., 17) with this prime. Response times for congruent and incongruent conditions were then compared to a neutral condition (e.g., X + Y, with target 14). For the high skilled group, significant facilitation in naming congruent multiplication and addition targets was found at SOAs of 300 and 1000 ms. In contrast, for the low skilled group, facilitation in naming congruent targets was only observed at 1000 ms. Significant inhibition in naming incongruent multiplication and addition targets at 300 ms, and addition targets at 1000 ms, was found for the high skilled group alone. This advantage in access to simple facts for the high skilled group was then further supported in a problem size analysis that revealed individual differences in access to small and large problems that varied by operation. These findings support the notion that individual differences in arithmetic skill stem from automaticity in solution retrieval and additionally, that they also derive from strategic access to multiplication solutions.

PsycINFO classification: 2343; 2346
Key Words: Simple Arithmetic, Automaticity, Individual Differences, Priming, Fluency

1. Introduction

Until recently, it was widely assumed that the majority of adults reached asymptotic performance on the retrieval of simple arithmetic facts such that they directly retrieved solutions from memory, most of the time (Ashcraft, 1992; Geary & Wiley, 1991; LeFevre et al., 1996; LeFevre & Kulak, 1994; LeFevre, Sadesky & Bisanz, 1996). However, a growing body of research suggests that the use of various solution procedures other than direct fact retrieval (e.g., counting or transformation procedures: 9 + 7 = 9 + 1 + 6) may be far more widespread than was first considered and that this may vary with arithmetic fluency (LeFevre et al., 1996a; 1996b). That is, those who are fluent arithmeticians are assumed to be more likely to rely on automatic access to simple arithmetic facts than to rely on alternative solution procedures (LeFevre & Kulak, 1994).

Support for the influence of fluency on access to simple arithmetic facts is provided in two main studies by LeFevre and colleagues. In these studies, accessibility was indexed by unintentional sum activation produced in the performance of a number-matching task. Participants were first presented with a pair of numbers (e.g., 3 + 6) and then following a short inter-stimulus interval, were required to decide if a target number (e.g., 3) was one of the original numbers presented. In the first study, by LeFevre, Kulak and Bisanz (1991), the presentation of the sum (i.e., 9) to a high skilled group led to significantly slowed processing in comparison to a neutral prime, at an SOA of 80 ms. In contrast, significant interference to the sum for low skilled participants was observed only at a lengthier SOA of 120 ms. In the second study, by LeFevre and Kulak (1994), the results again revealed significantly slower performance by high skilled participants in sum as opposed to neutral trials. This occurred at SOAs of 40 and 60 ms in the first experiment and 60 ms in the second experiment. For the low skilled group, small non-significant interference effects were observed that were again, delayed in comparison to the high skilled group, being found at somewhat longer SOAs of 120 and 160 ms, respectively. Obligatory activation therefore appeared greater for high skilled individuals and occurred earlier in the processing sequence than it did for low skilled individuals. These findings, according to LeFevre and Kulak (1994), supported the hypothesis that individual differences in arithmetic skill may originate in automaticity of fact retrieval. Unfortunately, a comparable study involving the multiplication operation was not undertaken.

Further support for the notion that individuals with stronger arithmetic fluency are more likely to rely on direct solution retrieval stems from a series of investigations employing self report measures. In these investigations (Hecht, 1999; LeFevre et al., 1996a; 1996b; see also Geary & Wiley, 1991), participants were first required to solve simple addition or multiplication problems and then to report on a trial by trial basis the strategy that they employed to obtain their solution. The results were consistent across all studies in showing a significant positive correlation between a high level of fluency and the reported use of direct retrieval. Moreover, in the studies conducted by LeFevre and colleagues the results indicated that less skilled participants showed greater effects of problem size i.e., as problem size increased, solution latencies increased more for these individuals than they did for high skilled individuals. This, according to LeFevre et al (1996a), was a direct consequence of less skilled participants relying on solution strategies other than direct retrieval (e.g., counting or transformation procedures: $4 + 7 = 7 + 3 + 1$).

However, the veridicality of self report measures has been called into question due to the possibility that the instructions employed within this method may lead to reactivity, which in turn, may be influenced by fluency (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003). Support for individual differences in reactivity was provided in an investigation by Smith-Chant and LeFevre (2003). Low skilled individuals were found to be more affected by speed (vs. accuracy) biasing instructions and responded more slowly and accurately on large and very large problems when asked to provide self-reports of their solution procedures. High skilled participants, on the other hand, revealed smaller effects due to biasing instructions and were minimally reactive to the requirement to provide self-reports.

More recently, Jackson and Coney (2005) offered an alternative approach to the investigation of automaticity in multiplication and addition performance by employing numerical stimuli in a priming procedure analogous to that utilised in the single word semantic priming paradigm. Participants were first presented with either of two prime types: one representing a single digit arithmetic problem (e.g., 6 + 8), the other employed as a neutral condition (e.g., 0 + 0). Following a given SOA (i.e., of 120, 240 or 1000 ms), they were then presented with a target that was either congruent (e.g., 14) or incongruent (e.g., 17) with the prime. In the addition, 240 ms SOA condition, for high skilled participants, the time taken to name congruent targets was significantly facilitated in comparison to the neutral condition. For the low skilled group however, facilitation merely approached significance. At the longest SOA, facilitation was significant for both groups but appeared greater for the high skilled group. The trend in the addition data identified in the Jackson and Coney (2005) study was, therefore, generally consistent with the earlier findings of the LeFevre et al. (1991) and LeFevre and Kulak (1994) studies in revealing earlier and greater levels of activation for high skilled participants. Furthermore, this trend was also evident in the multiplication data, with the level of facilitation approaching significance at the 240 ms SOA for the high skilled group, and reaching significance at 1000 ms. For the low skilled group, facilitation was not evident at the short SOA and barely reached significance at the long SOA.

In addition to the facilitatory effects, the Jackson and Coney (2005) study also revealed significant inhibition in naming incongruent targets for the high skilled group only. This was evident across all SOAs in the multiplication condition and at the shortest SOA in the addition condition. Furthermore, the inhibitory effect found at the long SOA was quite large and appeared to have increased in conjunction with an increase in facilitation, thereby, suggesting the use of expectancy in naming performance for this group alone. Unfortunately, the increase in inhibition at this SOA just failed to reach significance, a result possibly reflecting the use of a high skilled group who were not high enough in skill to be easily distinguished from the low skilled group. The investigation of individual differences in this study was a subsidiary aim. Hence, the sample was divided into skill groups on the basis of a median split and extreme groups were not selected.

The main aim of the present study was thus to re-examine individual differences in priming effects by replicating the earlier study using a larger sample size and more distinguishable skill groups. Additionally, unlike the earlier study, a lengthier short SOA condition of 300 ms was employed in an attempt to determine whether activation of multiplication facts also occurs for less skilled individuals (not previously found at the 240 ms SOA), but is delayed in comparison to that of high skilled individuals (LeFevre & Kulak, 1994). This study also employed neutral stimuli that differed from those used in the earlier Jackson and Coney (2005) study (i.e., X + Y and X x Y). This was done for two main reasons. Firstly, previous research indicates that the processing of zero stimuli may occur more slowly than other numerical stimuli and therefore the use of the 0 + 0 and 0 x 0 neutral stimuli in the earlier study potentially exaggerated the facilitatory effects that were identified (Stazyk, Ashcraft & Hamann, 1982). Secondly, the new neutral stimuli were employed to guard against artificial slowing of responses in this condition due to the incongruence between the prime and the target (e.g., 0 + 0 presented with 14). Finally, in view of the possibility that less skilled individuals show greater problem size effects because of their reliance on solution procedures other than direct retrieval (LeFevre et al., 1996a, 1996b) a second objective of the present study was to assess individual differences in access to small and large facts.

2. Method

2.1 Participants

Fifty-four undergraduate psychology and mathematics students, including 9 males and 45 females, from Murdoch University participated in this study. Participants either received credit toward partial fulfilment of course requirements or were reimbursed $10 for their time. The participants' ages ranged from 17 to 52 years, with a mean age of 26.

2.2 Design and stimulus materials

Three within group variables were examined. The first of these determined the arithmetic operation i.e., addition or multiplication. The second variable incorporated three prime-target relationships, including congruent (e.g., 2 + 4 = 6), incongruent (2 + 4 = 9) and neutral (X + Y = 6) conditions. The final within group variable was SOA with two levels: 300 ms and 1000 ms.

Two sets of primes originally utilised in the Jackson and Coney (2005) study were employed for each of the two operations (see Appendix A). The first set for each operation comprised 18 simple arithmetic facts selected from the 2s through 9s matrices (e.g., 2 + 3). The second set consisted of the reverse operand placement equivalents of the first set (3 + 2).

As in the previous research, arithmetic ties (e.g., 3 + 3 and 3 x 3) were excluded from use as primes, as these problems have been shown to be solved more quickly than others (LeFevre et al., 1988). Additionally, to ensure that each prime set was balanced in terms of operand placement; half of the arithmetic facts were produced so that the smaller of the two operands in each problem was placed on the left-hand side and half with the smaller operands on the right hand side. Finally, each stimulus set consisted of six smaller problems (i.e., with both operands of a magnitude less than or equal to five; e.g., 2 + 3), six larger problems (operands greater than or equal to six; e.g., 8 + 9), and six of mixed magnitude (e.g., 2 + 9). This enabled testing for the presence of the problem size effect.

The target sets for each of the congruent, incongruent and neutral conditions consisted solely of the correct solutions to the 18 simple arithmetic facts investigated in this study. These targets were then simply paired with an alternative problem for the incongruent condition. To guard against split effects in the multiplication condition, incongruent targets were paired with problems so that they differed by at least 16 from the correct solutions to these problems. For the addition condition, incongruent targets differed by at least three from the correct solutions. Further constraints on the incongruent target sets were included to address possible confounding relationships between the prime and the target. Firstly, incongruent targets were not permitted to be one of the operands or the numbers plus or minus one from those used in the prime. Secondly, where possible, multiples or factors of the operands and number series relations were excluded. Finally, incongruent targets were paired with primes in such a way that they could not be the correct solution using a different operation, a double-digit number containing the operand, or a number containing the correct solution (i.e., if the correct solution was 7, then numbers such as 17 and 70 were also excluded).

Neutral conditions have been useful in assessing facilitation and inhibition and hence distinguishing automatic from conscious processing in word priming research but to date have not been widely utilised in the study of arithmetic (Neely, 1991). The neutral condition stimuli (i.e., X + Y for the addition condition and X x Y for the multiplication condition) were thus chosen in accordance with three main recommendations outlined in a review of the word priming literature by Neely (1991). The first of these was that neutral primes should be equated with other primes in relation to their value as a warning signal that a target will soon appear. Secondly, neutral primes should be unassociated to the target so that they are a neutral baseline by which to assess spreading activation between related stimuli. Lastly, in order to provide a baseline by which to compare expectancy effects, neutral primes should not offer any information as to the semantic nature of the target to follow. In the present study the prime X + Y can be likened perceptually to the other numerical primes such as 2 + 3, with both consisting of two common individual symbols separated by an arithmetic operator. Additionally, with X and Y often used in the place of numbers to denote separate unknown quantities, the recommendations against any association between prime and target, and any indication of the semantic nature of the target, were also met. Unlike the previous Jackson and Coney (2005) study that employed a 0 + 0 and 0 x 0 neutral condition, the expectation of the target 0 being presented was avoided.

2.3 Psychometric testing

The arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) was used to identify two arithmetic fluency groups. This test incorporates 60 arithmetic problems that variously include the addition, subtraction, division and multiplication of single, two and three digit numbers (ACER, 1984). The participants were instructed that they had five minutes to answer as many questions as accurately as they could. They were instructed to begin with the first question and without omitting any, to work through each in turn (ACER, 1984). Rough working out could be undertaken anywhere on the page and participants were advised that if they finished the first column that they should immediately go onto the second one (ACER, 1984).

Participants were placed into high and low skilled groups based on the number of problems that they solved correctly. Twenty eight participants formed the low skilled group, with a mean correct score of 12 (*SD* = 1.73). This score corresponded to a percentile rank of 0 in a normative sample of 124 tertiary graduate/diplomates, 7 in a sample of 973 administrative officer or administrative assistant applicants, and 2 in a sample of 1270 bank trainees (ACER, 1984). Twenty six participants constituted the high skilled group, with a mean correct score of 31 (*SD* = 5.22). These scores corresponded to a percentile rank of 35 in the sample of tertiary graduate/diplomates, 83 in the administrative applicant sample, and 74 in the bank trainee sample (ACER, 1984).

2.4 Procedure

Participants were individually tested on the computer task in a well-lit cubicle room containing an Amiga 1200 microcomputer, with 1084S monitor. This system controlled stimulus presentation, trial sequencing, timing and data collection. Individual operands within each problem did not exceed dimensions of 5 x 15 mm on the screen and were separated by 5 mm from the arithmetic operators (i.e., the x or + sign), which did not exceed 5 x 10 mm. Stimuli were presented centrally, white against an amber background. A chin rest stabilised the participant's head at a viewing distance of 60cm from the screen.

Participants each completed four blocks of 54 experimental trials (i.e., two for each of the addition and multiplication operations corresponding to the two levels of SOA). Addition and multiplication trials were blocked separately so as not to produce cross operation or relatedness errors. Half of the participants started with the addition operation first and half started with multiplication. In the first 300 ms block, of the participants assigned to the addition condition first, half were exposed to addition Set 1, whilst half were exposed to addition Set 2 (see Appendix A). Similarly, half of the participants assigned to the multiplication condition first were exposed to multiplication Set 1, whilst the remaining half were exposed to multiplication Set 2. Participants were then exposed to the exact same set that they saw in the first block in the second 1000 ms block. Repetition of these trials at the longer SOA allowed for a level of familiarity with the stimuli, drawing attention to the prime-target relationship. This process was then repeated in the third and fourth blocks using the operation not tested in the first two blocks. Exposure to individual sets and all stimuli was counterbalanced across participants, with the computer randomly generating the order of presentation of the individual congruent, incongruent and neutral trials within each block.

Before testing, participants were advised to respond both quickly and accurately. Each trial began with the participants focussing their gaze on a 1 x 1 mm blue central fixation dot that was exposed for 600 ms. After a 150 ms period in which the screen remained blank, the prime was presented for 100 ms. The target number appeared following the given SOA and remained exposed until the participant named the number. A two-second interval separated the participant's response and the start of the next trial. Participants' vocal responses were detected using a microphone connected to a headset. The microphone triggered an electronic relay that was interfaced to the computer and stopped a hardware timer. The value of the timer was accurate to 1 millisecond and measured the participant's vocal reaction time from the onset of the target. Padded ear guards attached to the headset prevented external noise intrusions. The experimental session, including debriefing, lasted approximately 30 minutes.

3. Results

The mean response latency for each participant in each condition was recorded. These data were screened for outliers using an exclusion criterion of +/- 2.5 z-scores. This led to 0.77% of all scores being replaced using mean substitution. The resulting reaction time data are presented in Table 1. Due to the negligible error rates produced in target naming performance they were not considered in the present analysis.

Table 1.
Mean Reaction Times (ms) for all Prime-Target Relationships as a Function of SOA, Operation and Fluency.

	Low Skilled		High Skilled	
Addition	300 ms	1000 ms	300 ms	1000 ms
Congruent	476(50)	477(56)	423(50)	442(54)
Neutral	479(54)	493(59)	437(50)	458(47)
Incongruent	481(55)	492(57)	448(51)	468(51)
Multiplication				
Congruent	486(51)	492(63)	450(54)	446(50)
Neutral	493(55)	505(57)	464(48)	481(48)
Incongruent	487(47)	509(58)	476(54)	481(48)

Note. Standard deviation in parentheses.

These data were initially entered into an overall split plot analysis of variance used to assess the presence of operation differences. A significant main effect was found for operation, with reaction times to addition-related targets found to be 16 ms faster overall than to multiplication-related targets ($F(1, 52) = 10.6$; $MSe = 4088.6$, $p = 0.002$). This finding is consistent with operation differences recognised in earlier studies and possibly reflects differences in solution magnitudes between the two operations (ranging from 5 through 17 for addition and 6 through 72 for multiplication) (Jackson & Coney, 2005; Zbrodoff & Logan, 1986). Previous research indicates that it takes longer to perform number naming tasks when numbers are large than when they are small (Brysbaert, 1995; Jackson & Coney, 2005). In view of this difference in processing, the two operations were analysed separately.

3.1 Multiplication analysis

A split plot analysis of variance, involving SOA and prime-target relationship as within group variables and fluency as the between group variable, was used to analyse the multiplication data. Significant main effects were found for all three variables. Firstly, responses to the short SOA condition were 9 ms faster than to the long SOA condition ($F(1, 52) = 4.4$; $MSe = 1652.2$, $p = 0.041$). Secondly, responses to the congruent condition were 20 ms faster than to the incongruent condition and 17 ms faster than to the neutral condition ($F(1.8, 91.3) = 39.5$; $MSe = 357.9$, $p < 0.001$). Finally, high skilled participants responded 29 ms faster overall than did low skilled participants ($F(1, 52) = 4.9$; $MSe = 2321.3$, $p = 0.031$). These main effects were then further qualified by two significant two-way interactions. The first of these occurred between SOA and prime-target relationship ($F(2,104) = 4.7$; $MSe = 348.4$, $p = 0.011$). Paired sample t-test comparisons involving the short SOA condition revealed significant facilitation (i.e., neutral – congruent) of 10 ms ($t(53) = 3.1$, $p = 0.003$) that increased to 24 ms at the long SOA ($t(53) = 5.9$, $p < 0.001$). The overall pattern of performance to multiplication-related targets was thus one of increasing facilitation over time.

The second and more important significant interaction in the context of the present study was that between prime-target relationship and fluency ($F(2, 104) = 10.7$; $MSe = 314.1$, $p < 0.001$). Paired sample t-test comparisons involving the low skilled results revealed significant facilitation of 10 ms ($t(27) = 4.6$, $p < 0.001$) but no inhibition. In contrast, analysis of the high skilled results revealed significant facilitation of 24 ms ($t(25) = 7.7$, $p < 0.001$) and a 6 ms inhibitory (incongruent - neutral) effect that approached but did not quite reach significance ($t(25) = 1.8$, $p = 0.088$). The advantage in facilitation for high skilled participants was significantly greater than that observed for low skilled participants ($t(43.9) = 3.9$, $p < 0.001$).

No significant three-way interaction was observed in the multiplication analysis however, in view of the particular interests of the present study in changes in facilitatory and inhibitory effects over time, planned comparisons between all prime-target relationships were undertaken for each group at both SOA's. The facilitatory and inhibitory effects for these analyses are presented in Fig. 1.

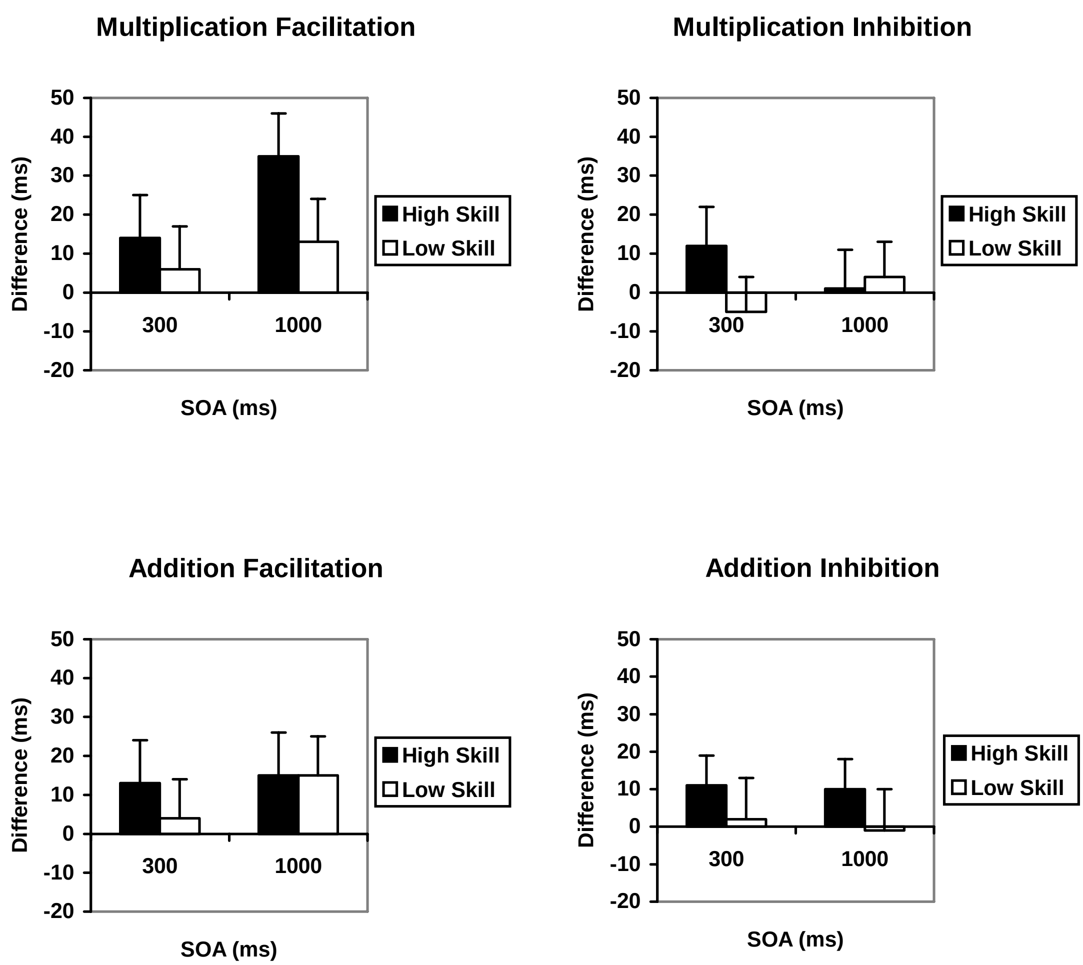


Fig. 1 Showing facilitatory and inhibitory effects for high and low skilled groups as a function of SOA. The 95% confidence intervals were calculated based on the *MSe* term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects for each group.

For the high skilled group, facilitation was found to be significant at both the short ($t(25) = 3.3$, $p = 0.003$) and long ($t(25) = 6.2$, $p < 0.001$) SOAs, and increased significantly over time ($t(25) = 2.8$, $p = 0.010$). Significant inhibition was evident at the short SOA ($t(25) = 2.4$, $p = 0.022$) only. In contrast, for the low skilled group significant facilitation was observed only at the long SOA ($t(27) = 2.6$, $p = 0.014$). Thus, no obligatory activation of multiplication facts was identified for the low skilled group using the short 300 ms SOA employed in the present study.

3.2 Addition analysis

A split plot ANOVA was then performed on the addition data. Significant main effects were again found for all three variables. Firstly, a significant main effect of SOA was found, with responses to the 300 ms condition found to be 14 ms faster than to the 1000 ms condition ($F(1, 52) = 6.8$; $MSe = 2410.8$, $p = 0.012$). Secondly, a significant main effect of prime-target relationship was found ($F(1.6, 84.4) = 25.4$; $MSe = 417.7$; $p < 0.001$). Responses to the congruent condition were facilitated by 12 ms, with responses to the incongruent condition inhibited by 6 ms. Finally, a significant main effect of arithmetic fluency ($F(1, 52) = 8.4$, $MSe = 13235.9$, $p = 0.005$) was found. High skilled participants responded 37 ms faster overall than low skilled participants did. This finding was then further qualified by a significant interaction between prime-target relationship and fluency ($F(2, 104) = 4.6$, $MSe = 338.8$, $p = 0.013$). For the low skilled group, facilitation of 9 ms approached but did not quite reach significance ($t(27) = 1.9$, $p = 0.066$) and no inhibition was evident. In contrast, for the high skilled group, significant facilitation of 14 ms ($t(25) = 4.7$, $p < 0.001$) and inhibition of 11 ms ($t(25) = 4.6$, $p < 0.001$) was found.

No significant interaction between SOA and prime-target relationship was observed in the data ($F(2, 104) = 1.0$; $p = 0.364$) and, as in the multiplication analysis, no significant three-way interaction involving fluency was found. Nevertheless, planned comparisons of changes in facilitation and inhibition effects for each group (see Fig. 1) over time were again undertaken. For the high skilled group, significant facilitation ($t(25) = 3.6$, $p = 0.002$) and inhibition ($t(25) = 3.4$, $p = 0.002$) were found at the short SOA. These effects then persisted over time with similar facilitation ($t(25) = 2.6$, $p = 0.015$) and inhibition ($t(25) = 2.6$, $p = 0.016$) effects found at the long SOA. The only significant effect observed for the low skilled group was facilitation that again occurred only at the long SOA ($t(27) = 2.8$, $p = 0.009$).

In summary, the findings of the present study demonstrated individual differences in target naming latencies as a function of arithmetic fluency. Priming using both multiplication and addition problems led to earlier access to correct solutions for high skilled participants than it did for low skilled participants. Moreover, it produced significant inhibition in naming incongruent targets at 300 ms in both the addition and multiplication conditions, and at 1000 ms in the addition condition, for the high skilled group alone. The present results therefore extend the previous findings of the LeFevre et al (1991) and LeFevre and Kulak (1994) studies that demonstrated interference effects in a number matching task, involving the addition operation only.

3.3 Problem size analysis

A subset of the data including reaction times to small and large problems (consisting of operands ≤ 5 or > 5, respectively) was selected for use in determining the influence of problem size on arithmetic processing. These data were initially screened for outliers using a cut off score of +/- 2.5 z-scores. This led to mean substitution of 1.47% of all scores.

With the selection of only a subset of the data in this analysis, a mis-match was created between the solutions in the congruent and incongruent conditions, and between problems and solutions of differing magnitudes. For example, congruent targets for small multiplication problems ranged between 6 and 20, whilst incongruent targets for these problems largely ranged between 30 and 63 (i.e., except for the incorrect solution ‘6’). Thus, any differences found in direct comparisons between the two problem sizes may have resulted from a confound of target magnitude. In order to remove any confounding influence of this kind, the raw data for all problems within the original stimulus set were entered into regression analyses to first ascertain any effect of magnitude and then to adjust for it in the obtained reaction times. Pearson correlation coefficients for each group and the best fitting model between the mean overall reaction time and number magnitude are presented in Table 2.

Table 2.
Pearson Correlation Coefficients (*r*) and Models of Best Fit between Reaction Time and Number Magnitude.

	High Skilled		Low Skilled		Overall	
	300 ms	*1000 ms*	*300 ms*	*1000 ms*	*300 ms*	*1000 ms*
Addition						
Congruent	0.48	0.31	0.13	0.66*	0.36	0.59*
Neutral	0.06	-0.19	-0.36	0.06	-0.18	-0.02
Incongruent	0.25	0.30	-0.19	0.44	0.03	0.43
Model of Best Fit	Reaction Time = 0.83(Number Magnitude) + 455**					
Multiplication						
Congruent	0.76**	0.52*	0.65**	0.55*	0.79**	0.60**
Neutral	0.65**	0.31	0.54*	0.46	0.73**	0.52*
Incongruent	0.44	0.20	0.34	0.43	0.52*	0.42
Model of Best Fit	Reaction Time = 0.46**(Number Magnitude) + 467**					

Note. *$p < 0.05$, two-tailed. **$p < 0.01$, two-tailed.

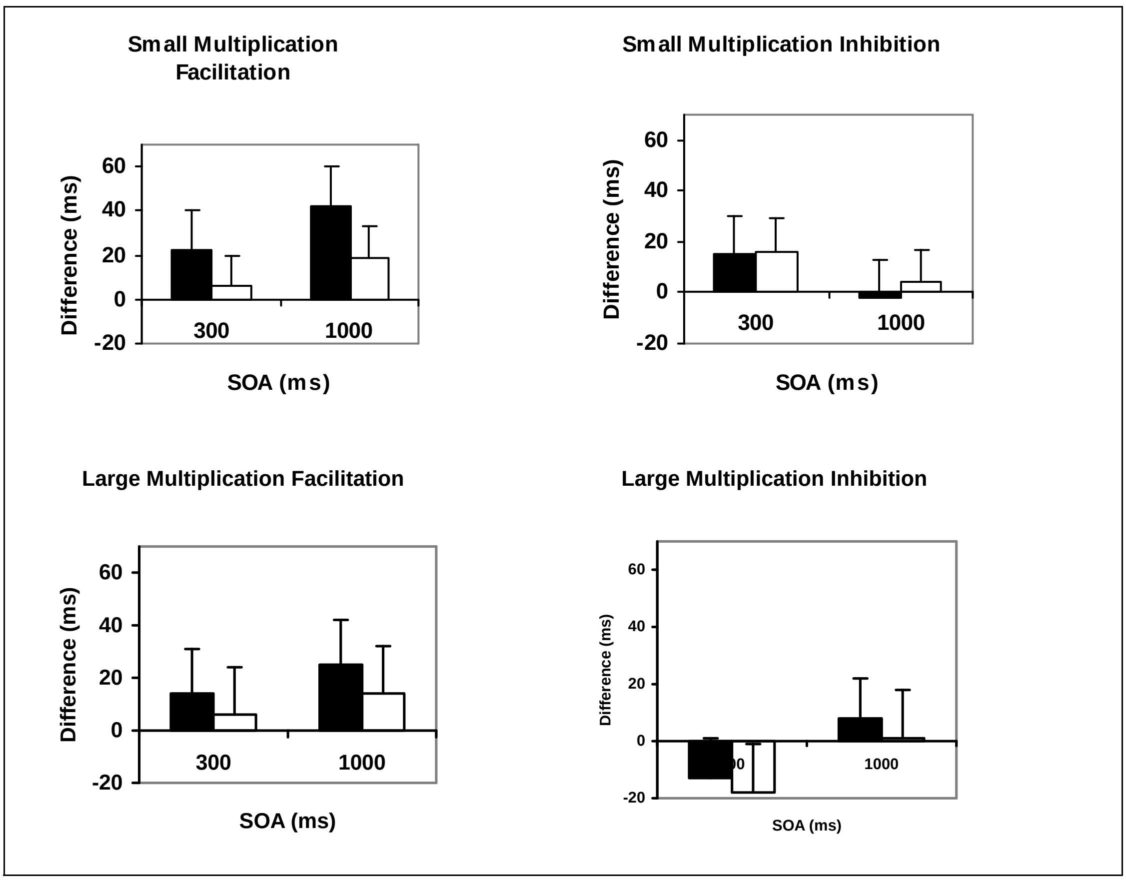
A strong positive correlation between naming latencies and number magnitude was found in the congruent addition condition for the low skilled group only. This finding is consistent with previous research indicating greater increases in solution latencies with problem size for low skilled individuals than for high skilled individuals in addition performance (LeFevre et al., 1996a, 1996b). However, a negative relationship between fluency and the problem size effect was not indicated in the present multiplication data, with similar patterns of correlations found for both fluency groups.

As in the earlier Jackson and Coney (2005) study, both models were employed to compute predicted reaction times scores for their respective operations. Residual reaction time scores were then computed by subtracting observed reaction times from predicted ones. Following this, mean residual reaction time scores were determined for each of the small and large problem sizes, for all participants. These were calculated for each operation by averaging the residual reaction times for the six smallest and the six largest problems. These data were then entered into separate split plot analyses of variance for both the multiplication and addition conditions.

3.3.1 Multiplication analysis

As in the previous analyses, the split plot ANOVA involving the residual multiplication data revealed significant main effects of SOA ($F(1, 52) = 6.3$, $MSe = 2895.0$, $p = 0.015$) and prime-target relationship ($F(2, 104) = 16.8$, $MSe = 1595.6$, $p < 0.001$) and a significant interaction between these two variables ($F(2, 104) = 4.5$; $MSe = 906.8$, $p = 0.014$). Additionally, a significant two-way interaction between size and prime target relationship ($F(2, 104) = 4.4$, $MSe = 1405.7$, $p = 0.015$) and a significant three-way interaction between SOA, size and prime target relationship ($F(1.8, 95.5) = 4.6$, $MSe = 1059.8$, $p = 0.015$) were found. For small problems at the short SOA, significant facilitation of 13 ms ($t(53) = 2.1$, $p = 0.037$) and inhibition of 16 ms ($t(53) = 2.2$, $p = 0.030$) was found. At the long SOA, significant facilitation of 30 ms ($t(53) = 5.5$, $p < 0.001$) was found. In contrast, for large problems, significant inhibition of 15 ms was observed at the short SOA ($t(53) = 2.4$, $p = 0.022$) and significant facilitation of 19 ms was observed at the long SOA ($t(53) = 3.0$, $p = 0.004$).

No four-way interaction involving fluency was found in the data. Nevertheless, in the interest of locating individual differences in facilitatory and inhibitory effects over time, planned paired sample *t*-test comparisons were undertaken at each SOA, for both problem sizes. The facilitatory and inhibitory effects for these comparisons are presented in Fig. 2.



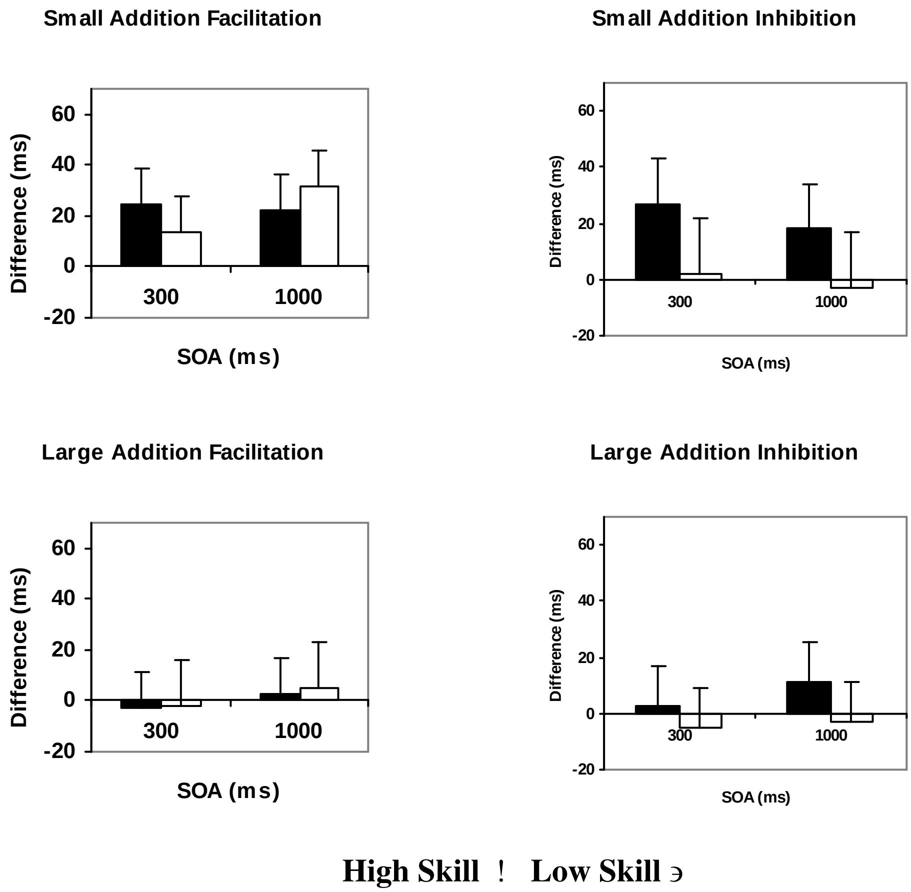


Fig. 2 Showing facilitatory and inhibitory effects for high and low skilled groups as a function of problem size and SOA. The 95% confidence intervals were calculated based on the *MSe* term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects for each group, at each problem size.

For the high skilled group, in the small problem condition, facilitation approached significance at the 300 ms SOA ($t(25) = 2.0$, $p = 0.052$) and reached significance at the 1000 ms SOA ($t(25) = 5.3$, $p < 0.001$). In the large problem condition, facilitation was observed only at the long SOA ($t(25) = 2.9$, $p = 0.007$). For the low skilled group, in the small problem condition, significant facilitation was observed at the long SOA ($t(27) = 2.7$, $p = 0.013$) and a marginally significant inhibitory effect was observed at the short SOA ($t(27) = 2.1$, $p = 0.048$). The facilitation found for the high skilled group for small problems at the long SOA was significantly greater than that observed for the low skilled group ($t(52) = 2.1$, $p = 0.037$).

3.3.2 Addition analysis

The split plot analysis of variance involving the addition data revealed significant main effects of SOA ($F(1, 52)$, = 9.5, $MSe = 3977.9$, $p = 0.003$), prime-target relationship ($F(2, 104) = 14.9$, $MSe = 1215.5$, $p < 0.001$) and fluency ($F(1, 52) = 7.6$, $MSe = 24775.6$, $p = 0.008$), and a significant interaction between prime target relationship and fluency ($F(2, 104) = 4.1$, $MSe = 1215.5$, $p = 0.020$). As in the multiplication analysis, a significant interaction between size and prime-target relationship was found ($F(2, 104) = 13.1$, $MSe = 1107.8$, $p < 0.001$). In the small problem condition, significant facilitation of 23 ms ($t(53) = 4.8$, $p < 0.001$) and inhibition of 10 ms ($t(53) = 2.2$, $p = 0.030$) was found. No significant facilitation or inhibition was observed in the large problem condition.

Planned paired sample *t*-test comparisons examining facilitatory and inhibitory effects for each group were again undertaken at each SOA, for both problem sizes (see Fig. 2). For the high skilled group, in the small problem condition, significant facilitation ($t(25) = 4.1$, $p < 0.001$) and inhibition ($t(25) = 2.9$, $p = 0.007$) was observed at the short SOA. These effects again reached significance at the long SOA (with $t(25) = 2.8$, $p = 0.010$; and $t(25) = 2.7$, $p = 0.011$, respectively). In contrast, for the low skilled group, facilitation reached significance at the long SOA only ($t(27) = 3.4$, $p = 0.002$). In the large problem condition, no facilitatory or inhibitory effects were observed for either group.

In summary, the problem size analysis revealed differences in access to solutions to small and large problems, as a function of arithmetic fluency. In the multiplication condition at the long SOA, facilitation observed in the small problem condition for the high skilled group was significantly greater than that observed for the low skilled group. Moreover, significant facilitation was observed in the large problem condition at this SOA, for the high skilled group alone. In the addition condition, significant facilitation and inhibition was observed in the small problem condition, at both SOAs, for the high skilled group only. In contrast, facilitation was only observed in the small problem condition at the long SOA for the low skilled group. Finally, pre-exposure to large addition problems resulted in no priming effects for either group.

4. Discussion

The present study employed a priming procedure and naming task to determine whether arithmetic fluency influences the ability to automatically access simple arithmetic facts from memory. The overall results showed that high skilled individuals access simple arithmetic facts earlier in the processing sequence than low skilled individuals do. At 300 ms, significant facilitatory and inhibitory effects in target-naming performance following exposure to multiplication and addition problems were observed for the high skilled group alone. At 1000 ms, significant facilitation was observed for both groups, in both operations. For the high skilled group, in the multiplication condition facilitation increased significantly over time. In the addition condition, significant inhibition was observed for the high skilled group only. Further analyses revealed individual differences in access to small and large problems that varied by operation. In the multiplication condition at 1000 ms, facilitation observed in the small problem condition was significantly greater for the high skilled group than for the low skilled group. Furthermore, significant facilitation was observed in the large problem condition at this SOA for the high skilled group alone. In the addition small problem condition, significant facilitation and inhibition was observed at both SOAs for the high skilled group, whilst facilitation only was observed at 1000 ms for the low skilled group. No priming effects were observed for either group in the addition large problem condition.

The findings of the present study are consistent with previous research involving number matching and priming procedures in demonstrating that high skilled individuals have earlier and, in some cases, greater access to simple addition and multiplication facts than low skilled individuals (Jackson & Coney, 2005; LeFevre et al, 1991; LeFevre & Kulak, 1994). Furthermore, given the use of a brief SOA and a task in which solution retrieval was not explicitly required, the findings of the present study support the hypothesis that individual differences in arithmetic skill stem from automaticity in solution retrieval (Galfano, 2003; LeFevre & Kulak's, 1994; Velmans, 1999). Additionally, the finding of a large and significant increase in facilitation over time for high skilled individuals in the multiplication condition indicates that individual differences in arithmetic skill also derive from strategic access to multiplication solutions.

For the high skilled group, the finding of equivalent levels of facilitation and inhibition at the short SOA, and facilitation dominance at the long SOA in the multiplication condition, is similar to the pattern of performance observed in the investigation of associatively related word primes and targets (Neely, 1991). This finding supports the notion that, in skilled arithmeticians, multiplication knowledge is represented in memory in a similar form to word knowledge and accessed through similar mechanisms (Ashcraft, 1992; Dehaene, 1992; LeFevre, Bisanz & Mrkonjic, 1988). Such a finding is not surprising when the reliance on verbal rote learning of associations between words in the acquisition of multiplication knowledge is considered (Jackson & Coney, 2005).

The absence of an inhibitory effect at the long SOA in the multiplication condition is notably at odds with the Jackson and Coney (2005) finding of a significant 16 ms inhibitory effect for this group. This result possibly occurred due to differences between the skilled samples used in each study. The scores obtained by the previous skilled sample on the ACER SCT ranged between 18 and 47, with a mean correct score of 25. In contrast, the scores obtained by the present skilled sample varied less, ranging between 24 and 47, with a mean correct score of 31. Thus, it may be the case that the more skilled sample employed in the present study were able to suppress interference to incongruent targets before responding at the lengthier SOA (LeFevre et al., 1988).

Alternatively, the lack of an inhibitory effect at the long SOA in the multiplication condition may have resulted from lengthier response times in the neutral condition created by the use of the letter stimuli (i.e., X + Y and X x Y). That is, it may be the case that numerical stimuli, such as the 0 + 0 and 0 x 0 neutral stimuli employed in the earlier Jackson and Coney (2005) investigation, actually primes responses to like numerical stimuli (i.e., cf. letter stimuli priming numerical stimuli). In support of this, a comparison of neutral condition reaction times at the 1000 ms SOA, which was employed in both studies, reveals a significant advantage ($p < 0.001$) in responding to the zero stimuli in both the addition and multiplication conditions, for both groups. Nevertheless, given the differences between the samples mentioned earlier, further research into the priming effects produced as a result of the use of the different number and letter stimuli, involving comparable samples, would be beneficial.

Tied in with the above interpretation of the lack of inhibition in the long multiplication SOA condition is the notion that responses to congruent targets might also be speeded by priming using like stimuli. Whilst this possibility cannot be ruled out, the influence of such a confound appears minimal when considered in light of the similar facilitation effects observed between studies. For example, the facilitation effects of 10 and 26 ms observed at the 240 and 1000 ms SOAs (respectively) in the previous study are comparable to the 14 and 35 ms facilitation effects observed for the more skilled participants in the two SOA conditions employed in the present study. Similarly, the inhibition of 10 ms observed at the 240 ms SOA for this group in the previous study is comparable to the 12 ms inhibitory effect observed at the 300 ms SOA in the present study.

The results of the addition analysis in the present study, employing a larger sample size and more distinct fluency groups, differ from those of the previous Jackson and Coney (2005) study that found similar patterns of performance, irrespective of fluency level. Furthermore, in contrast to the previous research, the patterns of facilitation and inhibition for the high skilled group were found to be both significant and constant over time. This difference between studies may have resulted from the use of a longer short SOA condition in the present study (i.e., 300 ms as compared to 240 ms), possibly leading to the use of strategic processing by high skilled participants who had already reached a ceiling in activation due to priming earlier in the addition processing sequence. In such a scenario, with the use of a short (300 ms) SOA condition that was too brief to allow for strategic processing to influence responses to the target, this processing would have to have taken place after presentation of the target. However, given the finding of the same pattern of performance for these participants in the 300 ms multiplication condition, an appreciable increase in facilitation might also be expected in the 1000 ms addition condition. Furthermore, such an explanation is at odds with the results of the previous Jackson and Coney (2005) study that indicated that the facilitation and inhibition effects observed in the number naming task derived from the workings of two independent mechanisms. That is, the facilitation appeared to result from the automatic activation of correct solutions that occurred prior to exposure to the target. In contrast, given that automatic spreading activation does not produce inhibition and that expectancy does not operate at SOAs of 240 ms or less, the inhibition appeared to result from processing that occurred after exposure to the target (Neely, 1991). Consequently, the inhibition in this study was explained in terms of the operation of an obligatory response validity checking mechanism that involves the comparison of a given target to the correct solution in memory and hence, hesitation in responding to the incongruent condition where the two do not match. In the present study, only the high skilled group, who might be more inclined to engage in such a process, demonstrated significant inhibition at 300 ms in both operations and at 1000 ms in the addition condition. Thus, the findings of the present study support and extend those of the earlier Jackson and Coney (2005) study in demonstrating the operation of this inhibitory mechanism in high skilled multiplication and addition performance.

Consistent with self-report data obtained by LeFevre et al. (1996a), a correlational analysis revealed a negative relationship between fluency and problem size effects in the addition condition. However, strong positive correlations between naming latencies and number magnitude in the multiplication analysis were found that were comparable between groups. These results, given that the participants were not required to retrieve solutions in the present naming task, suggest that explanations of the problem size effect based on the differential selection of solution procedures between groups are incomplete (LeFevre et al., 1996a, 1996b). Moreover, the absence of positive correlations between problem size and reaction time in the neutral condition for the addition operation, suggest that explanations of this effect based on the time taken to articulate solutions containing various numbers of syllables are equally inadequate (Brysbaert, 1995). At the very least, models of the problem size effect in adult performance should be revised to incorporate the operation differences identified in the present study.

The problem size analysis revealed greater access to solutions to small and large multiplication problems and earlier access to small addition problems for the high skilled group. Interestingly, the large addition problem condition was the only one in which no significant priming effects were observed for either group. This finding potentially results from a disparity in the frequency of exposure to small and large addition problems. Small numbers occur more frequently than large numbers in naturally occurring settings, and small problems are presented earlier in instruction and with far greater frequency than large problems (Ashcraft, 1992; Hamman & Ashcraft, 1986). What is more, given that rote learning is commonly employed in the learning of multiplication tables, it could reasonably be assumed that large multiplication problems are verbally practiced to a greater extent than large addition problems. Consequently, performance on large addition problems may be at a permanent disadvantage and, given the lack of priming effects observed in the present study (even for relatively skilled individuals), may rely on strategic processing in solution retrieval.

The present study revealed significant advantages in access to correct addition and multiplication solutions for high skilled arithmeticians that varied as a function of arithmetic operation, SOA and problem size. Furthermore, it extended the results of the earlier Jackson and Coney (2005) study by demonstrating the operation of an inhibitory response validity checking mechanism in addition performance. Finally, the present study showed that individual differences in arithmetic skill originate not only in automaticity of solution retrieval but also in strategic access to correct multiplication solutions (Galfano, 2003; LeFevre & Kulak's, 1994; Velmans, 1999).

References

ACER (1984). *ACER Short Clerical Test Forms C, D, E: Uses administration, interpretation*. Camberwell, Victoria: The Australian Council for Educational Research Limited.

Ashcraft, M.H. (1992). Cognitive arithmetic: A review of data and theory. *Cognition, 44,* 75-106.

Brysbaert, M. (1995). Arabic number reading: On the nature of the numerical scale and the origin of phonological recoding. *Journal of Experimental Psychology: General, 124*(4), 434-452.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1-42.

Galfano, G., Rusconi, E. & Umilta, C. (2003). Automatic activation of multiplication facts: Evidence from the nodes adjacent to the product. *The Quarterly Journal of Experimental Psychology, 56*(1)*,* 31-61.

Geary, D.C. & Wiley, J.G. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in young and elderly adults. *Psychology and Aging*, *6*(3), 474-483.

Hamman, M. S. & Ashcraft, M. H. (1986). Textbook presentations of the basic arithmetic facts. *Cognition and Instruction, 3*, 173-192.

Hecht, S.A. (1999). Individual solution processes while solving addition and multiplication math facts in adults. *Memory and Cognition, 27*(6), 1097-1107.

Jackson, N.D. & Coney, J.R. (2005). Simple arithmetic processing: The question of automaticity. *Acta Psychologica, 119*, 41-66.

Kirk, E.P. & Ashcraft, M.H. (2001). Telling stories: The perils and promise of using verbal reports to study math strategies. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 27*(1), 157-175.

LeFevre, J., Bisanz, J., Daley, K.E., Buffone, L., Greenham, S.L. and Sadesky, G.S. (1996). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General, 125*(3), 284-306.

LeFevre, J., Bisanz, J. & Mrkonjic, L. (1988). Cognitive arithmetic: Evidence for obligatory activation of arithmetic facts. *Memory & Cognition, 16*(1), 45-53.

LeFevre, J. & Kulak, A.G. (1994). Individual differences in the obligatory activation of addition facts. *Memory & Cognition, 22*(2), 188-200.

LeFevre, J., Kulak, A.G. & Bisanz, J. (1991). Individual differences and developmental change in the associative relations among numbers. *Journal of Experimental Child Psychology, 52*, 256-274.

LeFevre, J., Sadesky, G.S. & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 22*(1), 216-230.

Neely, J.H. (1991). Semantic priming effects in visual word recognition: A selective review of current findings and theories. In D. Besner & G.W. Humphreys (Eds.), *Basic processes in reading: Visual word recognition* (pp. 264-336). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Smith-Chant, B.L. & LeFevre, J. (2003). Doing as they are told and telling it like it is: Self-reports in mental arithmetic. *Memory and Cognition, 31*(4), 516-528.

Stazyk, E.H., Ashraft, M.H. & Hamann, M.S. (1982). A network approach to mental multiplication. *Journal of Experimental Psychology: Learning, Memory and Cognition, 8*(4), 320-335.

Velmans, M. (1999). When perception becomes conscious. *British Journal of Psychology, 90* (4), 543-566.

Zbrodoff, N.J. & Logan, G.D. (1986). On the autonomy of mental processes: A case study of arithmetic. *Journal of Experimental Psychology: General, 115*(2), 118-130.

Appendix A

Prime Sets and Congruent and Incongruent Targets for Multiplication Operation			
Set 1	Set 2	Congruent	Incongruent
2 x 4	4 x 2	8	30
3 x 5	5 x 3	15	42
3 x 7	7 x 3	21	48
4 x 5	5 x 4	20	63
5 x 6	6 x 5	30	10
5 x 9	9 x 5	45	27
6 x 8	8 x 6	48	15
7 x 9	9 x 7	63	56
8 x 9	9 x 8	72	24
3 x 2	2 x 3	6	54
4 x 3	3 x 4	12	6
5 x 2	2 x 5	10	40
6 x 4	4 x 6	24	8
7 x 6	6 x 7	42	21
8 x 7	7 x 8	56	20
8 x 5	5 x 8	40	12
9 x 3	3 x 9	27	45
9 x 6	6 x 9	54	72

Prime Sets and Congruent and Incongruent Targets for Addition Operation			
Set 1	Set 2	Congruent	Incongruent
2 + 4	4 + 2	6	13
3 + 5	5 + 3	8	16
3 + 7	7 + 3	10	15
4 + 5	5 + 4	9	13
5 + 6	6 + 5	11	8
5 + 9	9 + 5	14	7
6 + 8	8 + 6	14	17
7 + 9	9 + 7	16	5
8 + 9	9 + 8	17	6
3 + 2	2 + 3	5	14
4 + 3	3 + 4	7	10
5 + 2	2 + 5	7	14
6 + 4	4 + 6	10	15
7 + 6	6 + 7	13	9
8 + 7	7 + 8	15	12
8 + 5	5 + 8	13	10
9 + 3	3 + 9	12	7
9 + 6	6 + 9	15	11