Graphics calculators: A (brief) case of technology*

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Undergraduate service courses in mathematics are not generally regarded as providing fertile ground for experimentation, and nor are they usually sought after as stimulating educational environments, perhaps inaccurately on both counts. In many universities such courses rarely attract high profile staff, and are regarded as a necessary evil, providing 'bread and butter' work. In this paper, we suggest that the thoughtful use of graphics calculators in such an environment can not only have a positive effect on students and their learning, but can also act as a powerful stimulus to staff. Indeed in a recent interview (Trotter, 1991) the prominent US mathematics educator Bert Waits said, "It's the most exciting thing I've been involved in my lifetime and a career of 30 years." In this paper we report on our own experiences, generally positive, but always interesting and challenging, in using graphics calculators in *Fundamentals of Mathematics*, a first year, semester long mathematics course, at Murdoch University. The paper provides some background, a rationale for using graphics calculators in the way that we have, and outlines the issues of teaching, learning and assessment that have arisen from this experience. It also includes some detailed evaluative information from both students and staff.

Fundamentals of Mathematics revises basic concepts of algebra and trigonometry and introduces students to matrices and differential calculus. In many ways the course is similar in content to courses taken at the upper secondary school level. However there is a difference in the mode of delivery, in this case mainly lecture presentations to large groups as opposed to secondary school class teaching. Many students taking the course are mature age students returning to study after some years break; most of the rest are students straight from secondary school but with limited mathematical backgrounds. Although students take the course both internally and externally, this paper concentrates on teaching and learning outcomes in the internal mode. The outcomes of work with students studying in the external mode have been discussed in Bradley & Kemp (1993).

During the latter part of 1993, we were able to attract enough funding from various sources at Murdoch University to allow us to purchase 35 graphics calculators to use with students in the first semester of 1994. With just these 35 calculators, weekly access both in and outside tutorials was possible for all 150 internal students and would probably have been sufficient for 200 to 250 students.

Rationale

There were two main reasons for the introduction of this technology into *Fundamentals of Mathematics*. Firstly, it was anticipated that it would help students to learn mathematics better than would otherwise have been the case, due largely to the opportunities for providing students with active learning experiences. It was expected that these experiences would help students to develop concepts and procedures more effectively. The second reason related to the nature of mathematics itself. Students in the latter part of the twentieth century ought to learn something about the

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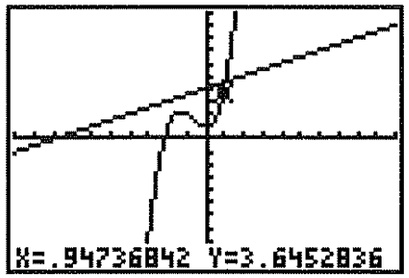
technologies that are commonly available to support mathematical work, and how to make intelligent choices regarding their use. Many students taking *Fundamentals of Mathematics* will use mathematics in a variety of fields, where they should find the appropriate use of technology helpful.

The choice of graphics calculators in preference to microcomputers was prompted mainly by practical considerations. Graphics calculators are much less expensive than microcomputers, making it feasible to provide substantial personal experience with technology for every student. In addition, microcomputers tend to be relatively large and cumbersome to move around, and so are usually located in laboratories in both schools and universities. In marked contrast, graphics calculators are about the same size as scientific calculators, and so are as easy to carry around. Unlike other computers, for which each separate item of software needs to be purchased, understood and maintained, requiring site licence arrangements and substantial staff and student time, graphics calculators come complete with inbuilt software, so that learning to use the calculator is essentially the same as learning to use the calculator's software. Finally, a further practical attraction of graphics calculators is the potential in the near future for individual student ownership. Indeed, this is already a reality for many students in parts of the UK and the USA.

Microcomputers do, however, have some advantages over graphics calculators. Whilst students may find a colour display helpful, a more significant advantage is the finer screen resolution which enables much smoother graphs to be drawn. For example, the picture below shows the image of a Texas Instruments TI-81™ graphics calculator screen on which the functions:

$$y = x^3 + 2x^2 - 1 \quad \text{and} \quad y = 0.5x + 4$$

have been graphed. The graphs have a 'chunky' appearance because of the relatively small number of screen pixels (95 x 63) used by the calculator.



Using this figure, the intersection of the two functions can be traced and the coordinates found to the required accuracy. Even low resolution microcomputer function graphers will provide smoother images than this. In addition, microcomputers are considerably more flexible than graphics calculators, since there is a large range of software available. But these advantages are not sufficient to outweigh the huge physical and economic advantages of graphics calculators. As the next section indicates, despite their limited flexibility, graphics calculators incorporate a number of features of potential value to students in a course like *Fundamentals of Mathematics*, as well as other features of less immediate importance.

Calculator features

There is a variety of models, with considerable price differences, available from the four graphics calculator manufacturers: Texas Instruments, Casio, Sharp and Hewlett Packard. We had been satisfied with the Texas Instruments TI-81™, which we had used in 1993 with external students, so we chose the most recent Texas Instruments model, the TI-82™ for use in 1994 with internal students. The particular features we found to be of value included:

- graphing
  (The graphing facility makes possible the graphing of functions and subsequent exploration of graphs by zooming and tracing; there is an automatic graphical calculation menu for finding roots, points of intersection, relative extremes and numerical derivatives at a point; there is also an easy list mechanism for graphing a *family* of functions.)

- tables
  (An alternative to a graphical representation of a function is provided in the form of a table of values, which allows the behaviour of a function to be studied numerically as well as graphically.)

- equations
  (Solutions of equations and inequalities, accurate to an appropriate number of decimal places, are accessible manually (by zooming and tracing) or automatically from the graphical calculation menu. In addition, there is a numerical equation solving command.)

- matrices
  (Matrices are easily entered, displayed, edited and recalled; matrix manipulations are readily carried out with intuitive commands, so that systems of linear equations are easy to solve.)

- calculus
  (Differentiation and integration are available both graphically and numerically; the numerical value of the slope of a line at a point is easily displayed, and tangents to curves can be drawn easily.)

- fractions
  (The calculator's capacity for converting decimal results into fractions is an attractive feature, allowing matrix inverses to be given with fractions rather than decimals, for example.)

As with its competitors, the TI-82™ has many other features, which have not been relevant to this course. These include polar coordinate graphing, graphing of the derivative function, data analysis, discrete mathematics, other uses of lists, drawing, recursively defined functions, elementary programming and the capacity to connect two calculators together with a cable for sharing of information. We have not found these other features to be a significant distraction to students. An overview of new features and their educational significance is given in Kissane (1994).

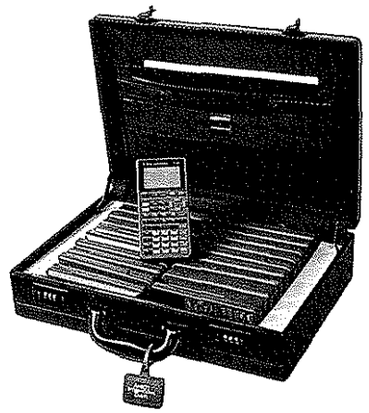
Course organisation

To take advantage of these graphics calculators features, some reorganisation of the course was necessary. Like many other undergraduate courses, course material was presented in whole-group lectures and followed up in whole group workshops and small group tutorials. During lectures, use was made of a graphing package on a microcomputer connected to an overhead projector. Workshops were operated as problem solving sessions and the tutorials were redesigned to include student use of the graphics calculators, guided by specially prepared activity sheets.

The assessment components for the course were assignments, short tests and a final examination. The assignments did not require the use of graphics calculators, although students were not prevented from using the calculators, which were accessible outside tutorials. One of the three short tests specifically required the use of graphics calculators, but students were prevented from using graphics calculators during the final standard three hour examination. Detailed information about the assessment for this course is contained in Kissane, Bradley & Kemp (1994).

Calculator access

A briefcase containing 20 calculators was passed from tutor to tutor for use in the weekly tutorials, which were deliberately scheduled at different times. Whilst the briefcase was a little on the heavy (and valuable) side, it enabled the technology to be taken to the students, instead of the students going to the technology, as is the case with other computer laboratories. (See the photograph.) A computer laboratory of this kind costs about the same amount of money as a single microcomputer with the appropriate software. Since an activity sheet was prepared each week guiding the students through the calculator's functions, the manuals were not routinely used by the students (although the manual is excellent, and can be used by students at other times to advantage).



Computer laboratory, *circa* 1994

The remaining 15 calculators were available for short loan at one of three locations on campus including the reserve section of the library. In addition the calculators in the library were available for overnight loan.

Tutorial activities

Students used graphics calculators in tutorials to help them understand the concepts and procedures covered in the lectures. The focus was on the learning of mathematics, rather than on the calculators themselves. The weekly calculator activities were directly related to the material covered in the lectures of the previous week to give students the opportunity to explore the material for themselves. Although the lecture presentation included the use of an overhead projector facility to give a more dynamic view of the material, the use of graphics calculators allowed the students more personal involvement.

Part of each tutorial, approximately 20 minutes, was allocated to an activity using the graphics calculators. Most of the activities were designed to be attempted by one person at a time, but many students found it helpful to collaborate with a partner and compare observations with others, finding that the discussion aided learning. Each student was expected to complete the weekly activity sheet, finishing it in their own time if necessary. Students were provided with ample access to the calculators outside tutorial hours.

Graphics calculator activity sheets

Part of the process of designing the calculator activities included becoming aware of the features of the calculators and the educational benefits of them. Since the main purpose of the use of the calculators was to aid student learning it was important to make the most of the appropriate features and so the preparation of the activity sheets was quite demanding.

The activity sheets took into account the need for students to become familiar with the use of the calculators. In earlier activities the sheets included quite detailed calculator procedures. As the students became more proficient, it was possible to give less specific calculator instructions for the procedures and it was assumed that students would refer to earlier sheets if necessary.

The sheets were designed to contain a set of developmental activities which generally progressed from structured to less structured. Students were asked to predict and check, to make up their own examples, to explain why something happened, and so on. Thus, there was an essential difference between these guided activities and a set of practice exercises.

Since the students had access to sophisticated technology it was possible to include activities which would normally be considered 'too hard' at this level. For example, it was possible to include

graphs that would normally be too complicated to plot easily by hand, equations and inequalities that would be too hard to solve algebraically and systems of simultaneous linear equations with non-integral coefficients and solutions.

At the start of the course some tutors were unfamiliar with graphics calculators. However, the tutors found the activity sheets sufficient to enable them to confidently use graphics calculators in their tutorials.

Assessment

The use of graphics calculators as a learning tool highlights only one aspect of their significance. Just as scientific calculators are seen as a standard tool available for use in every aspect of the course, so consideration needs to be given to the total integration of graphics calculators into the curriculum. This implies their use should be permitted, and even encouraged, in assessment situations such as assignments (investigations, projects), tests and final examinations. Although a major argument for using the calculators is that they have the potential to improve student learning, another substantial aspect concerns the extent to which students should acquire expertise with appropriate technologies for mathematics. These two issues together suggested that some part of the course assessment should allow students to use graphics calculators to accomplish mathematical ends. At the same time, such assessment procedures have the potential to provide information on how successfully students have acquired calculator skills. Consequently two new forms of assessment were trialed, namely a short test that required the use of the calculator and an examination question incorporating graphical output that looked for evidence of the students' ability to understand the relationships between graphs of functions, equations, roots and inequalities. In general terms, students handled both the test and the examination questions well, and seemed to have acquired substantial familiarity with the use of the graphics calculators for the mathematical procedures examined. Further discussion on the implications of graphics calculators for assessment and details of the test and examination question are given in Kissane, Bradley & Kemp (1994).

Student reactions

In trying to assess the success of the use of the graphics calculators in *Fundamentals of Mathematics* several data sources were used. These included informal feedback from tutors and students, interviews with individual students and a formal survey by the Institutional Research and Evaluation section of Murdoch's Academic Services Unit.

Throughout the course the informal comments from both students and tutors were very positive. The main criticism was that there was never enough time to do everything in the tutorials (the activities were designed assuming that the students would complete them outside the tutorial). However, the suggestions were always for longer tutorials rather than not using the calculators. The positive informal feedback was confirmed in the individual interviews conducted in the 'swot vac' week in June. Some students reported in the interviews that they had felt comfortable with the calculators from the start, whilst others found continued use eventually enabled ease of use and a better understanding of the abstract concepts in the course. A few students were openly hostile to the calculators from the start and admitted they had put no effort into their use and, therefore, had received no benefit.

Survey questions

The course evaluation questionnaires indicated a very positive attitude not only towards the use of the graphics calculators but also towards all aspects of the teaching of the course. The following table summarises the responses to the questions specifically concerned with graphics calculators on the formal survey towards the end of first semester 1994. Students were asked to indicate whether they strongly agreed, agreed, disagreed, strongly disagreed with the statements or were unable to judge. The summary indicates the percentage who either strongly agreed or agreed of those who felt they were able to judge.

	'Strongly agree' or 'Agree'
The graphics calculators were readily accessible outside the tutorials.	84
Overall I enjoyed using the graphics calculators.	69
Eventually I enjoyed using the graphics calculators.	88
Using the graphics calculators helped me to understand graphs of polynomial and rational functions.	85
Using the graphics calculators helped me to understand graphs of trigonometric functions.	83
Using the graphics calculator helped me to understand the relationship between graphs and solutions to equations and inequalities.	71
Using the graphics calculators helped me to understand matrices and their uses to solve systems of equations.	75
The "Activity" sheets provided appropriate activities.	73
Some assignment questions should require the use of graphics calculators.	68
It was a good idea to be able to use the graphics calculators in the test.	76
I think that we should be allowed to use graphics calculators in the final examination.	54

The response to the first three questions confirmed our informal impression that students were generally favourably disposed to using the calculators, although some were less enthusiastic at the beginning. The responses to the next five questions suggest that the tutorial activities helped students' learning in many parts of the course. The responses to the final three questions reinforced our view that graphics calculators need to be incorporated in assessment, although there is some ambivalence among students regarding their use in the final examination. In the light of these responses, we intend to give closer thought to the relationship between graphics calculators and assessment in the next presentation of the course. The issues associated with this kind of use are explored more fully in Kissane, Bradley & Kemp (1994).

Survey comments

All students were invited to make some general comment about the use of the graphics calculators and most did. The ratio of positive to negative comments was about four to one, which

is consistent with the overall level of support indicated in the responses to the survey questions. The following few examples of both negative and positive comments (copied exactly as the students had written them!), give the flavour of student views and echo the diversity of student opinion.

> *Although the graphics calculators helped 'visualise' graphs the diagrams in lectures did more so. I didn't like using the G. calculators at all.*
>
> *Tutorials were too rushed with assignment concepts and grafix calculator worksheets. Too much ground creating unnecessary burdens.*
>
> *I don't believe that the students should be assessed on using graphics calculators. ... They are an aid, NOT an assessable part of the course. I hardly used the calculators as I don't like them.*
>
> *I thought the graphics calculators didn't give me as much as I could get from them. We should be allowed to use the calculators (graphic) and normal ones in a test together.*
>
> *The graphic calculator is extremely helpful in my understanding of functions. It gives me a clear picture of the graphs of polynomial and rational functions. ... It is also helpful in solving equations.*
>
> *... I also felt that the graphic calcs. were very helpful in aiding my ability to draw graphs from the given equations and vice versa.*
>
> *The graphics calculators are an invaluable aid to the learning of concepts of this course. However, not enough information is provided about available functions ... such as TABLE.*
>
> *Once I understood how to use the graphic calculators they were relatively easy to use. I feel there should be a greater emphasis on their use.*
>
> *Use of graphics calculator and the maths package Jen used in the lectures helped to understand, and also gave meaning to what we were doing.*
>
> *... Graphic calculators are good as they help you to imagine what a graph looks like however I don't think they should be used in the exams (only exams that are particularly focussed on the TI-82) because maths is about using your understanding and perception and perhaps this ability will become obsolete if the calculator takes over, just as I'm unable to do simple sums without the calculator now.*
>
> *Graphics calculators did assist in many ways especially to provide quick and easy checks on your working. However if the calculator is to be integrated into the course as it is (ie having the test) I believe it should be available in the exam. With the clause that all working must be shown. Students would thus have to understand basic concepts and working yet have the calculator merely to verify your answer and to locate small calculation errors that may arise in working. It should not be used and allowed as other calculators are as this will encourage students to rely on calculators and not be able to devise answers without it.*

With an apprehensive group of non-mathematics specialists there will always be mixed reactions but we were heartened by the 70-80% positive response from the students, both from the formal questions and the individual comments. Taken together these encourage us to continue with curriculum development using graphics calculators.

Conclusions

Even with rather modest resources, it is clear to us that the use of graphics calculators can have quite a substantial impact on a mathematics course. It seems that it is a considerably better investment in technology than would be possible with microcomputers for the same amount of

money. To maximise the benefits of this technology, careful attention needs to be paid to the design of appropriate student activities that emphasise the place of the calculators in concept development.

The results of the graphics calculator test and the trial of a different style of examination question were very pleasing and reinforced our view that modifications to the assessment program can be made in ways that support the mathematical intentions of a course rather than undermining them. Time allocation for the different aspects of the teaching and learning process, especially within the tutorials, is very important and careful thought will need to be given to this aspect in future presentations of the course. We see as the logical next step an integration of the calculator use into assignments and the final examination.

There is always a danger of novelty effects with new technologies, with the risk that the early flushes of enthusiasm will cloud our judgements. As the title of this paper acknowledges, the use of graphics calculators in a single course for a single semester is a brief case, when contrasted with our interest in long term effects. However the experience of several months of student use is long enough for the novelty effect to be reduced in significance, and for us to be confident that further experimentation of this kind is desirable.

It seems inevitable that graphics calculators will become more accessible to Australian students in forthcoming years, and are likely to become the personal property of many students. The implications of this for course design, content, pedagogy and assessment need to be considered now, rather than waiting until many, if not most, of our students arrive on campus with their own graphics calculator.

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