Fuzzy Linear Assignment Problem: An Approach to Vehicle Fleet Deployment

Minh Ngoc Ngo, Kiam Tian Seow, and Kok Wai Wong
School of Computer Engineering
Nanyang Technological University
Nanyang Avenue
Singapore 639798
Email: ngoc_nm@yahoo.com, asktseow@ntu.edu.sg, askwwong@ntu.edu.sg

***Abstract* - This paper proposes and examines a new approach using fuzzy logic to vehicle fleet deployment. Fleet deployment is viewed as a fuzzy linear assignment problem. It assigns each travel request to an available service vehicle through solving a linear assignment matrix of defuzzied cost entries. Each cost entry indicates the cost value of a travel request that "fuzzily aggregates" multiple criteria in simple rules incorporating human dispatching expertise. The approach is examined via extensive simulations anchored in a representative scenario of taxi deployment, and compared to the conventional case of using only distances (each from the taxi position to the source point and finally destination point of a travel request) as cost entries. Discussion in the context of related work examines the performance and practicality of the proposed approach.**

I. INTRODUCTION

The problem of vehicle fleet assignment is at the heart of service vehicle deployment. By fleet assignment (e.g. as in taxi deployment), we refer to the concurrent allocation of one different vehicle in the available fleet for every travel request. Technically, it is a linear assignment problem (LAP) [2], where in an instance of LAP, a *mxn* matrix of assignment entries is given, with each entry indicating the cost value of one of the *m* resources (e.g. vehicles) for one of the *n* tasks (e.g. travel requests). Thus, optimal vehicle fleet assignment (i.e., assignment with minimum total cost) can simply be solved by treating it as a linear assignment problem. However, any linear assignment method based on a single performance criterion (either *min* distance or *min* time, i.e., with distance or time as cost entries), as proposed in existing works [11-13,16,17], is practically limited since this fails to incorporate not only other important considerations (e.g. vehicle utility), but also the vagueness that is inherent in these considerations. Linear assignment with multiple criteria is normally a difficult task, especially when we have clearly conflicting criteria, there is normally no optimal solution simultaneously satisfying all the criteria. Besides, some criteria are easily formulated with words but cannot easily be cast as concise formulas.

To incorporate these considerations while retaining the simplicity of it being solved essentially as an LAP, for which highly computationally efficient algorithms (e.g. Jonker-Volgenant algorithm [2]) exist, we propose a fuzzy logic approach to determining cost entries "aggregated" with multiple criteria incorporating human dispatching expertise. The result is a fuzzy linear assignment problem (FLAP) approach based on *min* cost, where the cost is a fuzzy "aggregation" of multiple criteria. Fuzzy logic, in this application context, is an ideal choice since it can handle multiple variables as well as the vagueness of their values expressed in rules that is characteristic of the human expertise [3].

To the best of our knowledge, unlike the proposed FLAP approach, very few other approaches [4,10] take into consideration simultaneous multiple requests. The ones that do consider often treat this problem as a linear assignment problem in which only one criterion – usually *min* distance or *min* time - is used. Importantly, the proposed FLAP approach has the following advantages, namely, it is simple and intuitive, allows multiple criteria to be flexibly incorporated and expressed in simple rules, and yet is computationally manageable and easily extensible. Simulations in a representative scenario of taxi deployment and our discussions show that the FLAP approach, fuzzily aggregating multiple criteria defuzzied as cost entries, generally outperforms the conventional LAP (CLAP) approach that uses a single criterion such as *min* distance for cost entries.

The rest of the paper is organized as follows. Section 2 presents the fleet deployment problem. Section 3 presents a specific FLAP approach to the problem in the context of taxi deployment. Section 4 presents simulation results that compare the FLAP and CLAP approaches, along with some discussions that examine the comparative performance and practicality of the proposed approach. Section 5 discusses related work on taxi deployment in the operations research literature. Section 6 presents the conclusions.

II. FLEET DEPLOYMENT AND FLAP

A. Overview of Fleet Deployment

An objective of fleet deployment problem is to assign each travel request to a different fleet vehicle that is available in order to minimize costs. In most fleet deployment applications, it is found that fleet assignment is subjected to

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multiple criteria, which makes it difficult to formalize the assignment task using mathematical models or individual rules. This difficulty comes from the multi quantitative and qualitative criteria that must be weighted to achieve a good trade-off among conflicting criteria.

To address the issues, we formulate, in the next section, a fuzzy linear assignment problem (FLAP) as a practical approach to solving the assignment problem. The fuzzy rule-base approach to reasoning out each cost entry allows the incorporation of human expertise in evaluating candidate vehicles for servicing new requests.

B. FLAP Formulation

The FLAP is considered an extension of the CLAP for dealing with the multi-criteria nature of many assignment problems and can be stated as follows. Let

$$T = \{t_0, t_1, ..., t_{nT}\} \text{ and } R = \{r_0, r_1, ..., r_{nR}\} \quad (1)$$

denote a set of tasks and resources respectively and let

$$c_{ij} = c[t_i, r_j], \text{ for } t_i \in T, r_j \in R \quad (2)$$

be a measure of cost value of assigning task $t_i \in T$ to resource $r_j \in R$. Each cost value c_{ij} is a fuzzy "aggregation" of multiple criteria. Let

$$A = \{a_0, a_1, ..., a_k\} \quad (3)$$

be the set of k attributes, which is called a context for the fleet assignment, each attribute can be a crisp or linguistic variable and let

$$Rule = \{R_1, R_2, ... R_n\} \quad (4)$$

be the set of fuzzy if...then rules which represent criteria for the assignment task. Each rule is of the following form:

R_i: ***if*** a_{i1} *is* V_{i1} ***and/or*** a_{i2} *is* V_{i2}... ***and/or*** a_{im} *is* V_{im} ***then*** *cost is* U_{Ri} (5)

where $a_{ij} \in A$, V_{ij} $(j=1...n_i)$ is a crisp value if a_{ij} is a crisp variable or a fuzzy term if a_{ij} is a linguistic variable [19] and U_{Ri} is the output fuzzy set of R_i. Essentially, each rule can be represented by a fuzzy relation and is defined as:

$$Ri(a_{i1}, a_{i2}, \ldots, a_{im}) = V_{i1} \times V_{i2} \times \ldots \times V_{im} \rightarrow U_{ri} \quad (6)$$

Assume only *and* connective is used in fuzzy rules, (6) can be rewritten as:

$$\begin{aligned} &Ri(a_{i1}, a_{i2}, \ldots, a_{im}) \\ &= [V_{i1}(a_{i1}) \wedge V_{i2}(a_{i2}) \wedge \ldots \wedge V_{im}(a_{im})] \rightarrow U_{Ri}(cost) \\ &= V_{Ri}(A) \rightarrow U_{Ri}(cost) \end{aligned} \quad (7)$$

in which, $\wedge$ is the connective *and* operator. For a specific assignment context A_0, the degree of matching a rule's antecedents is define as:

$$V'_{Ri} = V_{Ri}(A_0) \quad (8)$$

The consequence fuzzy output U'_{Ri} is nothing else but the image of $V_{Ri}(A) \rightarrow U_{Ri}(cost)$ on V'_{Ri}:

$$U'_{Ri} = V'_{Ri} \circ Ri \quad (9)$$

An aggregated fuzzy set of all the output sets can be defined as:

$$U = Agg(U_{R1}, U_{R2}, \ldots, U_{Rn}) \quad (10)$$

where *Agg* is the aggregation operator and is chosen based on requirements of particular problems. The commonly used operators are *sum* and *maximum*.

Each aggregated cost value c_{ij} in (2) is a function from the fuzzy output space defined in (10) into a space of crisp values:

$$c_{ij} = defuzzifier(U) \quad (11)$$

Assume $N_T < N_R$, the objective of the assignment problem is to find a particular mapping:

$$\begin{aligned} &\prod : T \mapsto R \text{ such that for } t_i\ t_j \in T, \\ &\qquad i \neq j \text{ implies that } \prod(t_i) \neq \prod(t_j) \end{aligned} \quad (12)$$

such that the total aggregated cost value

$$C_{TOT} = \sum_{i=1}^{N_T} c[i, \prod(i)] \quad (13)$$

is minimised over all possible assignment sets induced by $\prod$.

The process described in (6-9) is called fuzzy inference or fuzzy reasoning. The fuzzy output set of each rule is deduced based on given inputs which match the antecedents of that fuzzy rule to some degree. The compositional rule of inference or generalized modus ponens described in (9) is the most commonly used fuzzy reasoning process. Intuitively, the condition specified in (12) is to ensure that no two different tasks are assigned to the same resource vice-versa. Essentially, the FLAP includes two main steps:

1. define aggregated cost value using fuzzy reasoning process, as described in (3-11).
2. perform linear assignment, as described in (12-13)

III. TAXI DEPLOYMENT AND FLAP

A. Overview

In the context of a taxi service company, the FLAP can be described as follows. The company receives calls from customers. Each customer specifies a pickup and destination location. As customers demand fast service, assigning task must be done in real-time. Moreover, scheduling of vehicle should be done every time new customers call into the taxi centre. It is often the case that there are more than one requests arrive at the same time. Assuming that at time t, there are m requests pending and n taxis idle. The company's taxi dispatching system (TDS) should find an assignment that matches each available taxi to a request, subject to the following multiple criteria: a balance workload among taxis, short average trip time, and short average distance. In the taxi fleet deployment, a the assignment context A in (3) is basically a set of attributes represent information about taxis and the traffic network at a particular moment such as positions and traffic density. The structure of a TDS applying FLAP consists of two main modules as shown in Figure 1.

The Fuzzy Inference System (FIS) [1] addresses the first step of the proposed FLAP approach and the Linear Assignment Module (LAM) solves the second step. Within the FLAP framework, the attribute of human reasoning and decision making can be formulated by simple *if...then* rules coupled with easily understandable and natural linguistic representations.

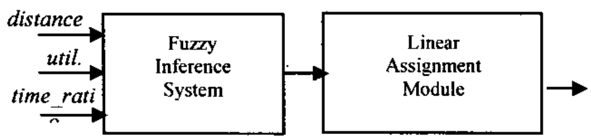


Fig. 1. Taxi Dispatching System

The linguistic values in the rule antecedents convey the imprecision associated with measurements such as the distance between two locations. Whereas, the linguistic values in the rule consequences represent the vagueness inherent in the reasoning process to generate each cost entry, based on which the assignment decision is made.

B. The Fuzzy Rule Base

The main goal of a fuzzy rule-base system is emulating a human expert and representing various criteria of the dispatching problem. In this situation, the knowledge of the human operator would be put in the form of a set of fuzzy linguistic rules The development of rules is time-consuming since expert knowledge is translated into fuzzy rule. Table 1 is the rule-base containing twelve essential rules for a taxi dispatching problem. The form of each rule has been specified in (5) using only connective *and*, *distance, utilization, time_ratio* are three linguistic variables and *cost* is the output variable. It is important to note that when *distance* changes one step, for example from *near* to *medium*, or when *time_ratio* changes one step from *small* to *big*, *cost* changes two steps, for example from *extremely low* to *low*.

TABLE 1
THE RULE BASE

distance	utilization	time_ratio	Cost
near	low	small	very low
		big	below average
	high	small	Low
		big	Average
medium	low	small	below average
		big	above average
	high	small	Average
		big	High
far	low	small	Average
		big	High
	high	small	above average
		big	very high

Whereas when *utilization* changes one step, *cost* also changes one step. This is to highlight that *distance* and *time_ratio* are of higher importance than *utilization*. This comes from the fact that any taxi company will put its profit and its customers' satisfactory to the first order. Only when these two criteria are satisfied, the company will consider benefiting its drivers.

C. Fuzzy Inference and Cost Entries

Inputs determination

A study [18] has been carried out to determine the k attributes of a context associated with a taxi and a request. The main goals of almost taxi companies are maximizing profitability and service quality, which is measured by customer waiting time. Thus three attributes are chosen: *distance, time_ratio and utilization.* These are actually three linguistic input variables for the FIS. *Distance* is simply the full straight line distance between current position of the taxi to the pickup location and then from the pickup location to destination. Distance is to ensure short average travelling distance for a trip. *Time_ratio* is the ratio between the time taken to travel to destination under current traffic condition and that under light traffic load condition. A big *time_ratio* reflects a heavy traffic condition. Finally, *utilization* of a taxi at time is defined as

$$Utilization(i) = \frac{\text{no. of times taxi } i \text{ is used}}{\text{total no. of requests}} - \frac{1}{N},$$

where N is the number of taxis. This parameter is considered to make sure a balance workload between taxis. Thus avoiding the case where a taxi is busy all the time while some others are almost idle.

Linguistic terms & membership functions

An essential step in developing a fuzzy inference system is to identify relevant states of linguistic variables by a set of linguistic terms with the corresponding fuzzy sets. The shape of a membership function is quite free. However, for computational efficiency and ease of data acquisition trapezoidal and triangular membership functions were used in this paper. The optimal partition of an input domain can be achieved by a heuristic method, but the basic principle might be to use our real-life linguistic terms such as *near, far, medium* for *distance.* The methods of constructing membership functions can be divided into direct and indirect methods. Direct method means that experts try to find answers to the following questions:

What is the membership degree of x in fuzzy set A?
Which elements x have the degree of membership A(x)?

By answering these questions, a set of pairs $\{x, A(x)\}$ can be defined, and the membership functions can be constructed using some curve-fitting methods such as trapezoidal approximation.

On the other hand, sometimes it is easier to compare the degrees to which elements belong to A than to give the actual degree of membership for each element as in the direct methods. An expert makes pair wise comparisons between elements $x_1, x_2, \ldots, x_n$ of the universal set U with respect to how much they belong in A [3].

Table 2 shows the linguistics terms used and the membership functions are shown in Figure 3 respectively. *cost* is the output of FIS, which is actually used as an entry for the cost matrix after defuzzification. *cost* can be understood as a weight, which reflects the likelihood that a taxi will be assigned to a particular request in the relationship with other taxis.

TABLE II
LINGUISTIC TERMS

Variable	Linguistic terms
distance	near(N), medium(M), far(F)
utilization	high(H), low(L)
time ratio	small(S),big(B)
cost	extremely low(EL), very low(VL), low(L), below average(BA), above average(AA), high(H), very high(VH), extremely high(EH)

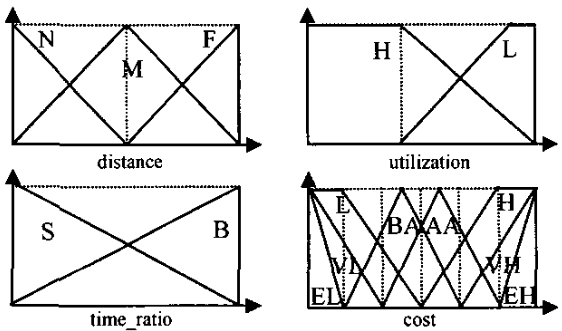


Fig. 2. Membership functions

Inference operators

The connective *and* is usually related to the notion of intersection *(min)*. However in this paper a softer interpretation of *and* could be achieved by using the algebraic product *(prod)*. Therefore, the computed truth degree of the premise should be dependent on any changes of each input

Product *(prod)* operator is also used for implication operator. The most significant advantage of *prod* over *min* operator is the fact that *prod* retains more information.

Since there is more than one rule, aggregation is necessary. As for aggregation operator, *sum* is used so that every rule will have some certain contributions to the final output fuzzy set.

Defuzzification method

Interestingly, the most important step, which affects the performance of the fuzzy inference system, is defuzzification. The literature presents different defuzzification methods [3] and clearly, each of them has its own features that are suitable for slightly different kinds of problems.

The Max Criterion Method is applicable for arbitrary fuzzy sets and for arbitrary domain of the output, which is not necessarily a subset of the real line. On the other hand, one disadvantage is that same output may be generated for different set of inputs since it does not specify which value of maximum membership function has to be chosen.

As for the Mean of Maxima Method, it is easy to realize that as long as one fuzzy set keeps dominating others, the output remains unchanged. This is the main disadvantage, which makes it unsuitable for this problem.

The main idea of the Centre of Gravity Method is to tale the rules into consideration according to their degree of applicability. One important advantage of this method is that it guarantees that if a certain rule was dominating in one step, it is not necessarily dominating again the next time. However, it will maintain a certain influence on the calculation of the centre of gravity. Because of this advantage, this paper makes use of this method as the defuzzification method to make sure that every rule has certain contribution to the final output. Moreover, a small change in any input will affect the crisp output, which is the "aggregated" cost value.

IV. SIMULATIONS AND DISCUSSIONS

A. Simulation Environment

The Intelligent Transportation Planning System (ITPS) software [20] was adapted to provide various emulations of the actual road conditions and events that occur in complex traffic road networks. Each road in the traffic network is associated with a maximum speed. The software is capable of simulating different traffic conditions including light, medium and heavy traffic load. To model these conditions, each vehicle when turning to a new road will set its speed to the maximum speed of that road

B. Experiments

For empirical comparison purposes, we carried out experiments for both the FLAP and CLAP approaches under the same dispatching situation and traffic conditions. For the CLAP approach, we take the distance of the shortest path between the taxi's current location and the destination as a cost entry, hence the approach is also called "Nearest Neighbourhood" or "nearest" for short. The traffic network used for these experiments was partly taken from Singapore traffic network with a total of 183 roads.

In each assignment, 20 taxis for 10 new requests were considered. It was carried out under two traffic conditions: *light* and *heavy* traffic load. Numerous 20x10 matrices were simulated for each approach on the ITPS.

Simulation data generated were evaluated based on three performance measures: average trip distance, average trip time and the maximum number of requests which are served by each taxi and deviates from the average number of requests. The last measure requires some clarification: it provides an indication of the relative utilization of a taxi; the smaller its value, the more balanced its utilization is relative to the other taxis'. Among the above measures, the average trip time and distance are of higher importance than maximum deviation.

C. Light Traffic Load

Figures 3a, 3b, 3c show comparisons of the two approaches under light traffic load with respect to three performance measures: average trip distance, average trip time and maximum deviation requests. As for the CLAP approach, a taxi always follows the shortest path to destination and it will not avoid traffic jam. Therefore in terms of distance, the CLAP approach always results in a smaller average trip distance as can be seen in Figure 3a. The average distance difference is about 0.6 kilometers, which is acceptable considering that the average distance of a trip is about 8 kilometers. However, Figure 3b shows that the average trip time in the case of the FLAP approach is slightly smaller from about 2 to 5 minutes per trip than in the case of CLAP. These results can be explained as follows: when the traffic load is light or during non-peak hours, the shortest path usually is the fastest path. Therefore, a significant difference cannot be seen in this case. Finally, as for utilization, the FLAP approach outperforms CLAP and always results in a more balance workload since the maximum number of requests, which is served by one taxi and deviates from the average number of requests, is small (see Figure 3c).

D. Heavy Traffic Load

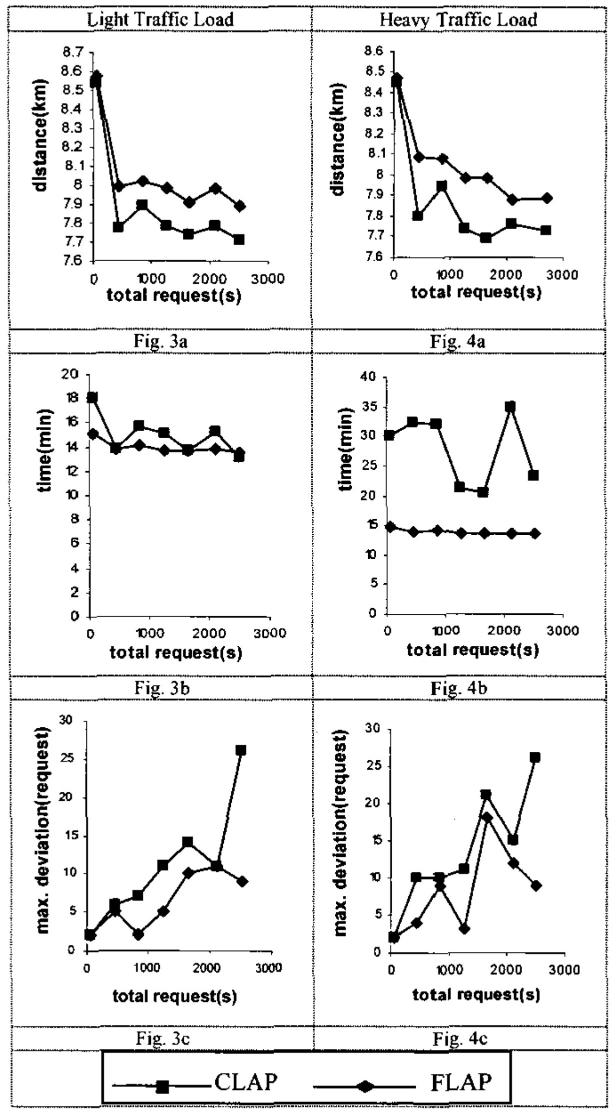
Figures 4a, 4b, 4c show comparisons of the two approaches under heavy traffic load with respect to three performance measures mentioned above.

The improvements of the proposed approach can be seen clearly in this case. In the CLAP approach, an assigned taxi always follows a pre-planned route, which is the shortest path, no matter how heavy is the traffic density of that path. Thus, when traffic load is heavy, it is expected that this approach always results in a much longer average trip time. Indeed, the experiment results from Figure 4b have proven this point. Average trip time is 10 to 20 min smaller under heavy traffic load with FLAP approach than that with CLAP approach. This is a significant improvement. Comparisons on average trip distance and maximum request deviation give similar results to the case of light traffic load (see Figures 4a, 4c).

Discussions

The simulation results show that the FLAP approach results in a slightly longer average trip distance but a much shorter average trip time, especially under heavy traffic loads (during peak-hours), while keeping the balance workload among taxis. In other words, it provides a combination of minimizing average trip distance and time as well as a compromise between minimizing traveling time/distance and minimizing number of requests deviate from average number of requests. This finding is consistent with the characteristic of most other applications of fuzzy systems such as the one in [14,15].

Another aspect worth examining is robustness. When the assignment is done by a human dispatcher, it is not always reliable and consistent in all cases because of errors in human judgment. Further more, fuzzy rules are modularly constructed and hence more rules can be added without changing the structure of the algorithm or altering the function of pre-existing rules. The result is a flexible system. Last but not least, the FLAP approach to vehicle deployment is computationally effective. As for the CLAP (or "nearest") approach, the shortest path between two locations must be constructed and information about road lengths must be known *a priori* for the LAP algorithm to function effectively.



However, with the FLAP approach, only straight line distances are needed, which is very easy to obtain over a geographical road network, since fuzzy logic can help to convey the imprecision involving measurements of distances.

V. RELATED WORK

There is an amazing amount of work attempting to solve the vehicle fleet assignment problem. In the simplest form, the assignment of vehicle can be formulated in terms of linear assignment and solved with network flow algorithms

[10] or assignment methods [2,11,12]. Many models are based on queuing theory (Green et al. 1995, Whitt 1992). In these cases, the proposed vehicle assignment models consider only a single objective or criterion. A different approach is described in [5]. This work employs a back propagation neural network model to learn the decision process of an expert dispatcher, thus considering the multi-criteria aspect. Through the learning mechanism, the system adapts to different dispatching environments. In the case of multiple requests, this approach only considers one request at a time. This alleviates some undesirable "myopic" behaviours. A learning approach based on linear programming is proposed in [6]. However, only linear or piecewise linear functions can be constructed to approximate the dispatcher's decision process. This approach is not shown to be competitive and flexible, but is better than single dispatching rule approaches.

However, relatively little work has addressed the problem of taxi dispatching. Shrivastava.M et al. [14] also made use of the fuzzy logic approach to the taxi dispatching problem with multi-criteria. However, the proposed approach is simple for real-situations (only one rule) and it did not solve the case with multiple requests at a time. Liao [4] discusses the experience of three Singapore-based taxi companies. The computerized dispatching systems used immediately detect the nearest taxi to a particular customer. This approach dedicated to single performance criterion, which is the *min* distance.

VI. CONCLUSIONS

This paper has presented a FLAP approach to service vehicle deployment that allows the incorporation of multiple criteria expressed with fuzzy rules, and hence admitting vagueness in decision-making that is a natural characteristic of human dispatching expertise. Defuzzifying these simple rules provides the "aggregated" cost entries for the linear assignment matrix for which a very efficient LAP algorithm exists. The result is a conceptually simple yet effective FLAP approach to dealing with multi-criteria vehicle fleet deployment that is also flexible (in that new criteria and rules can be easily added if necessary) and uniquely capable of handling vagueness inherent in rules relating these criteria. A comparative examination via extensive simulations in a representative scenario of taxi deployment shows that the proposed FLAP approach generally outperforms the conventional LAP (CLAP) that uses a single criterion such as distance for cost entries. One can infer that the FLAP approach can be extended to provide effective solutions to similar assignment problems including personnel assignments, vehicle and crew scheduling, assignments of jobs to parallel machines, ect,.

In future work, an adaptive vehicle deployment system based on the FLAP approach would be developed. The system would learn to "extract" new rules from real examples to complement those due to human expertise knowledge.

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