Mathematics, multimedia and higher level thinking skills

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Introduction

This paper emerges from our collective interest and experience in the use of multimedia in the science and mathematics classroom. We each approach this topic from a different direction. The first author, Martin, during his years as a classroom teacher, began to see the possibility of utilising multimedia to create a student centred learning environment in which a self paced curriculum would be delivered by the computer. Martin was aware, however, that there were many questions regarding the pedagogical value of using multimedia in the mathematics classroom that needed to be answered. Specifically, could the software address the vital issues of problem solving and higher-level thinking? The second author, Dorit, was involved in research in this area over the last 10 years. In her research she investigated the potential of computerised learning environments and constructivist teaching epistemology in promoting higher-level thinking skills in the science classroom.

The research described here arises out of Martin's particular interest in the possibility of using multimedia in the mathematics classroom to deliver a self-paced curriculum. The experiences of the two authors suggested that the computer alone could not do the job, but perhaps the teacher's role could be utilised as a facilitator, helper and problem solver (Maor & Taylor, 1995). Even given this idealised environment, serious questions regarding the computer's pedagogical abilities would need to be answered. One of these is the issue of the development of higher level thinking and problem solving skills.

Initially computer technology, within the objectivist setting from which computers evolved, was used to drill facts and practice basic skills. However, as time has gone by, the computer's role has become a tool to carry out these mundane skills-based tasks rather than a tutor of these skills. Jonassen & Reeves (1996), put it this way:

The learner should be responsible for recognising and judging patterns of information and organising it, while the computer should perform calculations, store information, and retrieve it at the learner's command.

The focus for the learner has become interpretation of the computer output. The computer may carry out some complex calculation or draw a difficult graph, but the user must interpret and apply these results. At the same time in society, we have seen a significant move toward the desire for students who are good problem solvers, creative thinkers and life-long learners. The question must be asked whether computer software is capable of encouraging the use of, and hence the development of, these higher level thinking skills, and if so, what aspects of the software tend to set this process in motion. Grabinger (1996), asks similar questions:

...So, how can we design machines to help people learn and think? Does this mean machines need to replicate human processes or that machines support processes? Can we use machines to help make the learning processes visible and more accessible? (p. 688)

To begin to answer these questions Martin collected intensive data detailing learner-computer interactions with multimedia mathematics software.

The study was carried out within a broadly constructivist framework. That is, in an environment where the student is viewed as an active constructor of his or her knowledge as opposed to a receiver of knowledge transmitted by the teacher (Solomon, 1987; Taylor, 1998). The reason for this choice was that constructivist approaches to teaching promote student involvement in their learning and enhance the likelihood of the development of higher level thinking skills (Grabinger & Dunlap, 1996; Maor, 1995).
The adoption of a constructivist framework within which to carry out the study also directed me to collect a broader data set than just student-computer interactions. Duffy and Cunningham (1996) make the point that constructivists view learning as "the activity in context". That is, the learning situation as a whole must be examined and understood in order to understand the learning. Hence, we could not ignore the learning environment in which the software is integrated and this environment's influence on the use of the software.

Specifically, we tried to create a learning environment similar to (within many constraints) the 'Rich Environment for Active Learning' as posited by Grabinger (1996). We believed the features of this environment would, not only be the most beneficial for the students, but also the most effective in encouraging higher level thinking. The attributes of the REAL which we tried to capture are described in Table 1.

### Table 1: Attributes of the REAL

<table>
<thead>
<tr>
<th>REAL Attribute</th>
<th>Explanation</th>
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<tbody>
<tr>
<td>Generative Learning Activities</td>
<td>Students learn through active involvement and use the tools available to them to develop and refine their understandings of the concepts being investigated. These type of activities encourage argumentation and reflection.</td>
</tr>
<tr>
<td>Cooperative Support</td>
<td>By working in groups the students can support each others learning. The discussion that goes on in the group can help the students in social negotiation of meaning. In addition, students working in groups may feel more confident to engage in the risks associated with tackling complex problems.</td>
</tr>
<tr>
<td>Authentic Learning Tasks</td>
<td>Students are provided with learning experiences which are as realistic as possible.</td>
</tr>
<tr>
<td>Student Responsibility and Initiative</td>
<td>The environment is student centred, placing a major emphasis on intentional learning. We wanted to encourage student questioning, self-reflection and self-monitoring.</td>
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</tbody>
</table>

**The research aims**

It was within this framework that the computer would be utilised by the students. As they worked with the multimedia we intended to investigate what attributes of the software stimulated them to higher-level thinking. In addition, we wanted to determine the attributes of the learning environment which were particularly supportive of this process. Overall, the goal was to combine the information gathered in addressing these aims to further the teaching and learning of mathematics in computerised learning environments.

**The research questions**

The broad questions that were investigated were:

- What design elements of interactive mathematics software encourage the utilisation of higher-level thinking skills?
- What elements of a constructivist-oriented learning environment support and/or encourage the processes involved in promoting the utilisation of higher-level thinking skills?
- How can this knowledge be translated into a vehicle for improving teaching and learning in mathematics?

Observation of a mathematics classroom with students using multimedia led us to focus upon particular aspects of the learner/computer interactions. We became interested in the way in which the students used the tools provided in the software. The software, described in more detail in the next section, consisted of a series of lessons in which students utilised various tools to manipulate and measure geometrical objects to form conclusions regarding the objects
properties.

Jonassen and Reeves (1996) suggest the use of the computer in education should be as a cognitive tool. According to them cognitive tools are:

The technologies, tangible or intangible, that enhance the cognitive powers of human beings during thinking, problem solving and learning.

Jonassen and Reeves emphasise that cognitive tools are unintelligent tools which require the learner to provide the intelligence, not the technology. The learner must plan, make decisions and regulate their own learning.

In the light of this we decided to focus on the unintelligent side of the software. That is, to look at the software from a cognitive tools perspective, and examine the students use of the tools provided therein.

To facilitate this, students in the study were provided with problems to solve which were independent of the software. As the students worked through the lessons they were learning how to use these tools. The interest was in whether they could now use these tools to solve complex, authentic problems, and if so, how they went about this task (cognitively and practically speaking).

Software selection and description

Reeves and Harmon (in press), have created two dimensions intended to be used for the evaluation of interactive multimedia (IMM) in education (see Table 2). These two dimensions are the pedagogical dimension and the user-interface dimension. Their definitions of these dimensions are as follows:

Pedagogical dimensions are those aspects of the design of IMM that directly affect learning. User interface dimensions are concerned with those aspects of IMM that ensure the learner can actually engage in a meaningful interactive with a program.

Table 2: The pedagogical and user-interface dimensions of interactive multimedia.

<table>
<thead>
<tr>
<th>Pedagogical Dimensions of IMM</th>
<th>User-Interface Dimensions of IMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epistemology</td>
<td>Ease of Use</td>
</tr>
<tr>
<td>Pedagogical Philosophy</td>
<td>Navigation</td>
</tr>
<tr>
<td>Underlying Psychology</td>
<td>Cognitive Load</td>
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<tr>
<td>Goal Orientation</td>
<td>Mapping</td>
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<tr>
<td>Instructional Sequencing</td>
<td>Screen Design</td>
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<tr>
<td>Experiential Validity</td>
<td>Knowledge Space Compatibility</td>
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<tr>
<td>Role of Instructor</td>
<td>Information Presentation</td>
</tr>
<tr>
<td>Value of Errors</td>
<td>Media Integration</td>
</tr>
<tr>
<td>Motivation</td>
<td>Aesthetics</td>
</tr>
<tr>
<td>Structure</td>
<td>Overall Functionality</td>
</tr>
<tr>
<td>Accommodation of Individual Differences</td>
<td></td>
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<tr>
<td>Learner Control</td>
<td></td>
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<tr>
<td>User Activity</td>
<td></td>
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<tr>
<td>Cooperative Learning</td>
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</tbody>
</table>
The pedagogical dimensions are 'scored' on continua which place the IMM, for any particular pedagogical dimension, on a range from objectivist to constructivist. While Reeves and Harmon define each of the pedagogical dimensions there is a subjective component in deciding where to place a specific piece of IMM on the continuum.

The intention to research within a constructivist environment required that we utilised IMM developed with concomitant principles in mind. A range of IMM software was examined before we selected the Geometry Inventor (LOGAL, 1997) from LOGAL Software Inc. (http://www.logal.net). The Geometry Inventor fitted well within our broad definition for multimedia and we discovered by writing to LOGAL Software that it had been developed with ideas of constructivism in mind. Following is a quote from a letter from LOGAL:

All of LOGAL's software is based on the constructivist philosophy that learning is the active construction of knowledge, and that knowledge comes to life when software helps students ask questions and learn through exploration.

![Diagram of pedagogical dimensions]

Table 3. An evaluation of the pedagogical dimensions of LOGAL’s Geometry Inventor.

Having received this information we used Reeves and Harmon’s pedagogical dimensions to evaluate the Geometry Inventor software. This evaluation is illustrated in Table 3.

As can be seen from Table 3, three of the pedagogical dimensions had a distinctly objectivist tendency. We believe this occurred because the software developers, in aiming at the educational market, made the software 'objective-based' to fit in with prescribed curricula. Associated with each lesson is a list of learning objectives or outcomes. These outcomes focused the lesson and tended to produce pre-defined learning sequences. It must be noted however, that within these predefined sequences there is a strong problem-solving approach to the learning and the students are exposed to a rich set of tools with which to work. As stated earlier, we decided to focus on these cognitive tools and the way students utilised them to solve problems rather that looking at students acquisition of the predefined knowledge goals.
The LOGAL Geometry Inventor

LOGAL's Geometry Inventor is 'internet enabled' meaning that the lessons can be accessed over the internet. This element also added an interesting aside to the study as previously we had not heard of any effective multimedia products which are delivered over the internet. LOGAL has its own application which allows it to run independently of Netscape or any other Web browser. The speed of access and use was quite acceptable throughout the study. It is necessary however, to download the initial application and copy it to individual computers before using it with a class. For this study Macintosh computers were utilised, however, the software is also available for Windows 95.

Geometry Inventor is part of a suite of Mathematics programs from LOGAL Software Inc. The entire suite is titled 'Tangible Math' and consists of the following components:

- Algebra Animator
- Geometry Inventor
- Function Investigator
- Probability Constructor
- Matrix Analyser
- Stats!

In liaison with Linda, the students' teacher, it was decided to use the Geometry Inventor. This module had a rich set of tools for the students to use and Linda felt it would complement her course well.

To give an idea of the nature of the software used and to place the observations regarding student thinking in context we have reproduced the screens from a lesson segment to examine. Each lesson is broken into a number of activities as follows:

- Warm Up
- Experiment 1
- Experiment 2
- Experiment 3
  (there are as many experiments as the developers saw fit for a particular lesson)
- Analysis
- Follow Up

As can be seen from the names given to the various activities are suggestive of the attempt at a constructivist approach by the software developers.

The following screen captures are from a lesson segment called 'Parallels and Transversals'. This lesson was chosen because it is relatively short and representative of the nature of the software. The lesson consists of the following parts:

- Warm Up (Dragging sets of parallel lines around on the screen and making some observations)
- Experiment 1 (Examining the distance between parallel lines)
- Experiment 2 (Examining angles formed by a transversal)
- Experiment 3 (Determining a method to ensure lines are parallel)
- Analysis (Summarising results and giving names to the various types of angles)
- Follow Up (Examining the slopes of lines, particularly parallel lines)

The screen captures are from Experiment 2 of the above lesson.

Figure 1, as well as being the first screen in this sequence, illustrates the basic structure of all the screens in LOGAL's Geometry Inventor. The far left column consists of a variety of tools with which the user can create and manipulate geometric objects as well as measure their attributes (angle size, length, area etc.). The user can also link object attributes to axes to produce graphs and/or tables of data. It is not our intention here to explain the use of each tool but rather to give an overall feel for the software. Basically, the rectangular box at the top left of the screen contains...
instructions and information for the student. This will be referred to as the *information area*. The student carries out the tasks from the information area using the dynamic geometry tools and answers any questions that may be posed. Often these 'questions' ask the student to propose theories or write down their own ideas. These ideas may be confirmed or challenged later in the sequence. The teacher has the option of allowing or not allowing the students to see the 'suggested answers'. We chose in this study not to allow students to view the answers. This was done in the hope that it would force students to think seriously about what they wrote and to attempt to justify their answers for themselves.

Looking again at Figure 1, at the bottom of the information box are some options, a student can return to Activity Central to choose an entirely new lesson, go to another experiment using the 'Chapter List', or use the arrows to move back and forth within a given activity. All in all the software does not restrict the users navigation but is still basically linear in design. The small question mark in the information area is active and may be clicked on if the student does not understand the terminology used in the instructions. The result of this can be seen in Figure 2.

![Figure 1](image1.jpg)

**Figure 1.** The first screen of the lesson 'Parallels and Transversals'.

![Figure 2](image2.jpg)

**Figure 2.** Illustration of the help material available in the lesson 'Parallels and Transversals'.
Figure 3. The second screen of the lesson 'Parallels and Transversals'. The diagram with the parallel lines is active and may be manipulated by the user.

On the screen shown in Figure 3 students manipulate the diagram. As they do this the table at the bottom of the screen in automatically filled in. Then on the screen shown in Figure 4 they search for relationships and put their answers in the answer box provided.

Figure 4. The students enter their answers in the answer boxes at the top of the screen.

In Figure 5 they also search for relationships in the table, this time between the interior angles. Once again clicking the small question mark will give them more information should they not understand the terms used in the information area. The result of clicking on this question mark can be seen in Figure 6.
Figure 5. As students manipulate the diagram, data appears in the table at the bottom of the screen. The students examine this data and write their conclusions in the answer boxes.

Figure 6. Another example of the help material available to the students. In this case the parallel line diagram is active. Manipulating it produces data in the table.

Methodology

A qualitative methodology (Erickson, 1986; Merriam, 1988) was adopted for this study as it would allow us to focus on the social situation in the classroom, as well as the student/computer interactions, and perhaps allow insights into the phenomena not afforded by a quantitative approach. It was decided to carry out the research in the complexities and uncertainties (Kennedy, 1997) of the school environment, as it would be in the classroom that any findings would be applied.

A class of 22 students used the software for twelve one hour sessions spread over a ten week period. During these lessons Martin attended to make field notes and collect student answer files at appropriate times. Martin conducted interviews with four pairs of students and pre and post-interviews with the teacher (as well as many informal discussions).

Table 4 illustrates the ways in which data was generated in order to address the research questions.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Question 1</th>
<th>Question 2</th>
</tr>
</thead>
</table>

Table 4: Methods of data generation.
The research site and class organisation

We located a private girls school with excellent computing facilities that appeared to be an ideal research site. Contact was made with the Head of Mathematics, Linda, at the school and she seemed enthusiastic right from the start. Linda told us she had a group of very able year nine students with whom she wished to do some extension/problem solving work. It was suggested to her that perhaps the students could work on the Geometry Inventor module of LOGAL Mathematics. We also spoke to her about the possibility of utilising a learning environment with a constructivist orientation, specifically based on Scott Grabinger's 1996 paper 'Rich Environments for Active Learning'.

Subsequent to this meeting LOGAL was installed on a computer at the school and we provided Linda with a copy of the Grabinger paper in order to help her visualise the learning environment we were hoping to establish. After several further discussions, Linda decided she would like to participate in the research and negotiations began over some of the details including time frame, LOGAL lesson selection, class environment, ethical issues of participation and lesson structure.

Overall it was Linda's enthusiasm plus the excellent facilities which made this an ideal research site. On the down side, the sample would consist of only female students attending a prestigious private school. This could hardly be considered representative of the general student population in Western Australia. However, at this early stage in the project we didn't want data generation to be marred by possible classroom difficulties which may have arisen at another site. Also we wanted a classroom environment not a laboratory approach which may have resulted if students had been removed from a school atmosphere and invited to work in the university computing laboratories. An advantage of this site was the excellent computing facilities available to the mathematics department, including internet access.

In the final analysis the sample consisted of 22 girls aged 13 and 14 and the study was carried out over a 10 week period. During this 10 weeks Martin met with them on 12 one hour occasions (during the rest of the time they worked on their regular maths work).

As part of trying to establish an environment in keeping with a 'Rich Environment for Active Learning' students were provided, at the outset, with an assignment problem, a booklet listing all computer lessons to be completed, and some exercises to be worked on at the end of each set of computer lessons. To promote cooperative learning, the students each selected a partner with whom they would work throughout the term.

The assignment was to play a critical role in the evaluation of the students use of the cognitive tools provided in LOGAL. Ideally we had hoped for authentic, realistic, relevant problems. How close we are to utopia in our imaginations! We ended up with much less. We searched for problems for a long time before finally accepting that we would have to settle with some fairly abstract and unrealistic assignment problems. A couple of what, at best, could be called pseudo-realistic situations were found, but they were rather contrived. The other problems were fairly typical Euclidean geometry fare involving relationships between medians and lengths in a triangle and some area/perimeter material. The problems, in hindsight, were adequate for the purposes of the research.
Presentation and interpretation of data

In order to interpret the qualitative data collected we began by making lists of what were considered to be points of interest from the data that related to higher level thinking. These points either came from the field observations, interviews (teacher and student), consideration of student answer files, or student assignments. Once this data had been reviewed several times some preliminary assertions were formulated. These assertions were put to the teacher (Linda) in a post interview, and to the students (in a slightly simplified form) in the form of a questionnaire.

Assertion 1: The students used the geometry tools in flexible ways to test their theories.

Supporting evidence for assertion 1

![Figure 7: Students' opinions regarding assertion 1.](image)

The histogram in Figure 7 clearly shows the students believed they had used the LOGAL tools to test their theories. Although this histogram appears to be strongly supportive of the assertion, one could question the students understanding of the statement. It would not be unreasonable to believe that the students interpretation of the statement given to them in the questionnaire would be superficial. There is also the question as to whether the fact that the statement reads as 'positive' may have affected the result. Taking all into consideration, we believe this graph supports the assertion although, perhaps, not as strongly as one may imagine at first glance.

In addition to the group opinion suggested by the histogram it is interesting to examine some specific instances of students reporting their use of the LOGAL tools in their own words. These quotations are take from student assignments. The students were asked to describe the process they went through in attempting to solve their problem.

Evidence from Shona and Tracey's comments

We drew a quadrilateral and then changed the lengths of sides etc. to make it as much as possible close to the one in the assignment. I don't know why we did that!...

Here we see an example of a common approach adopted amongst the students. They felt the need to draw and label their diagrams, as closely as possible, to mirror the diagram given on their assignment sheet. None of the assignment questions were about, or required, shapes of a specific size. A 'concrete' starting point seems to be important to the students and we believe this is why they begin the problem in this fashion.

Students demonstrated metacognitive activity as they considered further why they drew the diagram this way. This is described in the continuation of the quote:

...Probably so that we would relate the diagram to the problem. We gave the vertices the same letter names [as the assignment diagram] because we thought that would be easier. We drew in the diagonals and
marked our centrepoint.

The students give excellent reasons for drawing the diagram as they did. In doing so they illustrate the utilisation of a higher-level thinking skill. That is, the identification of a subgoal, in this case the construction of a clear and useful diagram. Continuing:

Phew, we thought, we have done the hard part, we have drawn the diagram using LOGAL tools.

No small effort had been invested by the students so far. On paper it may appear an easy, relatively short process, but they had already spent approximately 30 minutes to get this far.

As I said we got a bit off the topic and we were looking at relationships between anything. We measured all the angles and found exactly what we knew already. We then measured all the borders, the line segments and ratios between line segments, but nothing remained interesting and constant. (Shona & Tracey's assignment.)

Having achieved their first objective of a clear diagram the students now began a search for relationships. They started by examining angles within the quadrilateral. We can see that as they reviewed their approach the students realised that this was not productive: 'We got off topic', and 'found exactly what we already knew'. However perhaps they didn't realise that this type of 'floundering' is often found at the beginning of an attempt to solve a complex problem. They then began measuring other dimensions and searching for something 'interesting and constant'. We can add, from the class observations, that the students were engaging in a much more advanced thinking process than appears from a bare reading of this quote. They had definite ideas which they were pursuing. For example, at one point they measured the heights of the triangles formed by the diagonals of the quadrilateral they had drawn. They had in mind that the area of these four triangles equalled the area of the quadrilateral. Having examined these heights they varied the size of the quadrilateral (and hence the triangles) and searched for some relationship between the heights. This was a great idea but, unfortunately, unsuccessful. This is an excellent example of how the students' partnership with the software enabled them to engage in higher-level thinking by giving them the power to test their ideas. This would have been extremely difficult with pen and paper. One may also hypothesise that through this process of testing the students are generating knowledge constructs regarding quadrilaterals that would be otherwise unattainable for them.

Counterbalancing this, there were other times when the students were more random in their choice of properties to examine, although what appears random to an observer may actually not be so in the mind of the person carrying out the action.

Evidence from Mary and Sharon's comments

Mary and Sharon initially exhibit some similarities to Shona and Tracy in that they wrestled with the construction of a suitable diagram that would be similar to the one on their assignment sheet. Part of this process involved bisecting some lines and illustrated a useful application of the LOGAL tools:

We then weren't sure how to cut the lines in half, to try and join the halves to separate the corners. Sharon discovered the bisecting tool, in line segments. We then checked the diagrams accuracy by measuring each segment against its other half. We then measured the area of the hexagon, the corners and the middle area. using our own calculators we worked out the percentage of the area boarded and unboarded. (Mary & Sharon's assignment.)

This description shows that these two students were not willing to accept 'by faith' the diagram constructed by them using the computer tools. They actually checked to ensure the diagram was correct. This ability to double check illustrates how students can use the tools in the Geometry Inventor to review, reflect on, and verify aspects of their work. We consider this checking procedure, particularly where it relates to the students solutions to a problem, to be an example of higher-level thinking. In the simple example above students were saying to themselves 'Is this correct? Does it match the requirements of the problem?, Am I sufficiently sure of my progress to continue on to the next stage?'
Evidence from Joy's comments

In her assignment Joy wrote:

When I received my assignment, I read through it to determine what it was asking me to do and construct a way that I might solve it using LOGAL.

This student began very practically with the formulation of a plan to solve her problem.

It seemed reasonable and quite simple to solve but after I had created a triangle with no specific dimensions and put it in the basic shape of the one on my page, to make it easier to work with, I tried to place the lines in.

Joy, as did the others, creates a diagram similar to the one on her assignment sheet. However, she seems to be thinking in an abstract way in that she isn't concerned about the specific dimensions of the triangle she creates. Most of the students began at the concrete (with specific dimensions being important) and in the end spoke about in a more abstract way. 'Is the relationship I have found true for any triangle?' In some cases students did not make the transition completely to an abstract form preferring to give multiple examples rather than state a generalisation. However, in looking at many specific examples of the property they were examining they had moved in the direction of abstraction. We believe that the use of the tools in the Geometry Inventor can help students feel more comfortable with working with shapes whose dimensions are variable. Initially they may find dimensions important but later relationships within the shape become the focus and the size of the particular figure can be dynamically varied. This ability to vary the size of the figure while examining relationships may help the students to begin to think in an abstract manner.

Joy goes on:

This became a difficulty as at that stage I was not aware that you could subdivide a line into sections. I tried bisecting an angle and continued along this line until I realised that bisecting its opposite and did not always bisect the line and so I could not get the correct answer. (Joy's assignment.)

Joy had a theory that if you bisect an angle in a triangle the bisecting ray would cut the opposite side of the triangle in half. The fact that Joy utilises the Geometry Inventor tools and eventually rejects this theory independently (without teacher interaction) illustrates the power of this type of learning environment. Joy had the tools available to test the theory and convince herself that she was incorrect. It is precisely this type of partnership that we believe is capable of enhancing students higher-level thinking skills.

As well as these direct quotes from student work samples there were many occasions on which students were observed to be engaged in the process of testing their theories using the LOGAL tools.

Evidence from Field Notes

As implied in the above discussions of student assignment work, observations of interactions with the software also afforded many opportunities to view students utilising the Geometry Inventor tool set. Two specific and one more general example should prove to demonstrate this.

The students were not familiar with the concept of the gradient of a line so the lesson on this topic provided the first opportunity for watching the students use the tools in an unfamiliar situation.

They spent a while working on when a line would have a positive gradient and when a negative gradient. They actively tested theories using the tools and diagrams given. (Field notes 26/2/98)

The essence of the activity mentioned in the quote is that students are asked to generalise about when a line has a positive, negative, zero, or undefined gradient. We were surprised by how much time the students spent on this activity. Although, some of the students failed to come to the desired conclusions, all of the students spent time coming up
with, and testing, possible ideas using the tools.

The second example involves the students manipulating a triangle and trying to get the sum of two sides of the triangle to equal the third.

The students played around with this for a while. Using the tools to get the sum of two sides of a triangle equal to the third. When they finally did they were a bit embarrassed. They said 'That's so obvious!' and 'It's not really a triangle.' This screen certainly prompted thought and experimentation. (Field notes 5/3/98)

Again there was surprise at the length of time the students used and the amount of discussion this particular activity generated. The impact when the students finally managed to achieve the desired result was quite amazing. They won't forget this property of triangles! One negative aspect here. When some students have a manipulable diagram they tend to keep on changing the diagram in a search for a result without stopping to review their progress.

Finally a more general observation regarding tool use:

I saw a pair of students using the graph tool! However, they discarded it in favour of using the table tool. Students are definitely searching for pattern. They use tables a lot for this, dividing, multiplying and adding numbers to search for relationships. (Field notes 2/4/98)

The use of the table tool was by far the most common way for students to represent their data. The exclamation in the first part of the above quote is there because this was perhaps the only time students were observed trying to use the graph tool independently to investigate an idea. Probably this was because the students, at only age 13 and 14, had had little exposure to the function strand of mathematics. The way the students used the tables is of interest. Basically, once they had a list of numbers pertaining to the figure they were investigating the carried out all sorts of arithmetic operations in a search for a relationship. Depending on the situation this approach varied from an apparently random search to a focused and efficient effort to find a relationship. The students had problems finding relationships such as 'the sum of two sides of a triangle must be greater than the third side' because they seemed determined to search only for relationships in which equality played a part. We felt the students should be warned in the text when the word 'relationship' referred to something other than equality.

Disconfirming evidence for assertion 1

Interestingly Linda, the teacher, wasn't so convinced regarding this assertion. She believed the students had used the tools in a fairly mundane manner. Following is an excerpt from the post-interview:

Linda (reading my statement in a doubtful manner): Students used the geometry tools in flexible ways to test their theories.

Martin: Yes, that's a hard one to know. I was hoping you had observed something about that.

Linda: They [students] did but I think the way that the lessons were structured in that you do this, you do this, and there wasn't a blank page to just go and play around on didn't lend itself [to using tools in flexible ways]. The lessons didn't suggest, 'why don't you go and try this?' I don't know. I think the students used the tools to complete the lessons. (Post interview 6/7/98)

It is interesting to observe that one of the reasons Linda gives for the students not using the tools in a flexible way is the linear nature of the software 'you do this, you do this, you do this'. This statement while not supporting the assertion seems to support the idea that the creative use of these tools would be enhanced by a less linear approach by the software developers.

Interpretation of evidence for assertion 1

It appears that the students certainly did use the tool set provided in LOGAL to solve their problems. But what does
this show? Firstly it supports some of the foundational issues with regard to cognitive tools research as stated in Jonnasen & Reeves (1996). That is, cognitive tools provide mindful, challenging learning and enable the learners to form an intellectual partnership with the software.

In this same paper the authors suggest future research should assess the ability of learners to solve original problems, diagnose situations, draw implications from problem situations, or predict the results of changes in any problem situation. This result goes some way toward demonstrating that at least some students can do the above tasks through the utilisation of cognitive tools. In general the assignment problems would not have been possible for students of this age or mathematical background were it not for the LOGAL tools. In addition the problems contained an extension section as well as asking the students to state their own further problem of a similar nature. Most students completed the extension but none of the students came up with a further more complex question that they could investigate.

With regard to higher level thinking the students certainly demonstrated it in the process of using the tools to solve the assignment question. Their descriptions of the way they solved their problems illustrate advanced forms of thinking such as visualisation and metacognition. In the case, from the LOGAL lessons, of the triangle with the sum of two sides being equal to the third, we think there is evidence of the movement from a concrete representation for the situation to an abstract understanding. Granted, this could have happened in a regular classroom but the tide of technology seems to be inevitable and it is comforting that these higher-level skills can be encouraged in a multimedia environment.

Assertion 2: The geometry tools enabled the students to see geometry in a way that is not possible with pen and paper.

Confirming evidence for assertion 2

![Figure 8: Students' opinions regarding assertion 2.](image)

In this histogram it is apparent that the students believe that the LOGAL tools help them to see geometry in a way they couldn't have otherwise. The students had some prior exposure to pencil and paper geometry, so this meant they were able to compare between the different approaches to learning. Once again the statement would probably be interpreted as 'positive' by the students and this may have slightly favoured the supportive response. However, we believe this graph confirms the assertion.

Linda, the teacher, reacted in an extremely positive manner to this assertion:

Martin: What about the assertion that, 'The geometry tools enabled students to see geometry in a way that wasn't possible with pen and paper'?

Linda: Yes. Absolutely. But also just to see it so quickly. I mean yeah I thought that was a really good one [assertion]. We've just brought a dynamic geometry program for the Maths Department. (Post-interview 6/7/98)
Perhaps the greatest support to add to the above evidence comes from field observations. As the students were watched manipulating concrete geometric objects on the screen and recognising patterns and formulating rules, it gave one the sense that their construction of that knowledge was entirely different to the same concept developed on paper. Take, for example, the lesson where students examined the gradient of a line. They had lines on the screen and simply dragged them around to see increases, decreases, changes in sign, zero and undefined gradients. Some students spent a considerable amount of time (40 min+ in one case, classroom observations), attempting to generalise about conditions for negative, positive, zero and undefined gradients. In our opinion the kind of fluid visual constructions they can make (based on perhaps 50 moves of the line) can't be compared to the static constructions make with pencil and paper from perhaps a handful of trials. Similar arguments could be made about other geometric properties.

**Disconfirming evidence for assertion 2**

Not all students managed to make sense of the gradient material or other portions of the course (Student answer files, observations). They, although constructing some knowledge, evident in their attempted answers, didn't achieve the conceptions intended by the lessons.

Although it is of concern that the some students did not manage to grasp some concepts from the lessons, we do not consider this evidence weighs in heavily against the assertion. These particular students may not have grasped the concepts with pencil and paper either. It is entirely possible that, although they did not arrive at the intended constructions, they still made useful progress in their overall construction of their geometry knowledge. Unfortunately this, cannot be verified.

**Interpretation of evidence for assertion 2**

Activity-based learning, often involving manipulating objects, has been a prominent feature in pedagogical changes in the mathematics classroom over the last 30 years. The advent of technology puts manipulable diagrams into the hands of students. The computerised tools provided for this manipulation are, in our view, a super-set of the traditional tools. Using the Geometry Inventor as an example and comparing it to pencil, straight edge, compass and protractor, we can see that the computerised tools include all of the original tools plus more. Not only can complex relations be represented quickly (and repeated many times) but in some cases the models can be animated or connected to graphs. This means that with computer technology, you can 'have your cake and eat it'. Students are not deprived of previous tools (and hence ways of knowing) but are provided with a richer tool set with which they can form a powerful intellectual partnership (Jonassen & Reeves, 1996). This can lead to new ways of knowing.

The disconfirming evidence here is unconvincing since it is commonplace for students not to master a new concept.

A limitation of this conclusion is that students knowledge constructions cannot be viewed directly. Oh that they could! Observation and comparison gives us clues to the ways in which the students are thinking but it should be kept in mind that it is difficult to make conclusive statements.

**Conclusion**

Looking back to the research questions we think a strong answer can be given to question one with regard to one element of interactive mathematics software which encourages the utilisation of higher level thinking skills. This element is the provision of a rich and powerful set of tools which the students can use, not only to progress through any lesson in the software, but to test and investigate theories of their own. Maor (1993) found that higher level thinking skills are often evidenced when students ask and investigate further, deeper questions about the topic they are working in. The provision of cognitive tools, while not in itself causing higher level thinking, provides students with the opportunity to exercise their investigative powers and engage in deeper thinking.

The computer-based lessons themselves must encourage the process of exploration and investigation, a task which the LOGAL Geometry Inventor did well, for it is in this process that the skills associated with the tools are mastered. It is this 'intelligent' part of the software (i.e. the lessons) that enables the students to utilise the 'unintelligent' part of the
software (i.e. the cognitive tools) in what Jonassen and Reeves (Jonassen & Reeves, 1996) have called a 'powerful intellectual partnership'.

Underpinning the whole process, and vital to the final result, is the learning environment within which the software is integrated. The aspects which provided the most fruitful conditions for higher level thinking were anchored learning, cooperative support, and student responsibility and initiative (Grabinger & Dunlap, 1996). Anchoring the learning with a problem (albeit not as authentic as one might desire) gave the students an overall direction as they progressed through the lessons and caused them to focus more on the tools which, since higher level thinking was the goal of this research, proved a successful ploy.

The students really appreciated and benefited from the cooperative learning environment. One quote which illustrates this, and is rather fun, follows:

Tracey and I work together. I have the wild ideas and Tracey tells me to get a life! It works well. (Shona's assignment.)

The third aspect of student responsibility and initiative gave the students the chance to explore in a free environment in which they could allocate their time as they saw fit. They did have a goal, as far as the content and number of lessons went, but they could apportion their time as they saw fit to each of the activities. Things they considered easy could be left and more time spent thinking about issues that they found more challenging.

Overall it was enlightening to observe the excellent support the constructivist environment gave to the goal of higher-level thinking through the use of multimedia.

The power of combining these two elements of cognitive tools and REAL's should be an incentive to teachers to utilise these aspects in their classrooms. If this is done, student higher level thinking and problem solving skills will be enhanced through an intellectual partnership with the computer technology.

References


