In recent years, technology has become an important influence on the mathematics curriculum. Some have even suggested that it is the major influence on our curriculum thinking at the moment, reflected in part by the recent national graphics calculator conference held by the Australian Association of Mathematics Teachers, although it is not the intention of this paper to argue this point.

This paper identifies and very briefly describes some of the mathematics curriculum issues associated with graphics calculators. To do so requires a view on what is meant both by ‘curriculum’ and by ‘issue’. In this context, ‘curriculum’ refers to all of a student’s experiences, intended and unintended, planned and unplanned, as they study mathematics at school. An ‘issue’ is a relevant matter that needs to be studied, debated and discussed, as it has not yet been resolved. The intention of describing these issues is to make clear that there are many of them to be considered, rather than to ‘resolve’ them by arguing for a particular point of view. Indeed, the large number of issues itself makes clear how pervasive are the questions associated with graphics calculators at this moment in time.

The issues need to be considered at a variety of levels, including the curriculum development level, the public examinations level, the school level, the classroom level and the individual teacher level. How we think about and respond to them at these levels will have considerable implications for the teaching and learning experiences of mathematics students. Although research may help with the resolution of some issues, by informing our thinking, we should not expect research to resolve issues of these kinds. Adequate resolution demands instead that the mathematics education community concentrates some attention on the issues and engages in free debate about them. The last few years have seen a number of papers in this and companion journals regarding technology and mathematics education, which also might help with resolving the issues, and will certainly help with expanding them from the brief snapshots provided here.

An earlier version of this paper was used primarily to stimulate such debate among people from a range of contexts and sectors in small workshop settings. The main purpose of this paper then is to help such debates among people by identifying suitable topics for conversation. While consensus is unlikely through conversation alone, we might at least expect that differences of opinion, philosophy or experience are brought into sharper relief by explaining our point of view to others and by listening in turn to their points of view.

The issues which follow are not arranged especially into an order of priority. In fact, many such groupings are defensible, depending on one’s point of perspective. Rather, they are presented as discrete collections of issues, as if they were independent of each other. In fact, of course, they comprise a rich and inter- connected web.

Scientific calculators

Scientific calculators contain a few features beyond arithmetic calculators, notably the evaluation of trigonometric, logarithmic and exponential functions, computational statistics and arithmetic with very large and very small numbers. Graphics calculators are also capable of all of these things and more. In the past, many schools have routinely put scientific calculators on their booklists for early secondary school; in some cases, this decision has not been reviewed in the light of graphics calculators, so that students are sometimes expected to purchase both. From what age is a scientific calculator useful in school? That is, which scientific calculator capabilities are essential to learners and when are they essential? Do students need to have both a scientific and graphics calculator? Are scientific calculators used in other subject areas? If so, what is the level of cross-departmental discussion?

Equity

In some situations, graphics calculators have been withheld from school use because of perceived equity problems. However, some have argued that when calculators are mandated for a course, equity problems are reduced, as schools are then obliged to deal with them. When calculators are optional equipment, less affluent communities cannot justify their use, while more affluent communities take advantage of them.

Calculators come in different models and makes. Some companies produce less expensive and less capable calculators as well as their top of the range models. How important is it for students to have the most capable, top of the range (but most expensive) calculators? Which students require the extra features offered? In what ways are students actually disadvantaged by having a graphics calculator with
a smaller range of mathematical capabilities? Should calculators be chosen to match particular courses of study, or should all courses of study use the same calculator in a particular school?

Some schools have experimented with hiring schemes and buy-back schemes and developed successful second-hand calculator markets. Do these measures reduce inequities or are they impractical to administer? What are the best ways of helping families who cannot afford a calculator for their children?

It has been suggested that the main equity issue concerns equity of access to a knowledgeable mathematics teacher, familiar with and able to use well a graphics calculator for teaching and learning mathematics.

Roles of calculators

Calculators can be deployed in several ways at school and have a variety of interpretations. Are they mainly useful for calculator activities, which would not be otherwise accessible? Are they most useful for projects and investigations, completed with a measure of student control? Are they predominantly important because of their assessment aspects, permitting students to answer some kinds of examination and test questions more easily? Are they principally useful because they force us to rethink what is in the mathematics curriculum, why it is there and when it appears? Are they mainly learning devices or mainly teaching devices or mainly assessment devices? Or do they have none of these roles in school mathematics and we would be better advised to construct a curriculum without any deployment of technology at all?

Content and sequencing

Are some things becoming more important these days, while other aspects of mathematics are declining in importance, because of technological changes generally? Or is mathematics, particularly at the school level, fixed for ever? Some mathematical ideas arise naturally on graphics calculators earlier than they did previously. (For example, complex numbers appear automatically on some calculators; similarly, practical answers to maximum and minimum questions can easily be obtained numerically, years before a student encounters differential calculus.) Is this kind of premature exposure to mathematical ideas a problem? For whom is it a problem? Should some content be added to the mathematics curriculum? (In place of what?) Should some things be deleted?

Text book design

How should graphics calculators be integrated into mathematics text books? (Or can they just ‘added in’ to an existing textbook?) To what extent is it appropriate to have special ‘calculator activities’, or should calculators just be used when it seems natural and appropriate to do so? To what extent should text authors instruct students to ‘Use your calculator to...’ do something and to what extent should they leave this decision to readers? Do existing textbooks need to be re-designed to make good use of graphics calculators or is it more a matter of using and interpreting today’s textbooks carefully with tomorrow’s curriculum in mind?

Learning to use a calculator

It seems in the past quite rare for students to be taught directly how (or whether or when) to use their scientific calculators, under the apparent expectation that such learning would take place incidentally. Is this also the case for graphics calculators? At what age should graphics calculators be introduced? A younger age gives more time for incidental learning and familiarity, while an older age is probably closer to higher mathematical needs.

What is the place of official calculator manuals and how best can they be used by students? How useful are other published materials, such as calculator-specific handbooks? How useful are teacher demonstration devices such as overhead projector models, video compatible models, posters of calculator keyboards and the like?
Worksheets

Many teachers (and their students) use published worksheets these days in mathematics classes. Indeed, there are available collections of reproducible black line masters for this purpose. A criticism of (some of) these is that they frequently are not much more educational than a set of practice exercises, that students just complete the instructions without much thinking, that they emphasise individual work to the exclusion of educationally useful collaboration and that they are used to construct the curriculum rather than selected to suit it. Are such criticisms valid for calculator worksheets? To what extent should worksheets give detailed calculator instructions (as distinct from mathematical and educational instructions)? To what extent should they give instructions for a particular model of calculator? How can investigative work be encouraged with calculator worksheets?

It has been suggested that calculators are general purpose tools, of relevance to most parts of the school curriculum, so that their use mainly with calculator worksheets undermines this importance. That is, calculators are interpreted mainly as useful for doing calculator activities rather than for learning and doing mathematics.

Student decision-making

One of the limitations of class sets of calculators and specific calculator activities is that the decision to use a calculator, as well as what to do with it, are both made by the teacher. How are students to learn for themselves when and how to use a calculator, if someone else generally decides for them? It seems logical that students can only make a choice to use a calculator if they could also make a choice to not use a calculator. Of course, they cannot make a choice to use a calculator if they do not already know how to use it. To what extent is calculator use an individual activity and choice and to what extent should students be expected to work collaboratively to make and carry out their decisions?

Calculator dependence

To what extent are students dependent on their calculators for mathematical tasks? Which particular tasks would we expect students to be able to complete without using their calculator? Why do we think these are important or even necessary? To what extent are the sources of calculator dependence a lack of confidence (as they frequently are with children's arithmetic)? To what extent is calculator dependence a consequence of the limited time devoted to helping students make a choice of whether and how to use a calculator? How can such dependence be prevented? How can it be reduced? What aspects of calculator dependence are improved by denying students access to calculators?

Assessment

For some time now, students have been permitted (in fact, expected) to use calculators in high stakes examinations, such as school leaving examinations in some states. What are the implications for the curriculum and for the examination of graphics calculator availability in formal assessment? How can different calculator models be accommodated in formal assessment without compromising equity? To what extent (if at all) should examinations change in style to accommodate graphics calculators? How can examinations be designed to ensure that mathematical thinking is paramount, rather than calculator use? Should there be calculator-free components of assessments? (Why? Why not?) What working should be recorded by students to convince us that their mathematical thinking is sound?

In recent years, mathematics examinations have increasingly moved away from rewarding memory (e.g. by the provision of mathematical formulae for students). Is the availability of calculator memories just another step in this direction, or is it a more worrying trend? To what extent should students be prevented from taking notes with them to examinations (including electronic notes) or encouraged to do so in a thoughtful way? To what extent should examinations reflect the normal availability of stored information or maintain the emphasis on independent work, devoid of aids that would otherwise be accessible?

Electronic algorithms

Some graphics calculator programs seem to have been devised mainly to answer routine mathematical questions, such as those that appear on tests and examinations. Should these be interpreted as a signal that such routine knowledge is not worth testing in an examination, or should we argue that being able to use such a program is important (and conceptually not much different from using a well-rehearsed and familiar 'type example')? While some might argue that student development of these kinds of 'electronic algorithms' is a powerful form of learning, requiring disciplined thought, others might suggest that the algorithms are usually transferred electronically between calculators, and thus require little thought by students. To what extent should formal assessment avoid asking questions that are procedural and routine, anyway?

Multiplying choices

One of the possible consequences of students having access to a powerful technology such as a graphics
calculator is that they may see that it is possible to complete mathematical tasks in more than the 'one, efficient, correct' way. To what extent are students encouraged to use their calculators to complete tasks in a variety of ways and to what extent are they encouraged to use them in the 'best' way from the outset? To what extent is the development of personal ways of doing mathematical things encouraged? prevented? celebrated? tolerated? Who decides which ways of doing things are best?

**Calculator programming**

A feature of graphics calculators is that all of them are programmable. That is, small calculator programs can be written to perform particular tasks. Students can use such programs that they have written or use programs transferred electronically from another student’s calculator. (Interestingly, the present generation of mathematics teachers is not very familiar with such programming, as programmable scientific calculators have been quite rare in schools — a consequence of the fact that they have usually not been allowed for examination use.) How relevant is the programmability of calculators to mathematical thinking? What aspects of mathematics can be addressed with programming that cannot be easily addressed otherwise? Is small scale calculator programming an appropriate thing for students to learn in a mathematics curriculum? Why? Why not?

Programs can be used to upgrade a calculator in small ways (e.g. by adding a quadratic equation solver to a calculator for which it is not in-built.), which may help to reduce the gap between more and less powerful calculators. Of course, such a practice is of no consequence if calculator memories are cleared, either for assessment purposes or because a calculator is one of a class set that regularly has the memories reset.

**Computer use**

Graphics calculators are best thought of as small, purpose-built computers, although they are much less powerful than desk top and lap top microcomputers. To what extent are computers still useful to mathematics students with good personal access to calculators? To what extent are they necessary, in addition to graphics calculators? To what extent do they overcome the limitations of graphics calculators? One of the most important arguments for the use of graphics calculators is their potential availability to students wherever they are and whenever they need them; in contrast, it is often necessary to travel to a computer laboratory to use computers (with all the attendant problems). But have computers become more available in everyday mathematics classrooms to students these days?

As well as overtly mathematical uses, students can use computers to interact with their calculators in various ways. For example, calculator programs and applets can be transferred via computers and students can use computers to include calculator screen dumps into their assignments. In addition, the Internet can provide advice about calculator use for both students and their teachers, as well as information about new calculator developments. How important are these aspects of computer use for calculators and do they justify the availability of the relevant hardware and software?

**Professional development**

In many settings, the provision of adequate professional development for teachers is critical. What are the actual needs for professional development related to graphics calculators? (And are these the same as people’s expressed needs?) What kinds of professional development are necessary, beyond the obvious introductory workshop sessions (sometimes called ‘Where’s the on button’)? Who are the best people to provide such professional development? Colleagues? Heads of Department? External consultants? Curriculum authorities? Commercial companies? Calculator manufacturers? Education systems? Professional associations? To what extent should employers provide help and to what extent should professionals be expected to be responsible for themselves?

**Conclusion**

The set of curriculum issues identified here is necessarily incomplete, but is intended to offer some starting points for debate rather than the end of a conversation. The breadth of the list highlights my belief that the graphics calculator — indeed, any aspect of technology — needs to be integrated within a curriculum, rather than being merely added on. Hopefully, the next few years will see steps taken towards the resolution of some of the issues, as the Australian community gathers more experience and evidence on the place of graphics calculators in school mathematics. Ideally, any resolution will come about through processes of dialogue and consultation among the various stakeholders, rather than by decree from individuals and organisations in positions of authority.