Chapter 10

Using ICT in Applications of Secondary School Mathematics

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Apart from easing some of the computational burden, an attraction of ICT for mathematics education is that we may have new opportunities for pupils to learn and teachers to teach. This chapter describes some recent examples of technologies that seem likely to be appropriate for use in Singapore, and analyses some of their potential for mathematical applications and modelling, especially in probability and statistics. Particular attention is paid to hand-held calculators because they are the most likely to be affordable and available, especially when the constraints of examinations are taken into account. Additionally, some computer software with particular strengths is acknowledged: spreadsheets, which are likely to be widely available where computers are available; 

Fathom and Tinkerplots, innovative commercial software for statistics. The Internet offers considerable potential for mathematics education, despite relatively high costs of obtaining access to it, and seems likely to grow in significance in Singapore in the next several years. Some ways in which the Internet might support increased attention to applications and modelling are identified. The chapter concludes with recognition of the pervasive significance of professional development of teachers.

1 Introduction

There have been remarkable changes in the past decade to the Information and Communications Technology (ICT) available to pupils, teachers and schools. These changes together suggest that a 21st century mathematics curriculum should be substantially different from curricula of earlier times. In this chapter, we review some of the changed possibilities for both pupils and teachers, and look particularly at the implications of these for applications of mathematics.

Applications of mathematics can be supported by ICT in a range of ways, including representing real-world situations, collecting and analysing data, representing mathematical models and undertaking computation. This chapter provides examples of some of the ways in which ICT use by pupils can contribute to their understanding of and competence in applying mathematics to real world situations, especially in the areas of probability and statistics.

In describing the aims of mathematics education in Singapore schools, the Ministry of Education (2006) has referred explicitly to the use of ICT and applications:

Mathematics education aims to enable students to:

Make effective use of a variety of mathematical tools (including information and communication technology tools) in the learning and application of mathematics. (p. 3)

As well as having some intrinsic importance, applications and modelling play a vital role in the development of mathematical understanding and competencies, as reflected in the Singapore secondary school syllabuses. It is important that pupils apply mathematical problem-solving skills and reasoning skills to tackle a variety of problems, including real-world problems. Their learning of mathematics and their understanding of its relevance to modelling will be supported by using mathematical ideas and thinking.

The Singapore secondary school syllabuses define mathematical modelling as follows:

Mathematical modelling is the process of formulating and improving a mathematical model to represent and solve real-world problems.
Through mathematical modelling, students learn to use a variety of representations of data, and to select and apply appropriate mathematical methods and tools in solving real-world problems. The opportunity to deal with empirical data and use mathematical tools for data analysis should be part of the learning at all levels. (Ministry of Education, 2006, p. 4)

It is helpful to think of this process diagrammatically. A typical representation of the thinking involved is shown in Figure 1, taken from the seminal Australian document, *A national statement on mathematics for Australian schools* (Australian Education Council, 1990, p. 61).

![Diagram of the modelling process](image)

*Figure 1. Mathematical modelling according to A national statement on mathematics for Australian schools*

It is important to note that the use of ICT in such a formulation occurs only on the right of the diagram: representing a model within an ICT context and using the ICT to “do the mathematics” are the key features. The rest of the process of mathematical modelling involves thinking skills. The diagram also makes it clear that modelling involves encountering and using real data of some kind.

## 2 A Variety of ICT Tools

There is a variety of ICT tools that pupils might find helpful for applications of mathematics. An object is only likely to be helpful as a tool, however, if attention is given to learning how to use it effectively, efficiently and autonomously, so that attention is needed for this in the classroom. Indeed, it seems important for attention (and time) to be given to the development of expertise with appropriate ICTs as part of the tools of the mathematical trade for all pupils. Brief comments about the range of possible tools is offered below.

### 2.1 Calculator

A standard calculator (sometimes – inaccurately – referred to as a four-function calculator) will allow pupils to undertake numerical computations when the tasks are beyond their mental capabilities, or when accurate results (rather than rough approximations) are needed. While in many cases, approximate results are sufficient at first, it will rarely be the case that such a calculator will be sufficient.

### 2.2 Scientific calculator

A scientific calculator usually allows for ‘table’ functions (such as trigonometric or logarithmic functions) to be readily evaluated, obviating the need for a book of mathematical tables. It will also extend the range of numbers that can be accommodated, to very large and very small numbers, because of the availability of scientific notation. In addition, they permit some more sophisticated mathematical operations such as raising numbers to powers and finding roots; recent models also allow for some elementary data analysis, usually restricted to finding sample statistics such as means and standard deviations. For these reasons, such calculators have been in widespread use in school curricula in Australia now for some thirty years or so. However, scientific calculators essentially do not provide tools that are much more sophisticated than elementary calculators and offer little more to applications and modelling than tools for calculation of numerical results.
2.3 Graphics calculator

In sharp contrast to scientific calculators, graphic calculators offer pupils significant opportunities to engage in mathematical modelling, as they permit pupils to examine information in a range of ways (Kissane, 2007) and, critically, experiment with mathematical objects. Thus, for example, not only can pupils evaluate a mathematical function at a point (as they can on a scientific calculator), but they can do so at a set of points to generate a table of values. Similarly, rather than being restricted to numerical values, pupils can represent a function graphically (of considerable importance in trying to choose a mathematical model for data). In addition, it is relatively easy for pupils to change a function and see the consequences for its numerical values or its graph. Collectively, these multiple representations of mathematical functions are often described as ‘the rule of three’, referring to the symbolic, numerical and graphical representations. Clearly, such capabilities extend the range of pupil activities for modelling very considerably.

Newcomers to such technology frequently think that the mathematical capabilities of graphics calculators are mostly concerned with this idea of representing functions, especially drawing graphs of functions; indeed, some refer to graphics calculators as ‘graphing’ calculators, sympathetic with this view. However, the technology of graphics calculators is much more powerful than this restricted view suggests, and offers pupils significant experimental possibilities in many domains other than graphing of functions. As a reflection of this, only about a quarter of Kissane and Kemp (2006) is concerned with function graphing. Thus graphics calculators offer significant capabilities in equation solving, sequences and series, elementary calculus, geometry and probability, independent of the idea of function graphing. For this reason, it is arguable that the term ‘calculator’ captures only poorly the range of capabilities, although it is difficult to imagine a new term gaining currency.

Most importantly for the present purpose, graphics calculators offer significant new opportunities for pupils to engage in data analysis, a key element of mathematical modelling with real data. Where a scientific calculator merely provides some numerical statistics for a data set, a graphics calculator allows for models for the data to be constructed, modified, examined and tested, not unlike the capabilities offered by a statistics package.

2.4 CAS calculator

In recent years in some countries (including Australia and the USA), graphics calculators with computer algebra system (CAS) capabilities have increasingly become available. These allow for exact mathematical processes such as manipulating algebraic expressions (e.g., factoring and expanding), solving equations, differentiating functions, evaluating integrals and solving differential equations to be handled by the calculator. In several Australian states, such tools are now regarded as standard equipment that senior secondary school pupils are expected to use well and are permitted to use in important examinations, such as those at the end of secondary school.

As far as modelling is concerned, access to such tools allows for experimentation with symbolic representations of models to be undertaken. This allows pupils to model situations in general rather than in particular, provided they have a sufficient understanding of the mathematical ideas involved.

Recent hand-held devices include a very wide range of capabilities, including CAS capabilities, making them self-contained mathematical tools of great sophistication. The best known examples at present are Casio’s Classpad and Texas Instruments’ TI-Nspire, both of which are popular in schools in Australia. Interestingly, neither of these is usually referred to as a ‘calculator’ by their respective manufacturers, reflecting the very many tools beyond mere ‘calculation’ that they offer to pupils. Such tools can gain currency in schools only when the school curriculum has adjusted to them, however, so that it seems prudent to restrict attention in this chapter to hand-held technologies that do not incorporate CAS.
2.5 Computer software

The major advantage that calculators hold over computers is that they are portable. Being small, light and battery-powered means that they can be easily taken from class to class, between home and school, from classroom to examination room. These are significant advantages for the design of a curriculum.

Computers tend to be larger, but with significant strengths not available to calculators. These include much larger memories, colour displays, appropriate software and telecommunications capabilities. These capabilities can be very helpful for mathematical modelling, and it seems highly unlikely these days that professionals engaged in applying mathematics to real world problems would do so without significant ICT use.

Generic software such as Microsoft Excel, has the advantage of being widely available (as it is often bundled with computer purchases) and also widely used outside schools (in businesses, for example). Although the software was not designed for mostly mathematical purposes, and certainly was not originally intended for educational uses, it has often been used creatively by teachers and even by statisticians to handle tasks involving modelling. A spreadsheet is especially useful when very large data sets are involved (although this is rarely the case for secondary school), as it has much better data handling capabilities than a calculator (although somewhat inferior to software designed for statistics).

In recent years, some mathematical software has been developed explicitly for educational use. Outstanding commercial examples for statistics include Tinkerplots and Fathom, while outstanding examples for geometry include Geometer's Sketchpad, Cabri Geometry and Cabri-3D. Because these have been designed for educational use, they also have associated materials of various kinds available for teachers, which is an important practical consideration. More recently, free software such as GeoGebra has been developed, and considerable work has been done internationally to share educational uses of this software with a wide audience of teachers.

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When schools have the necessary facilities, there is much to be gained by providing pupils with access to software packages of these kinds, although it seems unwise to rely entirely on such software to support pupil work, at least until such time as computers themselves become as ubiquitous, inexpensive and acceptable to examination authorities as calculators are.

For pupils engaged in applications of mathematics, computer presentation software (such as Microsoft PowerPoint, typically included as part of a computer bundle of software and thus widely available) is also important, as it allows them to describe their work to fellow pupils or to others. If mathematical modelling and applications are to move beyond `exercises' for pupils and become a more prominent part of pupils' work (for example, in the form of projects or activities), software of this kind will allow the pupils to consider the key elements of communication and justification of their findings to an interested audience.

2.6 The Internet

In recent years, the Internet has become a prominent part of people's lives in many countries, certainly including Singapore as well as Australia. Consequently, there are many ways in which the Internet might be used for educational purposes, with a typology of these offered by Kissane (2009); for this reason, the author maintains structured links to suitable websites for mathematics pupils and their teachers at the address above. As far as applications and modelling are concerned, as well as providing a range of current examples and some useful tools, the Internet may be used in schools to provide a source of authentic data that pertain to questions of interest to pupils.

There is not space in this chapter to exemplify all of these ICT tools for applications of mathematics. Instead, we focus attention on the important areas of statistics and probability. Partly, this recognises the primacy of these aspects of mathematics in the Singapore secondary school syllabus descriptions and partly it is because the use of ICT has transformed this area of mathematics in recent years.
3 Modelling with a Calculator

As their name suggests, calculators are helpful for doing calculations. Many of the everyday applications of mathematics referred to in the secondary school syllabuses (Ministry of Education, 2006) require relatively little computations and there is much to be said for pupils learning to apply their mathematics approximately and mentally to the everyday world. However, when the numerical data go beyond what can be comfortably handled mentally, when many calculations are needed, or when more precision is appropriate than good approximations can provide, use of ICT for computational help seems unavoidable. The alternative is to restrict applications of mathematics to invented data and further increase a common pupil view that mathematics is not really helpful for understanding the world.

Consider the use of mathematical ideas to model and to understand the changing population of Singapore. Statistics Singapore (2009) has provided relevant data, accessible to pupils. These data can be entered into a scientific calculator and a suitable line of best fit obtained, but the exercise of doing so is unlikely to add much understanding of what is happening. On a graphics calculator, data are stored, and thus can be manipulated in some ways. For example, Figure 2 shows some data and a least squares regression line of best fit for the total population from 1980.

![Figure 2](image)

Figure 2. Using a graphics calculator for analysing Singapore’s population

The analysis suggests that the data are reasonably well-approximated by a linear function, as reflected in the high value of $r^2$ and the visual appearance of the scatterplot and the least squares line.

The quality of the modelling in this case can be explored by imagining that the analysis was conducted from 1980 to 1990 and then using the results to predict the population at a later time (such as 2008, the most recently available data). Figure 3 shows the calculator being used in this flexible way, tabulating a linear function to arrive at a predicted population of 3.89 million people in 2008.

In fact, the population of Singapore in 2008 was substantially (almost a million) more than this prediction, suggesting some caution about using a simplistic linear model to describe something as complex as population growth, and helping pupils to realise some of the realities of dealing with real data. Once the data are stored in a graphics calculator, pupils can explore them in various ways, helping to understand more fully the real world they represent. In this case, further explorations might involve considering subsets of the data or involve applying more sophisticated models than the simple linear regression, consistent with the idea of successive improvements to a mathematical model reflected in Figure 1 above.

4 Simulation and Modelling

While many practical applications of mathematics rely on accessing authentic data, there are others for which the simulation of random data is more appropriate. A very useful facility on all mathematical ICT (including in particular scientific and graphics calculators as well as statistical and spreadsheet software) involves the capacity for random generation of data. Typically, this is based on pseudo-random numbers, uniformly distributed on the interval (0,1).

A good example of this, readily accessible to school pupils via simulation, involves the birthday problem. This interesting problem concerns the probability that a randomly chosen people group of a certain number of people will include at least two people with the same
birthday. Determining this probability is both practically difficult (because of the large number of calculations) and conceptually difficult (as it requires a good understanding of independence and laws of probability). To generate data that represent birthdays, a transformation of random numbers from (0,1) to random integers on \([1,365]\) is needed, ignoring for the sake of simplicity the unusual birthday of 29 February.

Although some devices handle such a transformation automatically, the elementary mathematical idea involved is too important to omit from any modern school curriculum. If RAN\# represents a number in (0,1), then the transformation:

\[
\text{Int}(365\times \text{RAN}\# + 1)
\]

produces a random integer on \([1,365]\), where Int refers to the integer part of a number. Generation of a set of 30 ‘birthdays’, each of which is the number of a particular day of the year, is relatively easy on a graphics calculator, as shown in Figure 4.

![Figure 4. Generation of 30 birthdays](image)

Once the data are generated, a sort of the birthdays facilitates a quick check for any matches. In the example shown in Figure 5, there is a match, indicating that two of the thirty simulated people in this set have the same birthday.

![Figure 5. Sorting data to find a match](image)

This example illustrates some important points: applications of mathematics using ICT do not always use the same mathematical approaches as more formal treatments; relatively unsophisticated mathematics can be used to address an everyday question; understanding how to use an ICT tool allows it to be used flexibly for modelling.

Simulation practices can be usefully extended to model more sophisticated situations. To illustrate, consider a Plane Bookings problem, of determining the most appropriate number of bookings an airline ought take for a small commuter plane, recognising that it is inefficient to fly the plane without all seats being occupied, but that some people make bookings and fail to turn up for the flight. Problems of this kind can be simulated, after making suitable assumptions, and again using an elementary transformation of random data. To be concrete, suppose that on average one out of six passengers is a ‘no-show’ (a figure not markedly different from some actual data) and that the plane has only 20 seats.

![Figure 6. Simulating flight bookings on a 20-seat plane](image)

Simulation can be used to explore the consequences of various possible strategies for this situation, which is quite difficult for school pupils to tackle using school mathematics. Figure 6 shows an Excel
spreadsheet exploring successive examples of booking 24 seats, to
determine the number of seats occupied, and how often the plane is
overbooked (so that there are unseated passengers). The mathematics
behind this spreadsheet comprises the simple transformation of Ran# to
Int (6Ran# + 1), which generates dice rolls. For the purposes of the
application, the spreadsheet regards a result of 5 as representing a no-
show.

Simulations of this kind allow pupils to see that, while theoretically
booking 24 seats will on average result in 20 passengers taking their
seats, there will be times when the planes are overbooked and also times
when the plane is still not filled. What the simulation provides that a
formal analysis of probabilities will not, is a sense of how often various
phenomena occur, so that a realistic appraisal of the proposal can be
considered. The consequences (both in the real world and in the model)
of various possibilities can be considered, and evaluated, using a
relatively simple tool of this kind.

5 ICT and Statistics

Applications of mathematics frequently rely on data, both for building
and for verifying models, which is why statistics and probability have
become prominent parts of school mathematics curricula in recent years.
The use of ICT has revolutionised the practice of statistics, so that data
analysis without ICT is very hard to defend. While calculators can handle
some aspects of data analysis quite well, dedicated computer software is
usually more powerful and more flexible, although is generally not
developed for school pupils. In recent years, educational software for
data analysis has been developed, with the two best examples being
Tinkerplots and Fathom (Key Curriculum Press, 2009a; 2009b).

Tinkerplots has been developed for use with pupils around the
middle years, and consequently has less sophisticated data analytic
procedures inherent in it than does Fathom. However, it provides pupils
with powerful ways of engaging in data analysis, with the aim of
understanding the data well enough to ‘tell the story’ associated with
them. This is especially important for analysing data obtained by pupils
themselves to answer questions of their own interest.

An excellent example of this concerns pupil backpacks, with data
collected from a school and made available for illustrative analysis with
the software. (Readers can download evaluation versions of the software,
including the associated datafiles, from the web link provided by Key
Curriculum Press (2009a).) Data comprise information obtained from 79
pupils of various classes, including their weight and the weight of their
backpack, and were obtained from pupils in a US school, using a set of
scales. Of key interest is the comparison between pupil weight and
backpack weight. The role of the software is to facilitate data analysis, so
that pupils are not distracted by the tasks of calculating statistics or
drawing graphs, but instead can focus on the meanings behind the data.
Each case provides information on various pupil attributes such as class,
body weight, backpack weight, and so on. Sorting the cases is a primal
act of analysis, and the software provides very intuitive ways for pupils
to do this to reveal important relationships.

![Figure 7. Exploring Backpacks (Key Curriculum Press, 2009a)](image)

The use of software of this kind allows for less emphasis on
sophisticated quantitative analyses and more emphasis on the story of the
data. The story can be constructed by choosing suitable ways of
representing the data visually and numerically. In this case, Figure 7 shows the extent to which the weights of backpacks, as a proportion of the weights of the pupils, seem to grow substantially as pupils progress through school, with an alarming number exceeding the recommendations of health authorities (of a maximum of 15%). The diagram shows mean percentages and also allows pupils to explore other questions of interest, such as differences between boys and girls.

While Tinkerplots is described by its publishers as suitable for grades 4-8, it arguably extends beyond both extremities. Fathom (an earlier statistics package from the same developer) is also designed for educational purposes, but with a slightly older focus, including inferential statistics and formal mathematical procedures such as linear regression.

Although some of these procedures are accessible via graphics calculators, as illustrated earlier, computer software provides more analytic opportunities and better visual representations. To illustrate, Figure 8 shows representations of the Singapore population data, including the least squares regression line shown previously. Notice how well the ‘least squares’ concept is demonstrated in the graph: where a calculator provided only the numerical results, the software also allows the underlying concept to be seen. The ‘line of best fit’ is the line that minimises the sums of the areas of the squares shown.

In addition to improving the understanding of key ideas, a tool like Fathom allows pupils to more easily explore underlying patterns in the data. In this case, graphs of the two population variables: the ‘Residential’ population and the ‘Total’ population illustrate that the former is much more linear than the latter, as shown in Figure 9. That is, it seems as if Singapore’s residential population can be predicted fairly well with a linear model perhaps because it depends mostly on a relatively stable rate of natural phenomena such as birth and death rates. In contrast, the total population is harder to predict successfully, as it presumably involves other factors such as migration, government policy decisions, economic influences, and so on. This accounts for the rather poor prediction noted with the graphics calculator model earlier. Although similar analyses can be conducted on the less sophisticated calculator, the environment provided by software like Fathom requires less effort on behalf of the user to engage in exploratory modelling of these kinds.

Data analysis is most powerful when pupils are addressing problems that matter to them. They can design and undertake their own data collection, then use software such as Tinkerplots or Fathom to store and analyse their own data. Use of the software does not require high-level
ICT skills, and pupils will incidentally learn important lessons associated with statistics such as data editing, dealing with missing cases, identifying and correcting entry errors, and so on.

6 Using the Internet

The Internet has become one of the most important aspects of ICT in recent years, and also has some connections with mathematical modelling and applications. Kissane (2009) explores some of the ways in which the Internet is significant for both pupils and teachers of mathematics, several of which are of particular importance for applications of mathematics and mathematical modelling. The author’s website systematically catalogues some of the ways in which pupils might learn about mathematics through use of the Internet. For example, one category of use involves ‘Reading interesting materials’, in which there are websites identified that describe current examples of mathematical modelling and the use of mathematics in applications of many kinds. The Plus magazine from the UK is one of these, with regular contributions from people using mathematics, ideally suited for sophisticated secondary pupils. Similarly, the Australian Academy of Science’s Nova site contains many examples of mathematical modelling, described in some detail. Websites of these kinds can provide recent material for pupils, reinforcing the perspective that mathematics is a valuable tool to address modern problems.

Applications of mathematics involving simulations are best handled by the use of ICT, as the earlier examples have illustrated. In recent years, a number of excellent tools and applications of simulation have been made available through the Internet. In Kissane’s (2009) typology, some of these take the form of ‘Interactive opportunities’, for which links are provided. For example, the outstanding National Library of Virtual Manipulatives provides several examples in Data Analysis and Probability for pupils to use for simulation, allowing data to be generated, organised and analysed efficiently and thus helping pupils to see how randomness can be understood and used for practical purposes. Similarly, a search of the NCTM Illuminations site in the Data Analysis area and Probability area provides a rich set of resources for both pupils and their teachers.

As a third example, the remarkable Nrich site also has many simulation tools among its regular resources for pupils and teachers. An important new kind of resource offered by Nrich are teacher and pupil packages, which can be downloaded for offline use, particularly helpful when access to the Internet is not readily available in a classroom. For example, one of these (Nrich, 2009) concerns probability, and offers a collection of several Flash applets that can be used by the whole class (e.g. on an interactive whiteboard or a whole class display) or used by small groups or individual pupils. Figure 10 shows one of these applets, allowing for efficient simulation of tossing a pair of discs, differently coloured on each side, and recording successive results in a powerful way (using the sheet developed by the DIME project). In this case, the use of ICT has made it easier to use good ideas for pupils to explore simulation in a way that was developed many years ago, but which was difficult to implement for practical reasons.

![Figure 10. Nrich Flash applet for simulating coin tosses and recording results](image-url)
data. Most data on the Internet are not case-based data, but are summary data, which often provide less opportunity for pupil exploration. An example of case-based data is the Census At School project, administered by the Australian Bureau of Statistics (2009) and which is intended for pupils themselves to use. After a large group of pupils has completed census-like surveys, a large body of data has been stored on the Internet. Pupils are able to download a random sample of these data for analysis. Significantly, since each random sample is different, not only do pupils thus get an opportunity to explore relationships among real data, but they also have an opportunity to see how different their conclusions are from those of other pupils, and in the process begin to learn about the pitfalls (as well as the benefits) of random sampling. Although there is not yet a Singaporean version of this project (and consequently the data may be less directly relevant culturally to Singapore pupils), much is to be gained by pupils using this kind of data set to understand differences between themselves and others.

Some data are available on the Internet for direct downloading into software packages like Tinkerplots and Fathom. Indeed, both the Tinkerplots and Fathom websites provide good links to various data sources for mathematical modelling, although these tend to be a little culturally biased towards the US context, not surprisingly. Although some of the examples are rather sophisticated statistically, the Data and Story Library (DASL) (2009) is a useful data source, as is The eeps Data Zoo (Erickson, 2007). With such materials, contexts of pupil interest need to be considered, as many of the individual data sets on these websites are not intended directly for school pupils. There would seem to be a good case for teachers to share data sources in this way, to supplement students collecting their own data for modelling purposes.

7 Final Remarks

A range of ICT is now available for pupils, and it is clearly appropriate for the mathematics curriculum to take advantage of this, as the Ministry of Education (2006) has noted. The use of ICT for applications and modelling is constrained by facilities, the curriculum and the time available. Some examples of ICT (such as calculators) are likely to be available everywhere, and to meet many needs. These are most likely to be accommodated by external examination constraints, an important consideration for classroom use. Other manifestations of ICT, such as specialist software, are more powerful, but usually require more resources to use effectively.

This chapter has suggested that ICT can play a substantial role in improving the statistics and probability components of the school curriculum in particular. In ways that were not previously possible, pupils can engage in genuine data analysis, and explore questions of interest to themselves. They can also explore chance phenomena through the use of simulation, not previously accessible without access to ICT.

While teachers do not need extensive backgrounds in the use of ICT to take advantage of these opportunities, they do need support, professional development and time to learn how to use the ICT effectively and efficiently. As calculators and software become more user-friendly, and as the Internet continues to offer many new materials for teachers and their pupils, teachers can expect that the opportunities for sound use of ICT for mathematical modelling and applications will increase.

References


