BEGINNING TEACHERS’ MATHEMATICAL KNOWLEDGE: WHAT IS NEEDED?

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Over the past decade there has been growing interest in describing and measuring the kinds of mathematical knowledge needed by teachers. Such efforts are in parallel with the development of national standards for teachers, indicating levels of expectation across the years of teachers’ careers. This presentation provides an opportunity for teacher educators and teachers to consider the nature of mathematical knowledge needed by beginning teachers at all levels of schooling. Discussion will be informed by data from an ALTC funded national project that aims to improve the quality of pre-service teachers’ outcomes in mathematics and by the AAMT Standards framework.

Introduction

Interest in beginning teachers’ mathematical knowledge is not new. At the first MERGA conference, Brown (1977) described growing concerns about the mathematics knowledge of pre-service teachers and what he described as “anti-mathematical” backgrounds. In response to these concerns a remediation program was described “which is almost identical to that necessary in the lower secondary or upper primary schools” (p. 45).

In 1987, Shulman’s seminal work identified three domains of teacher knowledge: subject-matter knowledge, pedagogical content knowledge, and curricular knowledge. Subject-matter knowledge includes all of those ideas fundamental to the domain, pedagogical content knowledge extends to such matters as useful forms of representation, explanations and examples of the domain, and curricular knowledge includes understanding of how the subject-matter is organised over the years of schooling (Shulman, 1987).

A number of studies have deepened understanding of the kind of knowledge that teachers need for teaching. Mewborn (2001) showed that crude measures of teacher knowledge, such as the number of mathematics courses taken, were insufficient to characterise teachers’ mathematical knowledge for teaching. Hill, Schilling and Ball (2004) developed measures of teachers’ mathematical knowledge for teaching (MKT) using multiple choice items that could be described broadly as mathematics content knowledge set in a classroom context. Watson (2001) used a profiling approach with a
range of questions that addressed all of Shulman’s (1987) knowledge types. Using a similar instrument, Beswick, Callingham and Watson (2011) demonstrated that the different knowledge types could be considered as a single domain, providing an holistic conception of teacher knowledge for mathematics teaching that included beliefs about and attitudes towards mathematics as well as classroom focussed mathematics understanding. Further, they showed that the domain had a hierarchical structure in which general pedagogical knowledge and pedagogical content knowledge (PCK) related specifically to teaching mathematics were at the upper end of the scale and everyday numeracy was at the lower end.

Callingham and Watson (in press) focussed on pedagogical content knowledge restricted to the area of statistics. They used items of two main types – those in which teachers were asked to identify likely responses from their students to a particular question, and then to suggest appropriate interventions to one of these responses, and secondly, those in which they chose their “next steps” in response to questions showing students’ actual answers. These items attempted to capture both the diagnostic element of teachers’ knowledge and their understanding of students’ learning in the domain of statistics. A four-level hierarchy of teachers’ PCK was identified which could be used to both identify teachers’ understanding and also measure teacher change.

The Australian Association of Mathematics Teachers (AAMT) developed a rich description of the characteristics of exemplary mathematics teachers (AAMT, 2002/2006) through a project that brought together teacher expertise and research findings. This description of Standards for excellence in teaching mathematics for Australian Schools has three domains: Professional Knowledge, Professional Attributes and Professional Practice. These domains address the various knowledge types described by Shulman (1987) and aim to provide a basis for identifying exemplary teachers of mathematics. More recently, the Australian Institute for Teaching and School Leadership (AITSL) (2011) published a set of generic teaching standards that described seven standards across three domains: Professional Knowledge, Professional Practice and Professional Engagement. Of particular interest is that the AITSL document included four levels to describe different career stages, including graduate standards. The graduate standards are particularly relevant to the project reported here, which has a focus on improving pre-service teachers’ mathematical outcomes.

These various recent developments describe a rich context in which the collaborative project described here takes place. An increased attention to the forms of knowledge required for teaching mathematics, along with explicit descriptions of teaching standards at various levels, and a new mechanism for the accreditation of teacher education courses together require thoughtful responses by those engaged in mathematics teacher education. The systematic use of evidence to support professional opinion in the shaping and refining of mathematics teacher education programs is a critical part of that response, and the major focus of the project.

**Background to the study**

Building the Culture of Evidence-based Practice in Teacher Preparation for Mathematics Teaching (CEMENT) is a two-year project that aims to produce:

1. Evidence-based changes to mathematics education teaching within participating universities;

2. Evidence-based changes to teacher education courses;

3. Evidence-based changes to the accreditation of teacher education programs.

These changes are expected to improve the quality of mathematics teacher education and to support the development of a new generation of effective mathematics teachers who are well-prepared to meet the needs of all students.
2. Recommendations about effective models of teacher education for teaching mathematics;
3. Processes for bringing about change at unit and course levels; and
4. Progress towards a national culture of evidence-based practice in relation to mathematics teacher education.

The project team (authors) represent seven universities across all states and the Northern Territory, which include diverse institutions delivering a wide variety of teacher education courses. The mathematics education taught within the differing programs varies in the amount of time allocated, the nature of the content and delivery and the placement within the overall course structure. In order to meet the aims of the project, data were needed about what pre-service teachers at the end of their course knew and understood about mathematics teaching. There were limitations on the nature and amount of data that could be collected. Because of time and manpower constraints and the national nature of the study, it was decided that an automatically scored web-based survey would be used, which in turn limited the nature of the items. The focus of the survey needed to go beyond content knowledge of mathematics alone, and to include aspects of pedagogical content knowledge. In addition 10 items addressing teacher beliefs about mathematics and its teaching were included. Collaboratively, the team developed items that included all of these domains. A selection of these items was piloted with students at the University of Tasmania who were undertaking mathematics education units over the summer semester. This pilot study is the focus of this report.

Method

Sample

The students in the sample were all undertaking a pre-service course for primary teaching. The majority \((n = 52, 86.7\%)\) were studying off campus and were split almost equally between part-time \((n = 29, 48.3\%)\) and full-time \((n = 31, 51.7\%)\) study. Of the respondents, one-quarter \((n=15, 25.0\%)\) were aiming to graduate in 2011, with a further 29 students \((48.3\%)\) aiming to graduate by 2013.

Students were asked about their previous educational experience. Of the 55 students who responded, 23 \((41.8\%)\) had secondary schooling only, and 25 \((45.5\%)\) had a certificate level qualification, possibly reflecting some vocational training prior to university entrance. When their mathematics backgrounds were considered, 21 \((38.2\%)\) had only studied mathematics to Year 10, 11 \((20.0\%)\) had studied a non-pre-tertiary mathematics subject and 17 \((30.9\%)\) had studied a pre-tertiary mathematics subject in Year 11/12.

The sample was, therefore, towards the end of pre-service teacher education and had educational and mathematics backgrounds that have been reported elsewhere as typical of pre-service teachers (e.g., Ainley, Kos, & Nicholas, 2008). No information was collected about gender but the enrolment in primary education is predominantly female.

Instruments

A 45-item online test was undertaken by 60 pre-service primary teachers at the University of Tasmania. The instrument consisted of 10 items addressing beliefs about mathematics, 13 items addressing mathematics content and 23 items that addressed pedagogical content knowledge (PCK). Examples of items are shown in Table 1.
Table 1. Examples of items used in the pilot test.

<table>
<thead>
<tr>
<th>Item category</th>
<th>Example</th>
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<tbody>
<tr>
<td>Beliefs</td>
<td>Mathematics is a beautiful and creative human endeavour</td>
</tr>
<tr>
<td>Beliefs</td>
<td>Students learn by practicing methods and procedures for performing</td>
</tr>
<tr>
<td></td>
<td>mathematical tasks</td>
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<tr>
<td>Content knowledge</td>
<td>Which one of the following contains a set of three fractions that are</td>
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<td></td>
<td>evenly spaced on a number line?</td>
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<td></td>
<td>A)  ( \frac{3}{6}, \frac{3}{5}, \frac{5}{4} )</td>
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<tr>
<td></td>
<td>B)  ( \frac{3}{4}, \frac{19}{24}, \frac{5}{6} )</td>
</tr>
<tr>
<td></td>
<td>C)  ( \frac{3}{24}, \frac{19}{8}, \frac{7}{6} )</td>
</tr>
<tr>
<td></td>
<td>D)  ( \frac{4}{5}, \frac{7}{8}, \frac{5}{6} )</td>
</tr>
<tr>
<td>Pedagogical Content Knowledge (PCK)</td>
<td>A Year 5 teacher asked her pupils to determine the value of the following</td>
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<tr>
<td></td>
<td>calculation on their calculators:</td>
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<td></td>
<td>( 2 \div 3 \times 4 = ? )</td>
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<td></td>
<td>The class was surprised to find that some student calculators gave a</td>
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<td></td>
<td>result of 14, while others gave a result of 20. Which of the following</td>
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<td></td>
<td>best matches your likely response to this situation?</td>
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<tr>
<td></td>
<td>A. Use the difference as a motivation to teach the students how to use</td>
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<td></td>
<td>the correct order of operations, highlighting an acronym such as BODMAS.</td>
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<tr>
<td></td>
<td>B. Show the students how to use parentheses or brackets when entering</td>
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<td></td>
<td>expressions into their calculators.</td>
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<tr>
<td></td>
<td>C. Check school booklists and supplies to make sure that only one kind</td>
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<td></td>
<td>of calculator was available to students in the class.</td>
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<tr>
<td></td>
<td>D. Ask the pupils to explain the different results, and use their</td>
</tr>
<tr>
<td></td>
<td>explanations to discuss the order of operations as an arbitrary</td>
</tr>
<tr>
<td></td>
<td>convention.</td>
</tr>
</tbody>
</table>

The Beliefs items used a five-point Likert scale from strongly disagree to strongly agree; content items were scored right or wrong. Following discussion among the project team, the PCK items were mostly scored dichotomously as right/wrong. Some PCK items, however, provoked considerable discussion and scoring was determined on the basis of an agreed hierarchy. The PCK item shown in Table 1, for example, was scored as A = 1, B = 2, C = 0 and D = 2 on the grounds that the responses for B and D both represented good “next steps” for developing understanding, and the response to A was reasonable but not of the same quality as the two scored at 2.

Data analysis

Data were analysed in various ways to provide a range of information. First the scored responses were analysed using Rasch measurement to provide quality control information about the items by a consideration of fit to the Rasch model (Bond & Fox, 2007). Three scales were produced: Beliefs about mathematics (BELF, 10 items), Mathematical Content Knowledge (MCK, 13 items); and Pedagogical Content Knowledge (PCK, 23 items). From each of these scales a measure of performance for each student was obtained in logits, the unit of Rasch measurement. These measures were used as a basis for comparisons between groups based on the background variables. Finally, frequency counts of students’ choices provided some diagnostic information.
Results

All of the three scales showed excellent fit to the Rasch model indicating that within each scale the items worked consistently together to measure a single construct that could be used to make inferences about students’ performances. Performance measures were obtained for every student on each one of the three scales and used for further analysis.

Between groups analysis

Comparisons were undertaken between groups based on full-time/part-time enrolment, education background and mathematics background. No comparison was made between distance and face-to-face students because of the low numbers of students studying on-campus. No statistically significant difference was found among any of the groups on any measure. This finding is not surprising given the homogenous nature of the sample.

Performance on different kinds of scale

Boxplots of the distributions of students’ performance measures on each of the three scales are shown in Figure 1. The scales show a monotonic decline in median score from BELF, to MCK to PCK indicating that of the three scales students found the pedagogical content knowledge more difficult than straight mathematics content knowledge, which was more difficult than endorsing beliefs about mathematics.

To explore this finding further, results from the Rasch analysis output were examined to identify specific items or groups of items that students found difficult. These findings are reported for each of the three scales.
Beliefs about mathematics

The most strongly endorsed items were those indicating a broadly student-centred view of mathematics learning, such as “The teacher must be receptive to the children’s suggestions and ideas” and “Teachers must be able to represent mathematical ideas in a variety of ways”. Students, however, also strongly endorsed “Acknowledging multiple ways of thinking may confuse children”, in apparent contradiction to the other two items. At the other end of the scale, students found it difficult to endorse “The procedures and methods used in mathematics guarantee right answers”, possibly reflecting an emphasis on process rather than product. “Mathematics is a beautiful and creative human endeavour” was also difficult for students to endorse, although it is not clear whether they disagreed with the beauty and creativity or with the human endeavour. “Mathematical ideas exist independently of human ability to discover them”, however, was also fairly difficult to endorse, suggesting that students understood mathematics as a human activity but did not see it as creative or beautiful.

Mathematical content knowledge

Among the MCK items, the most difficult was identifying the prime factors of 30. Unexpectedly, however, a majority of students (n = 30, 54.5%) chose the option listing all factors of 30 rather than the anticipated attractive distracter of “1, 2, 3, 5” suggesting that the students understand the notion of factor but not the idea of prime factor. The next most difficult item was the fraction item shown in Table 1. Only 15 (27.3%) students answered this correctly. Surprisingly, at about the same level of difficulty was “The product of an odd number and an even number is odd”, to which students had to choose from the options “always true”, “sometimes true” and “never true”. Only 16 students (29.1%) responded correctly. Whereas the prime number item was based on knowledge of mathematical language, both of the other two items were more conceptual in nature, raising issues about students’ underlying understanding.

At the other end of the scale, the easiest items were combinations based on a menu, a definition of congruence, identifying an incorrect representation of \(\frac{3}{4}\), and a two-step computation based on reading currency conversions from graphs. The remaining items were all at about the mean difficulty level and consisted of a number of items based on geometry including an angle calculation, and one requiring an algebraic expression to describe a linear pattern. It seems that for this group of respondents, work on geometry and algebra would benefit them in terms of their mathematical development.

Pedagogical content knowledge

The easiest PCK items included the item shown in Table 1 about teaching an algorithm, and an item about choosing an appropriate representation to develop children’s understanding of proportional reasoning that was also scored with multiple codes. It is possible that by rescoring these items to try to allow for all reasonable possibilities that the items have lost their discriminatory power.

The other items all tended to bunch together on the scale which means that they provided a lot of information across a narrow range. It is possible that with a larger and more diverse sample, this difficulty might be overcome.

Of all the items, those addressing teaching aspects of measurement and geometry appeared slightly more difficult. Respondents could not, for example, identify rhombi
from a collection of 2D shapes, and suggested incorrect teaching explanations for students. One surprising item addressed materials suitable for developing subitising skills. Students were provided with a description of subitising and a choice of five possible materials: number line, dominoes and dice, number expander, MAB, and a large collection of objects, all represented pictorially. Of the 48 respondents, 17 (35.4%) chose MAB and 18 (37.5%) chose the large collection rather than the dominoes and dice ($n = 9, 18.8\%$).

**Discussion**

This pilot study is part of a much bigger project that aims to provide useful tools to universities so that they can monitor their pre-service teachers’ mathematical development in three domains: beliefs and attitudes, mathematical content knowledge and pedagogical content knowledge. The items trialled produced coherent scales but additional work is needed on the PCK items to ensure that they discriminate more effectively. As a first attempt, however, the project team was relatively satisfied with the instrument.

The finding that PCK was more difficult than MCK and BELF is consistent with other research in the area (Beswick, Callingham & Watson, 2011). This finding raises issues for mathematics education about how best to develop PCK in pre-service teachers. Although MCK and PCK are inextricably linked, it seems that mathematics understanding alone is not sufficient.

The nature of the items that respondents found difficult provides information that can be used to revise courses in the relevant university. More work is needed in areas such as geometry and measurement, which have received little explicit focus compared with fractions and proportional reasoning, for example. Students appear to have difficulty with choosing appropriate representations and materials for teaching, and this could be addressed in workshops and online activities.

The standards and frameworks available at present (e.g., AAMT, 2002/2006; AITSL, 2011) provide useful information about desirable attributes but little support for developing these. The instrument described represents a starting point for providing data about some aspects of these attributes so that pre-service teacher education can develop courses and approaches based on information rather than solely on the opinions and beliefs of teacher educators. Further items have been developed by the project team, as well as similar instrument intended for high school mathematics pre-service teachers, including those who are likely to be teaching outside their specialisation. These instruments will be trialled and modified throughout 2011, and information provided to all participating universities to inform future course development.

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References


