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Sensitivity Analysis of Voltage Imbalance in Distribution Networks with Rooftop PVs

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Abstract—A comprehensive voltage imbalance sensitivity analysis and stochastic evaluation based on the rating and location of single-phase grid-connected rooftop photovoltaic cells (PVs) in a residential low voltage distribution network are presented. The voltage imbalance at different locations along a feeder is investigated. In addition, the sensitivity analysis is performed for voltage imbalance in one feeder when PVs are installed in other feeders of the network. A stochastic evaluation based on Monte Carlo method is carried out to investigate the risk index of the non-standard voltage imbalance in the network in the presence of PVs. The network voltage imbalance characteristic based on different criteria of PV rating and location and network conditions is generalized. Improvement methods are proposed for voltage imbalance reduction and their efficacy is verified by comparing their risk index using Monte Carlo simulations.

Index Terms—Voltage Imbalance, single-phase Rooftop PV, Distribution Network, Sensitivity Analysis, Stochastic Evaluation

I. INTRODUCTION

Current and voltage imbalance are the most severe power quality problems in low voltage (LV) distribution networks [1]. Voltage imbalance is more common in individual customer loads due to phase load imbalances, especially where large single-phase power loads are used [2]. Although voltages well balanced at the supply side level, the voltages at the customer level can become unbalanced due to the unequal system impedances, unequal distribution of single-phase loads or large number of single-phase transformers [2]. Usually, the electric utilities try to distribute the residential loads equally among the three phases of distribution feeders [3].

An increase in the voltage imbalance can result in overheating and de-rating of all induction motor types of loads and also the distribution transformers [4-5]. Voltage imbalance can also cause network problems such as malfunction of protection relays and voltage regulation equipment, and generation of non-characteristic harmonics from power electronic loads [3].

In recent years, there is a growing interest by the residential customers to install single-phase grid-connected rooftop Photovoltaic cells (PVs) due to new energy and incentive policies in several countries [6]. The most important characteristic of these PVs is that their output power being fed to the grid is not controlled and is dependent on the instantaneous power from the sun [7]. Several technical problems of these systems such as harmonics, voltage profile and power loss are already studied and investigated in [8-10].

Along with the increase of rooftop PV application for residential loads, which are usually distributed randomly among the customers of the network, one of the main power quality concerns is whether their random locations and ratings in the network might lead to an increase in the imbalance index of the network beyond the standard values [11]. References [11-12] have investigated the maximum allowable number of grid-connected PVs in European and UK distribution networks based on voltage imbalance standard limitations. However the penetration of rooftop PVs will increase the maximum limits.

In this paper, a comprehensive voltage imbalance sensitivity analysis based on the rating and location of rooftop PVs in a residential LV distribution network is performed. The voltage imbalance is determined at different locations along a feeder. In addition, the sensitivity analysis is carried out for voltage imbalance in one feeder while there are installations of PVs in other feeders of the network.

Due to the randomness locating of the PVs in addition to their instantaneous power generation and the residential power demand within 24 hour or summer-winter periods, a deterministic analysis is unacceptable. References [13-17] have proposed and applied probabilistic analysis for uncertainties in the network such as load flow, voltage sag, fault, reliability and voltage unbalance. Therefore, a stochastic evaluation based on Monte Carlo method is carried out to investigate and predict the network voltage imbalance for the uncertainties of rooftop PV power ratings and locations.

II. VOLTAGE IMBALANCE

Based on IEEE Recommended Practice for Utility Interface of Photovoltaic Systems, the PV inverters will be permitted to connect to grid in normal voltage conditions of the network which is between 88-110 % of the nominal voltage and shut down beyond these limits [18].

In addition, the PVs should operate at a power factor greater than 0.85 (lagging or leading) when their output is greater than 10% of rating. Most inverters of grid-connected PVs are designed to operate close to unity power factor; while specially designed systems that provide reactive power compensation may operate outside of this limit with utility approval. It is highly desired that they operate at unity power factor at full load but due to inverter components (e.g., output filters, transformers) it might be reduced up to 0.85.

Voltage imbalance in the three-phase electric system is a condition in which the three phase voltages differ in amplitude and/or does not have its normal 120 degree phase difference.
There have been several methods for definition, calculation and interpretation of Voltage Unbalance Factor (VUF) as proposed in [19-21]. IEEE Recommended Practice for Monitoring Electric Power Quality defines this as [22]

\[
VUF\% = \left| \frac{V_n - V_{+}}{V_n} \right| \times 100
\]

where \( V_n \) and \( V_{+} \) are the negative and positive sequence of the voltage, respectively. This will be referred to as percentage voltage imbalance in the paper.

According to [22], the allowable limit for voltage imbalance is limited to 2%. Engineering Recommendation P29 in UK not only limits the whole voltage imbalance of the network to 2%, but also limits the voltage imbalance to 1.3% at the load point and indicates that the design of the network should be based on achieving values less than that [23]. ANSI standard for “Electric Power Systems and Equipment Voltage Ratings (60 Hertz)” recommends that electrical supply systems should be designed and operated to limit the maximum voltage imbalance to 3% when measured at the electric utility end points under no-load conditions [24].

IEEE Recommended Practice for Electric Power Systems in Commercial Buildings (Gray Book) indicates that the single-phase power electronics based devices like the computers, entertainment equipment, etc may experience problems if the voltage imbalance (resulted from non-standard voltage amplitude) is more than 2-2.5% [25].

The voltage imbalance has adverse effect on the three-phase power electronic based equipment in the network [26-27], the most important of them, is the central speed variable air conditioners. It will also have adverse effect on the operating characteristics of three-phase induction motors used in water pumps, elevators, etc in residential type loads [4-5].

III. VOLTAGE IMBALANCE IN LV DISTRIBUTION NETWORKS WITH PVs

The utilities try to minimize the imbalance index in their network by distributing single-phase loads equally across all three phases or even by reconfiguration of the network. Probabilistic studies have shown that it is very rare that the residential and small business loads can result in higher values of the voltage imbalance in the network. The measurements done in a LV distribution network in US [26], Brazil [27] and Iran [28] conclude that the probability of the voltage imbalance to be more than 3% in the network is about 2-5%. This can only be achieved if the engineering judgements have been applied for selecting the appropriate size of the conductors and cables, transformer ratings and also if the load dispatch among the three phases are rectified by later observations and measurements. If the appropriate designs have not been done or there is a non-standard voltage drop in the network, it is highly possible that the network also suffers from higher voltage imbalance.

The PVs are usually installed randomly among the residential loads, therefore it is expected that the total power generation and locations can differ lot from one network to another. For example, it is not even surprising if 80, 50 and 10% of the whole customers on each phase have installed rooftop PVs. In such a condition, even if the voltage imbalance of the network was within the standard limits without any PVs, it is not guaranteed to remain so after PV installation. Therefore, the possible PV installation number or rating on such systems must be investigated in a way that the voltage imbalance is still kept within the standard limit.

A. Network Structure

A sample radial LV (415 V) residential urban distribution network is considered for voltage imbalance investigations. This network is supposed to supply electricity to a combination of residential and small business customers. It has 3 three-phase feeders with equal length (400 m) and equal number of customers on each phase and feeder. The poles are located at a distance of almost 40 meters from each other and at each pole, 2 houses are supplied from each phase. The feeders and their cross-section are also designed appropriately based on the amount of power and the voltage drop. The simplified equivalent single line diagram of one phase of one of the three feeders is illustrated in Fig. 1. The technical data of the network is given in the Appendix.

It is assumed that the total electric load of the network is almost 1 MVA where the studied LV network has a 360 kW load. It is dispersed non-identically among the three phase, i.e. phase a, b and c have 60, 120 and 180 kW load, respectively. Although, the utility has already distributed the loads with equal numbers among the phases, but the power consumption of the houses differ from each other. The rest of the network load is considered as a lumped load.

In this model, the roof top PVs installed by the householders have an output power in the range of 1-5 kW working in unity power factor. The PVs are all grid-connected and the surplus of the electricity generated will flow to the grid.

![Fig. 1. Schematic single line diagram of one phase in a feeder of the studied LV distribution network.](image)

B. Power Flow Analysis

For calculating the voltage imbalance, it is necessary that the network to be analysed and the voltages at the desired nodes to be calculated. Based on the KCL on each node, we have for the \( k^\text{th} \) node

\[
\beta(V_{PV,k} - V_i) + \frac{V_{L,i}^2 + V_i^2}{Z_f} + \frac{V_{L,i} + V_i}{Z_L} = 0
\]

where \( Z_f \) is the feeder impedance between two adjacent nodes, \( V_i, i=1,\ldots,N \) is the single-phase voltage of the \( i^\text{th} \) node, \( Z_{L,k} \) is the load impedance connected to \( k^\text{th} \) node, \( V_{PV,k} \) and \( jX_{PV,k} \) are PV voltage and impedance connected to \( k^\text{th} \) node. In (2), controlling constant \( \beta \) is equal to 1 when there is a PV connected to \( k^\text{th} \) node, otherwise, it is zero.

For finding \( V_{PV,k} \), the voltage at the output of the PV inverter, it is needed to be calculated as well. The simplified diagram of the PV connection to the grid is shown in Fig. 2. Based on this figure, we have
where $P_{PV,k}$ and $Q_{PV,k}$ are respectively the active and reactive power output of the PV connected to the $k$th node. Assuming $P_{PV,k}$ and $Q_{PV,k}$ to be constant and $|V_i|$ and $\delta_i$ are known, $|V_{PV,k}|$ and $\delta_{PV,k}$ can be calculated.

\[
P_{PV,k} = \frac{|V_{PV,k}|}{X_{PV,k}} \sin(\delta_{PV,k} - \delta_i) \tag{3}
\]

\[
Q_{PV,k} = \frac{|V_{PV,k}|}{X_{PV,k}} \cos(\delta_{PV,k} - \delta_i) - |V_i| \tag{4}
\]

Fig. 2. Schematic diagram of a PV connection to grid.

To calculate $V_i$ from (2)-(4), an iterative method is required. Starting with a set of initial values, the entire network is solved to determine $V_i$. Once the solution converges, the sequence components are calculated.

C. Sensitivity Analysis

The voltage at any node can be considered as a function of the location and rating of PV. Therefore the sensitivity of network voltage imbalance to a PV location and rating is explained by

\[
S_k = \frac{\partial VUF}{\partial P_{PV,k}} \frac{P_{PV,k}}{VUF} \tag{5}
\]

Since voltages at each node are calculated iteratively, the sensitivity is calculated numerically once the iterations converge as

\[
S_k = \frac{VUF(\gamma + 1) - VUF(\gamma)}{P_{PV,k}(\gamma + 1) - P_{PV,k}(\gamma)} \tag{6}
\]

where $\gamma$ defines the rating of the PV, i.e. $\gamma = 0, \ldots, 4$ kW.

V. NUMERICAL RESULTS

The voltage imbalance in the presence of rooftop PVs is calculated for the system in Fig. 1 and the data given in the appendix. Several studies are performed, which are discussed below.

A. Case 1 - Voltage imbalance in the distribution network

The utilities usually measure and monitor the voltage imbalance at the beginning of the feeder (i.e. secondary side of the distribution transformer). As described previously, the probabilistic studies of the measurement results show that there is low chance of having a voltage imbalance beyond 2% this location. Let us assume that the length of the three phases and number of customers per phase are the same. Even then, since the power consumption of the loads are different, there will be different voltage drops along the feeder. This will result in different voltage amplitudes at different locations along the feeder. This phenomenon can result in high voltage imbalance especially at the end of the feeders. To keep the voltage drop within the limit at the end of the feeder, the utilities install single-phase pad mounted capacitors or increase the cross-section of the feeder. However, voltage imbalance at the end of the feeder still remains higher than the beginning of the feeder. This might cause a problem if some three-phase induction motors or power electronics based equipment are located far from the beginning of the feeder.

For example, in the network under consideration, voltage imbalance at the beginning of the feeder has increased from 0.85 % to 1.84 % at the end. The voltage profile of feeder 1 in the network is shown in Fig. 3a. The voltage imbalance versus the length of feeder is shown in Fig. 3b.

Effects of application of different types of conductors, such as Steel Conductor Galvanized (SC/GZ), Aerial Bundle Cable (ABC) and Aluminium Conductor Steel Reinforced (ACSR), with different cross-sections on the voltage imbalance along the feeder is shown in Fig. 4. It can be seen that for conductors SC/GZ and ACSR, the voltage imbalance can go to values higher than 2% for a 400 meter line while for ABC with 70 or 95 mm$^2$ cross-section, it is remaining within the standard limit. Aluminium and copper conductors with similar impedance to ABC have similar voltage imbalance by ABC.
B. Case 2- Sensitivity analysis of one PV on voltage imbalance

The voltage imbalance variation due to the location of one PV with a constant rating will depend on the total load of the phase in which it is installed. Usually rooftop PVs can have ratings up to 5 kW for urban residential customers. It is also important to define the point at which the voltage imbalance will be measured.

If the PV is installed in a phase, the voltage drop will be reduced. Let us consider a 1 and 5 kW PV installation at the beginning, middle and end of a feeder. In Fig. 5, the voltage profile of phase A is shown in this situation. As expected the voltage amplitude increases as the PV has higher ratings or it is installed at the end nodes.

If the PV is installed in low load phase (phase A in this case), this increase of the voltage difference between three phases will result in the increase in network voltage imbalance at the end of the feeder but will have only minor effects on the voltage imbalance at the beginning of the feeder. This increase at the end of the feeder is more if the PV is installed at the end of the feeder compared to the early nodes or if the rating of the PV is high. The sensitivity analysis of voltage imbalance (calculated at the end of the feeder) versus the location and rating of rooftop PV installed in low load phase A is illustrated in Fig. 6a.

If the PV is installed in high load phase (phase C), this increase of the voltage, will reduce the difference between three phases will result in the decrease in network voltage imbalance at the end of the feeder but will have only minor effect on the beginning of the feeder. This decrease at the end of the feeder is more if the PV is installed at the end of the feeder compared to the early nodes or if the rating of the PV is high. The sensitivity analysis of voltage imbalance (calculated at the end of the feeder) versus the location and rating of rooftop PV installed in low load phase C is illustrated in Fig. 6b.

The numerical results of the investigations prove that a rooftop PV with a rating less than 5 kW have minor effect on the network voltage imbalance at the beginning of the feeder (less than 0.1%) and not a significant effect (more than 15%) at the end of the feeders of the LV network that supplies up to 1 MW load. Since this is the most probable situation for all the urban residential distribution networks, it can be concluded that a specific location for a rooftop PV will not affect the voltage imbalance significantly. This can only be invalid if higher rating PV is installed at the end of a low load phase. Through the analysis of case 2, installing a high rating PV at the end side of a high load phase will have the best effect on the voltage imbalance improvement.

C. Case 3- Mutual effect of PVs on Voltage imbalance

After studying the sensitivity analysis of network voltage imbalance for one PV, it is necessary to study the mutual effects of PVs which are installed simultaneously on one phase of the network. In this case, the number of a constant rating PV has been increased in one phase of all three feeders assuming that there is no PV on other two phases.

The first study is for increasing the number of PVs installed on low load phase (phase A) of feeder 1, feeder 2 and then feeder 3. The result of this investigation is shown in Fig. 7a. This figure shows that the voltage imbalance, calculated at the beginning and end of feeder 1, will increase by increasing the number of PVs in one phase of all feeders. This increase is more obvious when calculated at the end of the feeder compared to the beginning of the feeder. This increase is shown for different ratings of PVs separately. It can be noted that by increasing the rating, the voltage imbalance at the end of the feeder, increases significantly. This figure also shows when PVs are installed on phase A of feeder 1, the voltage imbalance has a dramatic increase but when they are installed on phase A of feeder 2 or 3, still the voltage imbalance at the end of feeder 1 increases but with a slower rate.

![Fig. 5. Variation of phase A voltage profile versus the location and rating of the PV in phase A.](image)

![Fig. 6. Voltage imbalance sensitivity analysis versus PV location and rating in (a) low load phase- Phase A, (b) high load phase- Phase C.](image)
For studying the variation of voltage imbalance on other feeders, the voltage imbalance at the beginning and end of all three feeders are shown in Fig. 7b for the 5 kW installation. It can be seen that installing any PV on each feeder, will cause the voltage imbalance at the end of all feeders to increase but this increase is more significant when the PVs are installed on the same feeder. But this increase is almost the same at the beginning of the feeders when the PVs are installed at any of the three feeders.

The second study is carried out for high load phase (phase C) of feeder 1, feeder 2 and then feeder 3. The result of this investigation is shown in Fig. 8a. This figure shows that the voltage imbalance, calculated at the end of feeder 1, will decrease by increasing the installation number of PVs in one phase of all feeders up to a point, beyond which it will increase. It can be seen that if the number of 1-2 kW PVs increase, the voltage imbalance still decreases (as expected) but for 3-5 kW PVs, after a specific number of PVs installation, the voltage imbalance will increase. This happens due to this fact that if the power injected by PVs is more than the consumed power by the loads on that phase, the power will flow back to the distribution transformer and therefore, the voltage angle will differ a lot from previous situations. Thus, the angle of the voltage among three phases will start to have significant difference. This angle difference is dominant to voltage amplitude difference and will result in the voltage imbalance increase as their numbers exceed a limit. This limit is dependent on their total rating of the PVs in the network. The same results are obtained for the beginning of the feeder.

All the previous calculations were for feeder 1. For studying the variation of voltage imbalance on other feeders, the voltage imbalance at the beginning and end of all three feeders are shown in Fig. 8b for the 5 kW installations. It can be seen that installing any PV on each feeder, will cause the voltage imbalance at the end of all feeders to first decrease and then increase. But in both conditions, the most severe effect is when the voltage imbalance is calculated on the same feeders on which PVs are installed. These variations are almost the same at the beginning of the feeders when the PVs are installed at any of the three feeders.

D. Case 4- Stochastic Evaluation of voltage imbalance

Stochastic evaluation is a methodology that combines deterministic results with stochastic data to produce a probabilistic assessment. Monte Carlo simulation is a powerful numerical method of stochastic evaluation based on random input variables [31].

The inherent characteristic of LV distribution networks includes random variation of residential load demand and PV power generation at different time periods. This variation is based on load demand in a 24 hour daily pattern and summer-winter periods. In addition, the random location and nominal
power of PVs increase the randomness of the network.

For investigating the voltage imbalance in the network when there is a random combination of rooftop PVs with different ratings of 1.5 kW and at different location on all three phases and three feeders on the network, a stochastic evaluation based on Monte Carlo method presented in [31-32] has been carried out. Three random inputs of the stochastic evaluation are number of householders with installed rooftop PVs, ratings of the PVs and their location along three phases of three different feeders. It is assumed that everyone has an equal probability of PV installation. It is also assumed that the ratings of 1.5 kW have equal probability when installed by the householders. But the number of the householders with installed rooftop PVs is different in three Monte Carlo scenarios (i.e. ¼, ¼ and ⅔ of the householders have installed PVs in each of the simulation studies.) The objective of this probabilistic case study is calculating network voltage imbalance at the beginning and end of the feeder for different rooftop PV installation scenarios in LV residential distribution networks.

The voltage imbalance results as the output of Monte Carlo method are later used to calculate the probability density function (PDF) of voltage imbalance using the Poisson distribution function which can be expressed as

\[
PDF(k; \lambda) = \frac{\lambda^k e^{-\lambda}}{k!}
\]

where PDF is a discrete function, \( k \) is the number of occurrences of a specific VUF and \( \lambda \) is the average (mean value) of all VUFs.

For investigating voltage imbalance diversity, the studies are performed for cases when the PVs are installed by ¼, ½ and ⅔ of the householders. The probability density function of the voltage imbalance in all three cases is shown in Fig. 9. As seen in this figure, the probability density is almost the same for all three scenarios and \( \lambda = 0.36\% \) at the beginning of the feeder and \( \lambda = 1.84\% \) at the end of the feeder.

From Fig. 9, it can be seen that, there is a high probability that the voltage imbalance at the end of the feeder is more than the 2% standard limit. This probability is referred to as the risk index \( R_i \). Risk index can be explained as the frequency of the cases in the shaded area in probability density function (shown in Fig. 9) for values greater than standard voltage imbalance and is calculated from

\[
R_i = \text{shaded area} \times 100
\]

While voltage imbalance risk index is zero at the beginning of the feeder, it is about \( R_i = 33.5\% \) at the end of the feeder.

### VI. IMPROVEMENT METHODS

Based on the probabilistic and numerical results of Section V, it is proved that the voltage imbalance at the beginning of the feeder, regardless of the location, number and rating of the installed rooftop PVs, is mostly to be less than 1%. But this value is more than 2% standard level at 33.5% of the circumstances at the end of the feeders.

For improving the voltage imbalance at the end of the feeders, four improvement methods can be proposed. These methods which are based on voltage profile improvement in the feeders, can be divided into conventional and new control methods.

#### A. Feeder Cross-section Increase

The first improvement method is increasing the cross-section of the feeders to a higher size. This will result in reducing the voltage drop along the feeder and therefore, there will be little difference among the voltage amplitude of three phases of a feeder at the end. For verifying the efficacy of this method, another stochastic study has been carried out for the network with two different LV feeders (i.e. ABC 70 mm² and 95 mm²). The results of the study are shown in Fig. 10a. As it is seen in this figure, \( \lambda \) at the beginning of the feeder is remained almost constant \( (\lambda = 0.36\%) \) but has decreased from 1.84 to 1.56% at the end of the feeder.

![Fig. 9. Probability density function of voltage imbalance.](image)

#### B. Capacitor Installation

The second effective method is installation of pad mounted switched capacitors in the LV feeders. Based on IEEE Guide for Application of Shunt Power Capacitors, for uniformly distributed loads, the capacitor should be placed two-thirds of the distance from the distribution transformer [33]. It is important to note that if a three-phase capacitor is installed on a LV feeder, the voltage imbalance will almost remain the same. But if instead, three single-phase switched capacitors utilized, where just a single-phase capacitor is connected to the phase with voltage amplitude less than 0.95 pu, with the help of proper operation command of capacitor regulator, the voltage profile on that phase can be shifted up and made closer to the voltage amplitude of other phases. In this case, the voltage imbalance can be improved effectively. The results of Fig. 10b for the case, show that \( \lambda \) is decreased from 0.36 to 0.28% at the beginning and from 1.84 to 1.41% at the end of the feeder when a 15 kVar capacitor with the described characteristics, is installed at the 2/3 feeder length from the beginning of the feeder. Some other methods can be used to define the best capacitor placement for this purpose.

#### C. Cross-section Increase and Capacitor Installation

The third improvement method is combination of the previous two methods, i.e. increasing the cross-section of the feeder while installing three switched single-phase capacitors on the feeder. The voltage imbalance improvement in this method has much effective results compared to when each of them employee separately. In the studied case, it is decreased from
0.36 to 0.28% at the beginning and from 1.84 to 1.18% at the end of the feeder, as shown in Fig. 10c.

The risk index values of all the stochastic simulations for different improvement methods are shown in Table II. Studying the results of this table, it is obvious that application of the proposed new control scheme for the PV has the least risk index and is the most appropriate technical and cost effective method comparing to other improvement methods.

Table II. Risk Indices of Voltage Imbalance at the End of the Studied LV Distribution Network Feeder Based on Stochastic Simulations

<table>
<thead>
<tr>
<th>Risk Index (R_I %)</th>
<th>Cross-section Increase</th>
<th>Capacitor Placement</th>
<th>Cross-section Increase and Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8</td>
<td>2.3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

VII. CONCLUSIONS

A comprehensive voltage imbalance sensitivity analysis and stochastic evaluation based on the rating and location of single-phase grid-connected rooftop PVs in a residential LV distribution network were presented. The study investigated the voltage imbalance at different locations along a feeder and on other feeders of the network. Through the studies, it was proved that rooftop PV installation will have minor effect on the voltage imbalance at the beginning of a LV feeder designed with engineering judgments. However, it might increase at the end of the feeder to more than the standard limit. It was also proved that depending on the load of the phase in which the PV is installed, the voltage imbalance will increase or decrease based on the location and rating of the PVs. It was also proved that if a PV is installed on one feeder, the voltage imbalance will be modified on all the other LV feeders of the network. Based on the numerical results, a generalized characteristic of voltage imbalance of LV residential networks due to rooftop PV installation was presented. The stochastic simulation demonstrated that the risk index of non-standard voltage imbalance in these networks is high (33.5%). Improvement methods were investigated and their efficacies were verified by the stochastic and numerical results.

APPENDIX

Table III. Technical Parameters of the Studied LV Distribution Network

<table>
<thead>
<tr>
<th>Transformer</th>
<th>11/0.415 kV, 500 kVA, ( \Delta / Y ) grounded, ( x=0.04 ) pu.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feeder</td>
<td>70 mm2 Aerial Bundle Cable (( z=0.55+0.088 ) ( \Omega ) km)</td>
</tr>
<tr>
<td></td>
<td>50 mm2 ACSR, 2 km overhead line, (( z=1.08+0.302 ) ( \Omega ) km)</td>
</tr>
<tr>
<td>Rooftop PV</td>
<td>1-3 kW, unity power factor, L=5mH</td>
</tr>
<tr>
<td>Loads</td>
<td>1 kW, ( \cos \phi=0.95, z=51.0840+jx17.0863 ) ( \Omega ) (All on phase A)</td>
</tr>
<tr>
<td></td>
<td>2 kW, ( \cos \phi=0.95, z=25.9920+jx8.5432 ) ( \Omega ) (All on phase B)</td>
</tr>
<tr>
<td></td>
<td>3 kW, ( \cos \phi=0.95, z=17.3280+jx5.6954 ) ( \Omega ) (All on phase C)</td>
</tr>
</tbody>
</table>

REFERENCES


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