ENG460: Engineering final year thesis project

Improvements on a Simple Model of The Global Carbon Cycle

Date submitted:
Wednesday, 16 December 2009

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Abstract

This report describes improvements and changes made to a simple model of the global carbon cycle originally developed by Bice (2007). The model simulation period is from 1850 to 2000, and the steady state for atmospheric CO$_2$ concentration changed from 280ppm in the original model to 285ppm in the improved model. The original model has 5 reservoirs: atmosphere, surface ocean, deep ocean, land biota and soil. An additional 'earth' reservoir is added to the improved model. These reservoirs have various fluxes between them, and all are assumed to have equally distributed carbon, analogous to 'well-mixed tanks'.

Changes are made to the half-saturation value (effecting the Michaelis-Menten kinetics describing photosynthesis); and it is found that the temperature sensitivity of photosynthesis is best parameterised to reflect average measured land surface temperature of around 8°C. An improved method for accounting for anthropogenic changes due to burning of land biota is developed.

A global energy balance is incorporated into the model. This energy balance uses a ‘grey atmosphere’ approximation, as developed by Lenton (2000). Results of this energy model are in line with historical measured average global surface temperature (GST) anomalies from the Climate Research Unit (CRU) CRUTEM3 temperature dataset. However, it is shown that this energy model is somewhat unreliable, as the same ‘global warming’ effect can be reproduced by making small changes to planetary albedo, and possible changes to solar irradiation and relative humidity are not accounted for. Regardless of this, the energy model output for future predictions is in the same range as those made by the Intergovernmental Panel on Climate Change (IPCC) with more complex General Circulation Models (GCMs).

Acknowledgements

This thesis has been completed on exchange at Horscule Furtwangen University. Thank you very much to Professor Detlev Ringer for supervising the project and providing much appreciated guidance and assistance.
## Units and constants

A note on units: When applicable, or where not clear, units are specified in brackets to the right hand side of an equation or next to a specific term in an equation. When an absolute unit is not used, units are specified in terms of “length”, “time”, “volume” and “amount of substance” or “mass”. (Parameters abbreviations are attached in Appendix G)

### Units

<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>ppm</td>
<td>parts per million volume</td>
<td>$1 \times 10^{-6}$ per 1 unit of volume</td>
</tr>
<tr>
<td>Gt</td>
<td>gigatonnes</td>
<td>$1 \text{ Gt} = 1 \times 10^{15}$ grams</td>
</tr>
<tr>
<td>GtC</td>
<td>gigatonnes of carbon</td>
<td>$1 \text{ Gt} = 1 \times 10^{15}$ grams of carbon</td>
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<tr>
<td>GtC/yr</td>
<td>carbon flux in Gigatonnes per year</td>
<td>$1 \times 10^{15}$ grams per year</td>
</tr>
<tr>
<td>°C</td>
<td>Degrees Celsius</td>
<td>$273.15 \text{ K} = 0 \degree \text{C}$</td>
</tr>
<tr>
<td>K</td>
<td>Kelvin</td>
<td>$273.15 \text{ K} = 0 \degree \text{C}$</td>
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<td>millimoles</td>
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<td>W</td>
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<td>$1 \text{ W} = 1 \text{ J.s}^{-1}$</td>
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<tr>
<td>J</td>
<td>Joule (Energy unit)</td>
<td>$1 \text{ J} = \text{N.m}$ (1 Newton metre) $1 \text{ kg.m}^2/\text{s}^2$</td>
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<tr>
<td>µm</td>
<td>micrometre (or micron)</td>
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<tr>
<td>ppbv</td>
<td>parts per billion volume</td>
<td>$1 \times 10^{-9}$ per 1 unit of volume</td>
</tr>
<tr>
<td>mmol/kg</td>
<td>millimoles per kilogram</td>
<td>$1 \text{ mmol/kg} = 1 \times 10^{-3}$ moles per kg</td>
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### Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$\text{W m}^{-2}\text{K}^{-4}$</td>
<td>Stefan-Boltzmann constant   (5.67 \times 10^{-8} \text{W m}^{-2}\text{K}^{-4})</td>
</tr>
<tr>
<td>$Q$</td>
<td>Energy emitted by sun</td>
<td>$3.87 \times 10^{26} \text{W}$</td>
</tr>
<tr>
<td>$A_E$</td>
<td>Surface area of the Earth</td>
<td>$5.101 \times 10^{14} \text{m}^2$</td>
</tr>
<tr>
<td>$r$</td>
<td>Average radius of the Earth</td>
<td>$6.371 \times 10^6 \text{m}$</td>
</tr>
<tr>
<td>$c$</td>
<td>Earth’s specific heat capacity</td>
<td>$4.69 \times 10^{23} \text{J. K}^{-1}$</td>
</tr>
<tr>
<td>$A_{LB}$</td>
<td>Surface area of the land biota reservoir</td>
<td>$1.33 \times 10^{14} \text{m}^2$</td>
</tr>
<tr>
<td>$A_S$</td>
<td>Surface area of the soil reservoir</td>
<td>$1.33 \times 10^{14} \text{m}^2$</td>
</tr>
<tr>
<td>$V_S$</td>
<td>Volume of the soil reservoir</td>
<td>$6.65 \times 10^{14} \text{m}^3$</td>
</tr>
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Introduction

“So it happens that every element says something to someone… One must perhaps make an exception for carbon, because it says everything to everyone.”

Primo Levi

Carbon is one of the key elements for life on Earth. The ability of the carbon atom to make bonds with itself and form a vast variety of different compounds is crucial to our existence. The term ‘carbon cycle’ refers to the dynamic network of carbon transfer, where carbon is exchanged between global reservoirs such as the atmosphere and surface ocean. This cycle rotates all carbon in existence; so, for example, carbon atoms present in a human body may one day have been in molecules of carbon dioxide (CO$_2$) in the atmosphere, or calcium carbonate (CaCO$_3$) in the shell of marine organisms. Understanding of the carbon cycle is important in the context of assessing the impacts of the anthropogenic carbon emissions since the industrial era. The diagram below illustrates the major reservoirs and fluxes operating in the carbon cycle.

The global carbon cycle (NASA, 2007)
This project is centred on the evaluation and possible ameliorations of a simple global carbon cycle model developed by Bice (2007) (the equations for this original model are in Appendix A.2). Bice’s simple model contains 5 reservoirs and only a single equation describing the energy transport. Evaluation of Bice’s simple global model leads to some adjustments to model parameters and various additions to formulate a new ‘improved model’.

Modelling of the carbon cycle has generally been carried out in simple box and box-diffusion models. The Bice model is of this general type, where reservoirs of carbon (such as the land biota or surface ocean) are represented as perfectly mixed ‘boxes’. Flows of carbon in and out of the reservoirs are dependent on carbon concentration in these boxes (analogous to a ‘well-mixed tank’). Numerous simple models have been developed in this way, either of ocean-atmosphere interactions, the terrestrial carbon cycle, or a combined global cycle. Some of these models include energy balances or relationships in addition to the mass flow of carbon, such as Kwon & Schnoor (1994) and Lenton (2000). The energy relationship of Kwon & Schnoor is used in Bice’s simple model, whereas the global energy balance of Lenton using a grey atmosphere approximation is incorporated into the improved model.

A reason for the development of ‘simple models’ is that they are easier to follow, require less computational power and are more straightforward, while still exhibiting the main characteristics and changes that a more complex model may show. Also, simple models may also better suit the assumption of ‘well-mixed tanks’. The intention of this project is to keep the model as simple as possible, while still producing output reflective of measured data.

The timescale for modelling the carbon cycle can vary greatly, from processes occurring over many millennia on a geological timescale, to those occurring within seconds or minutes, such as air-ocean gas exchange. The model discussed in this work is on a decadal timescale. Processes acting on a longer timescale, such as calcium carbonate dissolution or the weathering of carbonate rocks (effecting atmospheric CO₂ on a multi-millennial time scale), are ignored as they have little or no effect on the processes occurring over the timescale considered. Also, processes acting on a shorter time scale (such as seasonal changes) are assumed to be levelled out in the model.
A key assumption in this model is that of a ‘semi-steady-state’ for several hundred years prior to the onset of the ‘industrial era’ and its associated anthropogenic perturbations. This is valid as the simple model in question only looks at the cycle on a time-scale of decades, as opposed to centuries or millennia, in which elements of the cycle are changing. The ‘industrial era’ is generally considered to begin around 1750, as atmospheric CO$_2$ concentration hovered around 280ppm for a considerable period prior to this. From 1750 onwards, increased atmospheric CO$_2$ has mainly been attributed to anthropogenic effects. A slightly different steady-state has been used in this model, with simulations starting from 1850 at an atmospheric CO$_2$ concentration of 285ppm. The reason for this is that historical records indicate a concentration of 285ppm between 1800 and 1850. Temperature datasets for verification purposes are only available from 1850, hence selection of 1850 as the starting year for simulations, as opposed an earlier date where atmospheric CO$_2$ concentration was at the generally accepted pre-industrial steady state of 280ppm.

An issue with carbon cycle modelling is the availability of empirical data for model verification. A model requires measured data to compare with model outputs. The two main time series of measurements available are that of average global surface temperature (GST) and atmospheric CO$_2$ concentration. Values for carbon fluxes and reservoir sizes given in literature are not measurements – these are estimates based on some type of model or accounting procedure. Global average surface temperatures have little meaning, according to some sources (Essex et al., 2007), so atmospheric CO$_2$ measurements from recent times and historical methods of finding CO$_2$ such as ice-core dating are the primary way of validating the model. However, temperature data is essential for evaluation and verification of the energy balance.
1. Simple model evaluation

There are 5 carbon reservoirs in the simple model of Dave Bice: atmosphere, surface ocean, deep ocean, land biota and soil. An additional ‘earth’ reservoir has been added to these (Ringer, 2009).

The fluxes between these are as follows:

- Photosynthesis and plant respiration (between atmosphere and land biota)
- Litter fall (from land biota to soil reservoir)
- Upwelling and downwelling (between surface ocean and deep ocean)
- Soil respiration (from soil to atmosphere)
- Burial (from deep ocean to Earth reservoir)
- Runoff (from soil to surface ocean)
- The biological pump (from surface ocean to deep ocean)
- CO₂ exchange between surface ocean and atmosphere
- External additions to the atmosphere, such as metamorphism and volcanoes (earth to atmosphere)

The reservoirs and fluxes in the simple model are illustrated in Figure 1.1, with carbon fluxes due to anthropogenic effects highlighted in red and added to the existing natural network of carbon fluxes.

![Diagram](image)

**Figure 1.1:** Diagram illustrating the reservoirs and fluxes in Bice’s simple model with the addition of the earth reservoir, following Ringer (2009) (own illustration)
1.1. Mass balances

The changes of carbon in the reservoirs with time are derived from mass balance difference equations, where units of the result are in GtC/yr. (added anthropogenic carbon fluxes are indicated in brackets):

\[
\frac{\Delta SR}{\Delta t} = F_{LF} - F_{SR} - F_{run} \quad (-F_{SD})
\]

Eq. 1.1: Change of soil reservoir carbon mass (SR) with time

\[
\frac{\Delta LB}{\Delta t} = F_{P} - F_{LF} - F_{R} \quad (-F_{burn})
\]

Eq. 1.2: Change of land biota reservoir carbon mass (LB) with time

\[
\frac{\Delta Atm}{\Delta t} = F_{g} + F_{SR} + F_{EX} - F_{P} - F_{atm-ac} \quad (+ F_{FF} + F_{SD} + F_{burn})
\]

Eq. 1.3: Change of atmosphere reservoir carbon mass (Atm) with time

\[
\frac{\Delta Earth}{\Delta t} = F_{burial} + F_{EX} \quad (-F_{FF})
\]

Eq. 1.4: Change of earth carbon reservoir mass with time

\[
\frac{\Delta DO}{\Delta t} = F_{DW} + F_{BIO} - F_{UW} - F_{burial}
\]

Eq. 1.5: Change of deep ocean reservoir carbon mass (DO) with time

\[
\frac{\Delta SO}{\Delta t} = F_{atm-ac} + F_{UW} + F_{run} - F_{DW} - F_{BIO}
\]

Eq. 1.6: Change of surface ocean reservoir carbon mass (SO) with time

1.2. Carbon fluxes

As Bice does not explain the derivation of the model equations, these have been investigated with the purpose of determining conformity with general mass transfer principles. The original simple global model equations for Bice’s model are attached in Appendix A.2. The terrestrial component of Bice’s model is taken from Gifford (1993), and the oceanic component from that developed by Walker (1991). As it was not directly clear how Bice derived the terrestrial model following Gifford, this can be referred to in Appendix A.1
The diffusive fluxes (across a phase boundary) are photosynthesis, plant respiration, soil respiration (which soil disruption increases) and CO₂ exchange between the ocean and atmosphere. The remaining fluxes are not diffusive, and carbon transfer is caused through convective mass transfer or due to gravitational forces.

The general principles that need to be considered are that of Fick’s Law applied to gas diffusion, where the following equation applies:

\[
J = -D \frac{\partial C}{\partial x} \quad \left\{ \frac{\text{mass}}{\text{length}^2 \text{year}} \quad \text{eg.} \quad \frac{GtC}{m^2 \text{year}} \right\}
\]

**Eq. 1.7: Fick’s law as applied to gas diffusion (Kruse, 2003)**

\[ \Delta M = A \left( \frac{\Delta J}{\Delta t} \right) = A \left( -D \frac{\Delta C}{\Delta x} \right) \]

**Eq. 1.8: Fick’s law as used with the difference equations in the model**

The diffusive fluxes in the model include an implied area in the initial flux estimates in GtC/yr, or in the case of ocean-atmosphere transfer, the surface area is included with the transfer velocity (explained further below). The diffusion coefficient D is generally coupled with the length, \( \Delta x \), over which the transfer occurs, as this length is hard to define and is assumed not to change from one timestep to the next.

The expression Bice used for ocean-atmosphere transfer follows Fick’s law, as demonstrated below. The explanation of the carbonate chemistry which this transfer is dependent on is described in Appendix B.1. Expressions for soil respiration and photosynthesis/plant respiration are not explained in terms of mass transfer by Bice, so these are further investigated. The equations are verified to be compliant with mass transfer laws.
Convective mass transport and gravity are the driving forces for carbon fluxes other than those stated in the above section. Litter fall occurs simply by dead land biota products falling to become part of the soil reservoir. Downwelling occurs due to higher density water in cold regions sinking to the deep ocean, and upwelling is due to surface currents and wind moving surface waters, leaving a ‘gap’ for deep waters to move to the surface. The biological pump and burial are both due to the weight of substances causing them to sink.

1.3. Key model assumptions

Although not all specified directly by Bice, there are some key assumptions that apply to this model.

Firstly, the reservoirs are assumed to be ‘well-mixed tanks’ with carbon mass equally distributed across the area of volume and over the time step of the reservoir. This assumes very fast dynamics in the mass flows mixing with the rest of the carbon existing in a reservoir (usually several mixing periods within a timestep).

The reservoirs will be assumed to have a constant size, but a changing content of carbon. For example, the land biota reservoir is assumed to encompass the entire land surface area in which photosynthesis could potentially occur. If anthropogenic changes are made, such as burning of a particular area, this does not reduce the reservoir surface area, it only removes the mass of carbon from that surface area.

For photosynthesis, the expressions describing global CO₂ uptake are the same as what would be used in an individual plant. The assumption here is therefore that the equation is applicable on a global level. For convective fluxes dependent on carbon mass in a reservoir, it is assumed that the flux will change proportional to carbon mass changes – mass changes represent a concentration change as volume and area are considered constant.
This model uses a minimum number of reservoirs. The ocean reservoir, for example, may be divided into the warm surface ocean, cold surface ocean, intermediate waters and deep ocean, or even more substructures, as opposed to only two ‘boxes’ as used in the simple global model (surface and deep ocean). It is assumed that representing the reservoirs as the sum of these individual reservoirs lumped together will still capture the necessary dynamics of the system. This representation means the model will better fit the ‘well-mixed tank’ assumption.

1.4. Ocean-atmosphere carbon transfer

The ocean-atmosphere transfer is described in the model by the equation below, with \( p_{\text{CO}_2,\text{atm}} \) and \( p_{\text{CO}_2,\text{oc}} \) being the \( \text{CO}_2 \) concentration in the atmosphere and ocean respectively (in ppm). The values 0.278 GtC/yr/ppm for \( k_{\text{oa}} \) used by Bice was obtained experimentally

\[
F_{\text{atm/oc}} = k_{\text{oa}} \times (p_{\text{CO}_2,\text{atm}} - p_{\text{CO}_2,\text{oc}}) \quad \text{(GtC)} \quad \text{(yr)}
\]

Eq. 1.9: Bice’s equation for the CO\(_2\) flux across air-sea per unit area, in g/m\(^2\) (Bice, 2007)

This expression describes the transfer using Fick’s Law, in what is known as the ‘stagnant film model’ for ocean-atmosphere gas transfer (Johnson, 2000). This general equation is as follows:

\[
\text{Flux} = \frac{D}{\Delta z} \times (CO_{2,\text{atm}} - CO_{2,\text{oc}})
\]

Eq. 1.10: \( \text{CO}_2 \) flux across air-sea per unit area, in g/m\(^2\) (Bice, 2007)

The term \( D/\Delta z \) is referred to as the piston velocity or transfer velocity. In Bice’s expression (Eq. 1.9), the constant \( k_{\text{oa}} \) has the value of 0.278 GtC/yr/ppm, encompassing both this piston velocity (given as 0.768 g CO\(_2\)/m\(^2\)/year/ppm), and the area of the surface ocean, to give a result for Eq. 1.9 in GtC/yr, for the entire ocean area. The implied area is \( 3.6198 \times 10^{14} \text{ m}^2 \), (or 70.7% of the Earth entire surface area, approximately \( 5.101 \times 10^{14} \text{ m}^2 \)), which is similar to other estimates obtained of sea surface area.
As it is difficult to determine it theoretically, the piston velocity is estimated experimentally, and then the flat sea surface area used to define the overall $k_{\text{in}}$ constant for use in Eq. 1.9. The value for piston velocity is assumed to be an average value over long time ranges and total area, taking into consideration factors such as waves and wind speed, which affect the transfer rate.

The transfer of CO$_2$ is driven by the concentration gradient between the atmosphere and the surface ocean. Atmospheric CO$_2$ concentration is assumed to be equal over the globe (analogous to the well mixed tank). Over time, it is dependent on a number of factors – temperature, soil respiration, photosynthesis, plant respiration and anthropogenic changes all have a direct effect. The CO$_2$ concentration in the surface ocean is also assumed to be globally equal. It is dependent on the carbonate chemistry of seawater, an important component of the model.

### 1.4.1. Seawater carbonate chemistry

When atmospheric CO$_2$ is dissolved in the ocean, the reactions that occur involve carbonic acid (H$_2$CO$_3$), carbonate ions (CO$_3^{2-}$), bicarbonate ions (HCO$_3^-$) and salt ions dissolved in water:

$$\text{CO}_2(\text{g}) + \text{H}_2\text{O} \rightleftharpoons \text{H}_2\text{CO}_3 \rightleftharpoons \text{HCO}_3^- + \text{H}^+ \rightleftharpoons 2\text{H}^+ + \text{CO}_3^{2-}$$

These reactions go in both directions and are a primary reason for considering the ocean as a net sink or a net source of atmospheric CO$_2$. As the reactions are all relatively fast the system usually exists in equilibrium (Bigg, 2003). Additional CO$_2$ in the atmosphere is constantly dissolved as bicarbonate and carbonate ions form to be in equilibrium with the atmospheric gas phase of CO$_2$. The equilibrium is dependent on temperature, alkalinity and the total carbon concentration of the ocean. The two component reactions forming carbonate and then bicarbonate ions have reaction rates $K_1$ and $K_2$ (disassociation constants), which are further described in Appendix B.1.

These reaction constants are dependent on temperature, pressure and salinity and pH of the water. For modelling purposes, an expression for the equilibrium concentration of CO$_2$ gas in seawater is needed. This concentration is dependent on the ratio of bicarbonate and carbonate ions to one another in the water, among other factors.
The concentration of bicarbonate ions in much higher than any of the other components in the
reaction, with the reaction constant $K_1$ being several orders of magnitude greater than $K_2$, so the
essential overall net reaction is:

$$CO_2 (g) + H_2O + CO_3^{2-} \rightleftharpoons 2HCO_3^-$$

The expression for $CO_2$ partial pressure in the ocean resulting from these relationships is shown
below. The partial pressure of $CO_2$ is commonly represented in ppm as opposed to moles per m$^3$ of
water: (Bice, 2007)

$$pCO_2 = kCO_2 \times \frac{[HCO_3^-]^2}{[CO_3^{2-}]} \quad \{ppm\}$$

Eq. 1.11: Expression for partial pressure of $CO_2$ in the ocean in ppm (Bice, 2007)

The coefficient $kCO_2$, carbonate and bicarbonate ion concentrations in this expression for use in
the model are given by:

$$kCO_2 = 0.035 + 0.0016 \frac{T_{so} - 278}{K} \quad \{kg/mmol\}$$

Eq. 1.12: Expression for combined reaction rate (Bice, 2007)

where: $T_{so}$ is the global average temperature of the surface ocean, and 278 K represents
the reference temperature for the ocean, K represents Kelvin (ie. 0.0016 per 1 degree K)

The expressions for the bicarbonate ions $[HCO_3^-]$ and the carbonate ions $[CO_3^{2-}]$ are given as:

$$[HCO_3^-] = \frac{\sum CO_2 - \sqrt{\sum CO_2^2 - Alk(2 \times \sum CO_2 - Alk) \times (1 - 4 \times K_2/K_1)}}{(1 - 4 \times K_2/K_1)} \quad \{mmol\}$$

Eq. 1.13: Expression for bicarbonate ions in the ocean (Bice, 2007)

$$[CO_3^{2-}] = \frac{Alk - [HCO_3^-]}{2} \quad \{mmol\}$$

Eq. 1.14: Expression for carbonate ions in seawater (Bice, 2007)

where: Alk is the average alkalinity of the seawater, 2.22 mmol/kg for $T_{so} = 15^\circ C$; $\sum CO_2$ is
the sum of inorganic carbon in the surface ocean

The parameters for these equations and further explanation are contained in the Appendix. The
expression for ocean-atmosphere exchange (Eq. 1.9) therefore does apply for Fick’s law for
diffusion, with the necessary concentration of $CO_2$ in the ocean for the concentration difference
being calculated from the carbonate chemistry of seawater.
1.5. Photosynthesis carbon transfer

Photosynthesis is the process in plants through which CO\(_2\), water and light energy are converted to carbohydrates, in reactions such as:

\[
6\text{CO}_2 + 6\text{H}_2\text{O} + \text{light energy} \rightarrow \text{C}_6\text{H}_{12}\text{O}_6 + 6\text{O}_2
\]

Atmospheric CO\(_2\) is taken into the plant leaves, and the rate at which this happens depends on a number of factors. Generally, if the plants have sufficient water, the rate of photosynthesis will increase with a greater atmospheric CO\(_2\) concentration. This effect is referred to as CO\(_2\) fertilisation. The rate of photosynthesis also depends on temperature and the presence of nutrients in the soil. Further details regarding the process of photosynthesis are detailed in Appendix B.2.

There are two main types of plants – C\(_3\) plants, in which photosynthesis increases with atmospheric CO\(_2\) to a fairly high CO\(_2\) level, and C\(_4\) plants, in which photosynthesis does not increase in high CO\(_2\) levels. Approximately 95% of land biota is composed of C\(_3\) plants, with the remaining 5% C\(_4\) plants. (Lenton, 2000) In the model it is assumed that all of the land biota is composed of C\(_3\) plants, in which photosynthesis rate increases with atmospheric CO\(_2\).

The photosynthetic carbon transfer is modelled by Bice as follows:

\[
P_{\text{max}} = \frac{K_{hs} + (p\text{CO}_{2,\text{atm, ini}} - p\text{CO}_{2,\text{atm, min}})}{(p\text{CO}_{2,\text{atm, ini}} - p\text{CO}_{2,\text{atm, min}})} \ast F_{p,\text{ini}} \ast \left\{ \frac{\text{GtC}}{\text{yr}} \right\}
\]

**Eq. 1.15: Equation to calculate \(P_{\text{max}}\), the maximum possible carbon transfer by photosynthesis, for use in the equation below (Bice, 2007)**

where:  
- \(K_{hs}\) = half-saturation value in ppm, \(p\text{CO}_{2,\text{atm, ini}}\) = initial atmospheric CO\(_2\) concentration in ppm (pre-industrial level); \(p\text{CO}_{2,\text{atm, min}}\) = minimum atmospheric CO\(_2\) for photosynthesis to occur (compensation point) in ppm; \(F_{p,\text{ini}}\) = initial photosynthetic carbon transfer at the initial conditions

\[
F_p = P_{\text{max}} \ast \frac{p\text{CO}_{2,\text{atm}} - p\text{CO}_{2,\text{min}}}{(p\text{CO}_{2,\text{atm}} - p\text{CO}_{2,\text{min}}) + K_{hs}} \ast \left\{ \frac{(\theta_p \ast \Delta T)}{\text{yr}} \right\}
\]

**Eq. 1.16: Photosynthesis expressions used in Bice’s model (Bice, 2007)**

where: \(p\text{CO}_{2,\text{atm}}\) = atmospheric CO\(_2\) concentration in ppm; \(\theta_p\) = temperature sensitivity of photosynthesis °C\(^{-1}\); \(\Delta T\) = change in reference temperature from initial conditions
The part of the equation describing the effect of increased atmospheric CO$_2$ concentration (CO$_2$ fertilisation) does not include any explicit term for the effective CO$_2$ concentration inside the leaf (where the carbon is transferred to), or the carbon concentration in the reservoir, so there is no obvious concentration difference in this sense.

When considering the photosynthesis process, there is some form of CO$_2$ mass transfer from the atmosphere. Fick’s law describes the rate of physical diffusion of CO$_2$ gas from the atmosphere to the site of carboxylation in the chloroplast, where photosynthesis occurs:

$$P = \frac{C_a - C_c}{\sum r} \left\{ \frac{\text{amount of substance}}{\text{length}^2 \cdot \text{time}} \right\}$$

Eq. 1.17: Carbon flux between the air and chloroplast (Lommen et al., 1971)

where: $P = \text{flux of CO}_2$ from atmosphere to chloroplast; $C_a = \text{CO}_2$ concentration in atmosphere; $C_c = \text{CO}_2$ concentration in chloroplast (site of carboxylation); $\sum r = \text{sum of resistances in pathway}$.

The rate of photosynthesis (chemical process of CO$_2$ fixation) is then dependent on the CO$_2$ concentration in the chloroplast (where the CO$_2$ fixation occurs) and may be described through what is known as a Michaelis-Menten equation:

$$P = \frac{P_M}{1 + \frac{K}{C_c}} = \frac{P_M C_c}{C_c + K} \left\{ \frac{\text{amount of substance}}{\text{length}^2 \cdot \text{time}} \right\}$$

Eq. 1.18: Chemical rate of CO$_2$ fixation described by M-M kinetics (Lommen et al., 1971)

where $P_M = \text{rate of photosynthesis at saturating } C_c$; and $K = \text{half saturation value, a constant equal to } C_c \text{ when } P = P_M/2$.

Following Lommen (1971) and rearranging Eq. 1.17 in terms of chloroplast concentration ($C_c$) and substituting into Eq. 1.18 gives the following result in terms of $P$, where $R$ is the sum of resistances in the pathway:

$$P = \frac{(C_a + K + RP_M) - [(C_a + K + RP_M)^2 - 4C_aRP_M]^{0.5}}{2R}$$

Eq. 1.19: Rearranging in terms of $P$ (Lommen et al., 1971)

As $R \to 0$, the limit of this equation is given by:

$$\lim_{R \to 0} P = \frac{P_M}{1 + K/C_A} = \frac{P_M C_A}{C_A + K}$$

Eq. 1.20: Limit of $P$ as $R \to 0$ (Lommen et al., 1971)
The limit shown in the above equation is the same expression used to quantify the CO₂ fertilisation effect on photosynthesis rate in Bice’s equation. This shows the photosynthesis rate equation does comply with Fick’s law for diffusion, but only when considering the resistance to CO₂ transfer between the atmosphere and chloroplast to be zero.

However, observing Bice’s equation, it can be seen that the initial rate of photosynthesis is set through $P_{\text{max}}$ by the parameter $F_{\text{p,ini}} = 100 \text{ GtC/yr}$. This figure is assumed to account for resistance in CO₂ transfer as it is the gross value for global photosynthesis or CO₂ fixation in the initial state of the model. If the assumption is then made that there will be no changes to overall resistance, multiplying this factor by the rate changes due to atmospheric CO₂ levels with no resistance factor is appropriate, as the resistance is not changing.

The above equation for photosynthetic carbon transfer considering resistance to be zero (Eq. 1.20) has units mass per unit area, per unit time (eg. GtC/m²/year). In Bice’s expression, the units of the photosynthesis carbon transfer are in units of GtC/yr. This is because the initial value for photosynthetic carbon transfer, $F_{\text{p,ini}}$, encompasses an area – it is 100 GtC/yr over the entire land biota surface area. Thus, it is shown that the photosynthesis expression used by Bice does comply with mass transfer laws, and is adequate for use in the model.

The assumption is made by Bice that, with sufficient water and nutrients, photosynthesis will increase if atmospheric CO₂ concentration increases – indeed, this is a generally accepted hypothesis, as long as atmospheric CO₂ is above the compensation point ($p_{\text{CO}_2,\text{min}}$). The CO₂ compensation point is dependent on temperature. Bice uses a constant value for this, 30ppm, calculated following Gifford (1993) using the following equation evaluated at a temperature of 17°C:

$$CO_{2,\text{min}} = (0.511 \times 10^6) \cdot e^{(-\frac{2815.5}{T+273.2})}$$

Eq. 1.21: Expression for finding the CO₂ compensation point (Gifford, 1993)

The rate of respiration is assumed to be a constant proportion of the photosynthesis rate, thus is effected by the parameters in the photosynthesis equations in the same way.
1.5.1. Photosynthesis temperature dependency

Bice models the temperature dependency of photosynthesis with a linear expression:

\[ F_p = F_{c,0} (1 + (\theta_p \times \Delta T)) \]

Eq. 1.22: Photosynthesis temperature dependency used by Bice (Bice, 2007)

where: \( \Delta T \) is temperature change from the reference temperature; \( \theta_p \) is the temperature sensitivity (\(^{\circ}\text{C}^{-1})\); and \( F_{c,0} \) is the flux of photosynthesis calculated using only the CO\(_2\) fertilisation part of the photosynthesis expression.

Experimental observations show that at low-light levels, temperature does not have much effect on photosynthesis, but if light intensity is high enough for it not to be rate-limiting, photosynthesis will increase with increasing temperature (Huxman, 2008). This is because high temperatures give the enzymes responsible for photosynthesis more energy, causing the CO\(_2\) fixation reactions in the chloroplasts to occur faster. The assumption in this model is therefore that light intensity is constant at a sufficient level for photosynthesis to increase with temperature. In addition to this, the temperature sensitivity parameterises precipitation to a degree. Higher temperatures are generally assumed to mean increased precipitation, which contributes to higher levels of photosynthesis, as water availability is often a limiting factor.

At very high temperatures, photosynthesis will decrease for two main reasons: the enzymes responsible for CO\(_2\) fixation will deform and lose their shape, not being able to function properly any more; and the stomata (pores allowing CO\(_2\) into the chloroplasts from the atmosphere) will close to retain water, inhibiting the diffusion of CO\(_2\) from the atmosphere. The optimum temperature for photosynthesis is generally around 25\(^{\circ}\text{C}\) to 30\(^{\circ}\text{C}\). Average global temperatures of levels higher than this (that would start to inhibit photosynthesis) will not be reached in the scope of this model, thus approximating the temperature sensitivity with a linear function is acceptable for the temperature ranges expected in this model (Quist, 2007). If the model incorporated, for example, the varying temperature with latitudinal belts, this would not be appropriate as temperatures in some regions may exceed this optimum temperature.

Bice uses a value of \( \theta_p = 0.04 \) in his model. Some adjustments have been made to the modelling of temperature sensitivity parameters, which are further described in section 3 (Model Improvements).
1.5.2. Photosynthesis and the land biota reservoir

In Bice’s simple model, photosynthesis is not dependent on the size of the land biota reservoir, and does not increase if the carbon mass in the land biota increases. When not considering any other factors, it might be a reasonable assumption, that the whole plant (volume) takes part in photosynthesis and that therefore an increase in carbon in the reservoir should result in a proportional increase in photosynthesis. Addition of carbon implies addition of plants able to carry out photosynthesis, so if there are more plants, it would be expected that photosynthesis increases. A suggestion is made in Bice’s discussion to experiment with having photosynthesis dependent on the land biota reservoir carbon mass.

This is included by having a factor in the photosynthesis expression for the relative change in land biota size from the initial conditions (highlighted in bold), so that the photosynthesis expression would be:

\[ F_p = P_{max} \times \frac{pCO_{2,atm} - pCO_{2,min}}{(pCO_{2,atm} - pCO_{2,min}) + K_{hs} \times (1 + (\theta_p \times \Delta Global Temp))} \times \left( \frac{LB}{LB_{ini}} \right) \times \left( \frac{GtC}{yr} \right) \]

Eq. 1.23: Photosynthesis flux dependent on changes in land biota (Bice, 2007)

LB refers to the size of the land biota reservoir and LB_{ini} refers to the initial size. Although the reasoning that photosynthesis should increase with LB size appears reasonable, considering the fact that the land biota reservoir is already quite large, and that it is limited by other factors such as nutrient availability means that the land biota will not increase like this, so making photosynthesis (and therefore respiration) dependent on LB is not representative of what would actually happen.

The comparison of atmospheric CO₂ model output with photosynthesis dependent and independent of land biota size is shown in Figure 1.2 (the model used is a version of the improved model).
When photosynthesis is set to be dependent on the size of the land biota, it increases more rapidly, as shown in the graph of photosynthesis flux. The $F_p \propto \text{LB/LBini}$ results for atmospheric CO$_2$ are much further away from the measured data than photosynthesis independent of land biota size. $F_p$ will therefore be left as independent of land biota size in the model. This will adequately represent the situation for the current conditions. Obviously, if for some reason all land biota was removed, then having photosynthesis at a value independent of the size would not be adequate. For the time scale and conditions in this model, ignoring the change of reservoir size in calculating photosynthesis is adequate.
1.6. Soil Respiration

In the soil reservoir, the many micro-organisms present respire and thus create a flux of carbon to the atmosphere. This respiratory output is dependent on a number of factors. Temperature, water availability, nutrient availability and carbon content of the soil are four key parameters. A diagram of the factors affecting CO$_2$ efflux from the soil in a more complex model is shown in Figure 1.3

![Diagram of factors affecting CO$_2$ efflux from the soil](image)

**Figure 1.3: Factors effecting CO$_2$ flux from the soil in a more complex model (Nakayama et al., 1994)**

Bice’s model simplifies what is happening significantly, as shown in his expression for modelling the carbon flux due to soil respiration:

$$F_{SR} = 49.4 \text{ GtC yr}^{-1} \times \frac{SR}{SR_{initial}} \times (1 + (\theta_{SR} \times \Delta T)) \times \frac{\text{GtC yr}^{-1}}{\text{yr}}$$

Eq. 1.24: Expression for soil respiration rate used in Bice’s model (Bice, 2007)

SR refers to the size (in GtC) of carbon mass in the soil reservoir, and $SR_{initial}$ is the initial size of this reservoir (1580 GtC). The temperature effect is modelled by $\theta_{SR}$, the temperature sensitivity of soil respiration to the change from the reference temperature, $\Delta T$.

The initial value for soil respiration ($F_{SR_{ini}} = 49.4$ GtC/yr) is assumed to be a function of all the effects shown in Figure 1.3. This initial value is then multiplied by factors relating to temperature
and change in carbon mass in the soil (change in carbon mass is equivalent to change in concentration, as volume is constant). This implies that all factors effect soil respiration in the same way for a change in temperature or change in carbon mass. That is, none of these other factors are assumed to limit the soil respiration, so it will increase with temperature or carbon content. The factors influencing the change in soil respiration in Bice’s model are shown in Figure 1.4.

Figure 1.4: Factors effecting soil respiration in Bice’s simple model

The change in carbon content of the soil reservoir is a parameter (SR/SR\text{ini}) – so respiration increases with carbon concentration if the soil surface area is considered fixed, but there is no term describing the concentration gradient with the atmospheric reservoir the CO\textsubscript{2} is transferred to.

Most organic carbon exists in the upper layer of the land surface to a depth of about 5m, (Kwon & Schnoor, 1994) but there is no expression for surface area or volume in the soil respiration expression. This is implicitly included in the equation, which effectively just calculates the increase/decrease in respiration from the steady state value of 49.4 GtC/yr. Looking at a general expression for soil respiration in terms Fick’s law, it would be:

$$J_{SR} = -D_{SR} \left( \frac{m^2}{year} \right) * \frac{\partial C}{\partial x} = -D_{SR} * \left( \frac{C_{\text{mass, soil}}}{V_{\text{Vol, soil}}} - \frac{C_{\text{mass, atm}}}{V_{\text{Vol, atm}}} \right) \Delta z$$

Eq. 1.25: Fick’s law as applied to CO\textsubscript{2} transfer through soil respiration

where: $J_{SR}$ is the flux in Gt/m\textsuperscript{2}/year; $D_{SR}$ is the diffusion coefficient (unknown but assumed constant); $C_{\text{mass}}$ refers to the carbon mass in the soil and atmosphere reservoirs; $\Delta z$ is the distance over which the diffusion occurs
\[ F_{SR} = J_{SR} \left( \frac{Gt}{m^2 \text{ year}} \right) \times A_{SR} (m^2) \]

**Eq. 1.26: Flux of carbon through soil respiration using Fick's law and surface area**

As there is assumed to be no resistance on the side of the atmosphere (CO\(_2\) concentration in the atmosphere is too low to have any effect) the term “C\(_{\text{mass atm}}\) / Vol\(_{\text{atm}}\)” is effectively zero. In the model, the initial value of \(F_{SR}\) is used and the only parameter changing in Eq. 1.25 is the mass of carbon in the soil reservoir. The mass of carbon in the atmosphere is zero as it is too low to have an effect on the diffusion of CO\(_2\). The effective diffusion coefficient, \(D_{SR}\) (m\(^2\) / year) is assumed to be constant.

The surface area and depth of the soil reservoir are also assumed constant, although not all of this volume may contain carbon to contribute to soil respiration. The term ‘\(\Delta z\)’ refers to the distance or resistance (units length) over which the transfer will occur. This value would be part of the initial value for soil respiration flux, and assuming that this stays the same, will not need to be included when calculating changes to soil respiration.

To show this, if we consider the change in soil respiration for a change in total carbon mass in the carbon reservoir:

\[ J_{SR1} = -D_{SR} \times \frac{\partial C_1}{\partial X} = -D_{SR} \left( \frac{C_{mass1_{soil}}}{Vol_{soil}} \right) \]

**Eq. 1.27: Rate of soil respiration per unit area**

The only value changing in this expression is the carbon mass in the soil reservoir (volume of the reservoir is constant). The CO\(_2\) concentration in the atmosphere is very small, and if the \(\Delta z\) term and diffusion coefficient are considered constant, then the expression for flux with increase carbon in the reservoir will be:

\[ J_{SR2} = -D_{SR} \times \frac{\partial C_2}{\partial X} = -D_{SR} \left( \frac{C_{mass2_{soil}}}{Vol_{soil}} \right) \]

**Eq. 1.28: Carbon flux per unit area with an increased in carbon mass in soil reservoir**
If we relate the first and second fluxes by a factor, N, it can be seen that the fluxes are related by the change in carbon mass in the reservoir:

\[ F_{SR2} = N \cdot F_{SR1} \]
\[ J_{SR2} \cdot A_{SR} = N \cdot J_{SR1} \cdot A_{SR} \]
\[ -D_{SR} \cdot \left( \frac{\text{Cmass}_{soil}}{Vol_{soil}} - \frac{\text{Cmass}_{atm}}{Vol_{atm}} \right) = N \cdot -D_{SR} \cdot \left( \frac{\text{Cmass}_{soil}}{Vol_{soil}} - \frac{\text{Cmass}_{atm}}{Vol_{atm}} \right) \]

\[ N = \frac{C_{2mass_{soil}}/Vol_{soil}}{C_{1mass_{soil}}/Vol_{soil}} = \frac{C_{2mass_{soil}}}{C_{1mass_{soil}}} \]

Eq. 1.29: Relationship between carbon fluxes for a change in carbon mass in soil reservoir

Therefore, multiplying the initial soil respiration value (49.4 GtC/yr) by the change in the mass of soil in the soil reservoir is adequate for modelling soil respiration. This initial value encompasses the parameters in the diffusion equation, which are assumed not to change and not be limiting to soil respiration. The initial value is over the entire potential area in which soil respiration occurs, or could occur (i.e., Eq. 1.25 evaluated at the initial conditions). Changes in the surface area (and therefore volume) are not a factor because the surface area is constant. Only the carbon mass (and therefore carbon concentration) in the reservoir is changing.

Atmospheric CO\textsubscript{2} concentrations (which create the concentration gradient) are assumed to have no direct effect on soil respiration in Bice’s model (there is an indirect effect through photosynthesis and litter fall adding to the soil reservoir). It is assumed that the atmospheric CO\textsubscript{2} concentration is too low to have any impact on the rates of CO\textsubscript{2} transfer from soil to the atmosphere. Other simple models of the carbon cycle (Kwon & Schnoor, 1994; Lenton, 2000) treat soil respiration in the same way.

One finding (Fang & Moncrieff, 1999) stated that there are two major influences on CO\textsubscript{2} flux through soil respiration: firstly the production of CO\textsubscript{2} in the soil, and then transport from the soil to the atmosphere. Another review of this area (Raich & Schlesinger, 1992) found that the rate of soil respiration is controlled by the CO\textsubscript{2} gradient with the atmosphere, but do not give any specific
examples of modelling this. Soil respiration in the improved model will be treated as in the simple models mentioned, by assuming that the concentration of CO$_2$ in the atmosphere is too low to have any effect on respiration rates.

1.6.1. Temperature dependency of soil respiration

Soil respiration, like photosynthesis, is related linearly to temperature change in Bice’s model. The temperature sensitivity of soil respiration carbon flux is given by:

$$F_{SR} = F_{SR,0} \times (1 + (\theta_{SR} \times \Delta T))$$

Eq. 1.30: Expression for soil respiration temperature sensitivity (Bice, 2007)

A value of $\theta_{SR} = 0.1^\circ$C is used for soil respiration temperature sensitivity. This is taken from Gifford (1993), who finds this value by parameterising his simple model with the results of the more complex ‘Rothamsted model’ (Jenkinson et al., 1991). The term ‘parameterising’ refers to the definition and selection of parameters in Gifford’s model in such a way as to produce output reflective of this more complex model.

The Rothamsted model predicts CO$_2$ emissions from the soil for assumed temperature rises. There are five compartments to the model (humus, decomposable and resistant plant material, microbial biomass and inert soil). Temperature, soil moisture, plant cover and clay content are used to predict CO$_2$ emissions. Using the outputs of this model to find the parameters for the simple model means that these effects are included to a degree. However, using this more complex model does not imply that Bice’s soil respiration parameters are validated against actual data – it is simply another model, prone to errors and inaccuracies, being used.

The method used appears reasonable, and predicting increased soil respiration linearly is adequate for the temperature ranges in the model, so this equation and value for soil respiration will be used. Reported $Q_{10}$ values (factor that describes how much a biological process will increase for a temperature change of 10$^\circ$C) for temperature sensitivity of soil respiration are found to be between 0.77 to 3.3. (Kwon & Schnoor, 1994). The equivalent $Q_{10}$ factor for $\theta_{SR} = 0.1^\circ$C is 1, which is within this range.
1.7. Other terrestrial fluxes

Plant respiration

Plant respiration is modelled to be directly dependent on photosynthesis, and thus is effected by increased atmospheric CO$_2$ and temperature in the same way as photosynthesis. The initial value is 50 GtC/yr (photosynthesis is initially 100 GtC/yr), so the equation for calculating the rate of plant respiration is as follows: (where $F_p$, the photosynthesis flux, is changing as described in the photosynthesis equations)

$$F_R = F_p \times \frac{50 \text{ GtC/yr}}{100 \text{ GtC/yr}} \left( \text{i.e. } F_p = F_{p,\text{ini}} \times \frac{F_{R,\text{ini}}}{F_{p,\text{ini}}} \right)$$

Eq. 1.31: Expression for plant respiration (Bice, 2007)

The assumption that plant respiration is 50% of photosynthesis is a broad generalisation. This is an average value for the globe – in reality, plants have different respiration rates. For older plants, with more mass to maintain, respiration can be higher than 50%, but for young plants it can be less than half the rate of photosynthesis (Gifford, 1993).

Making the assumption that plant respiration is a constant proportion of photosynthesis obviously implies that this relationship does not change with temperature, atmospheric CO$_2$ or any other factors. Gifford (1993) found that the relationship between photosynthesis and respiration did not change significantly with temperature in controlled studies of several plant species. In this study, the rate of respiration to photosynthesis ranged between around 0.43 to 0.53. Gifford used a value of 0.5 to generalise for a global value, which Bice follows, and this is deemed acceptable for use in the improved model.
Litter fall

Litter fall is dependent on the size of the land biota reservoir, and is assumed to have half the value of photosynthesis at initial conditions (50 GtC/yr). For steady state, considering that plant respiration and litter fall are the negative fluxes from the land biota reservoir, the sum of these must equal the positive flux of 100 GtC/yr due to photosynthesis. The expression for the rate of litter fall is as follows:

\[ F_{LF} = \frac{50 \text{ GtC}}{\text{yr}} \times \frac{LB}{LB_{ini}} \]

Eq. 1.32: Rate of litter fall (Bice, 2007)

Runoff

Runoff is a minor but necessary flux in the model, with an initial value of 0.6 GtC/yr, dependent on changes in carbon content of the soil reservoir. This models the carbon transfer from the soil reservoir to the oceans through water runoff, such as river flows. It balances the long term carbon input into the atmosphere by volcanic emissions.
1.8. Downwelling and upwelling

Some of the biggest carbon carriers in the system are the water movements within the oceans. Downwelling represents the movement of water from the surface to the deep oceans, occurring primarily in colder surface ocean regions. This occurs because colder temperatures increase the density of seawater, causing it to sink due to gravity effects. This is ‘balanced’ by the upwelling occurring in warmer regions and the rims of the continents. Surface conditions such as currents and winds move surface waters, creating a ‘gap’ which deeper ocean water rises to fill. Bice uses initial estimates for each of these fluxes, which are then dependent on carbon in the deep ocean (for upwelling) and surface ocean (for downwelling). The expressions for these are:

\[
F_{\text{uw}} = 100 \frac{\text{GtC}}{\text{yr}} \times \frac{\text{DeepOcean}}{\text{DeepOcean\_ini}}
\]

\[
F_{\text{dw}} = 90.6 \frac{\text{GtC}}{\text{yr}} \times \frac{\text{SurfaceOcean}}{\text{SurfaceOcean\_ini}}
\]

Eq. 1.33: Expressions for upwelling and downwelling (Bice, 2007)

The values used in Bice’s model appear to be in line with other estimates, and will continue to be used in the improved model.

1.8.1. Biological pump

The biological pump refers to the movement of carbon from surface oceans to deep ocean due to organisms and marine biota. Organisms in the surface oceans extract inorganic carbon from the water to form calcium carbonate (CaCO₃) shells, as well as using dissolved CO₂ to photosynthesise and respire, producing organic matter. When these organisms die, the majority of the organic matter is quickly decomposed, but some will remain, and this, along with the CaCO₃ shells, will sink to the deep ocean. This flux of carbon is referred to as the biological pump (or biopump) and is important in controlling atmospheric CO₂. The removal of carbon from surface water to deep waters reduces the alkalinity of surface waters due to the depletion of Ca²⁺ ions via the movement of CaCO₃ shells. It is estimated that without the presence of the biopump, atmospheric CO₂ could rise to 500ppm, due to the decreased ability of the surface ocean to absorb atmospheric CO₂. (Bice, 2007).
This photosynthesis, respiration and sinking of carbon are all part of this biological pump process. This is modelled with an initial value for the sinking carbon, which is then dependent on the change in mass in the surface ocean reservoir:

\[
F_{\text{BIO}} = \frac{10 \text{GtC}}{\text{yr}} \times \frac{\text{Surface Ocean}}{\text{Surface Ocean}_\text{Ini}} \quad [\text{GtC/yr}]
\]

Eq. 1.34: Flux due to the biological pump (Bice, 2007)

Bice’s model for marine life does not include the photosynthesis and respiration of the organisms in the model. The reservoir of marine biota effectively has carbon fluxes due to photosynthesis and respiration, as well as the sinking to the deep ocean. Photosynthesis and respiration are not controlled by atmospheric CO\(_2\) in the same way as plant photosynthesis. Instead, the nutrients necessary for marine photosynthesis (such as P and N) are generally the limiting factor. These nutrients are not part of the simple model, so Bice chooses not to include the photosynthesis and respiration, instead only modelling the biological pumping of carbon to the deep oceans. The assumption can be made that this NPP (net primary productivity = photosynthesis − respiration) is in equilibrium, and does not add or remove any net CO\(_2\) from the atmosphere.

### 1.9. Burial

The ‘burial’ flux refers to the amount of carbon deposited on the sea floor in sediments. This is modelled as a flow of carbon from the deep oceans to the earth reservoir. This is due to the (organic and inorganic) carbon moved to the deep oceans through the biological pump settling out on the sea floor. As such this flow is dependent on the biological pump and is given as:

\[
F_{\text{burial}} = 0.6 \times \frac{F_{\text{BIO}}}{F_{\text{BIO,ini}}}
\]

Eq. 1.35: Burial carbon flux (Bice, 2007)
1.10. External additions

‘External additions’ refers to the naturally occurring processes of volcanism and metamorphism adding carbon to the atmosphere. External additions are modelled as having a constant value of 0.6 GtC/yr. This is removed from the ‘earth’ reservoir and added to the atmosphere. (Fossil fuel and thus fossil fuel emissions are also assumed to come from this earth reservoir).

1.11. Anthropogenic Effects

Historical anthropogenic effects data is as an input for the model, so that results can be compared with historical measured data for temperature and atmospheric CO$_2$ concentration. Data for fossil fuel emissions and land use changes are used to represent the anthropogenic impact in the model. The datasets used for this are discussed in section 2.3.

Land use changes data is available in annual estimates of carbon flux to the atmosphere that include soil disruption, deforestation, agricultural changes such as conversion of forest to cropland. This accounts for a net carbon flux by including carbon sinks in such things as recovering agricultural land returned to forest. Bice assumes that 75% of land use changes flux is attributed to “burning” (a carbon flux from the land biota reservoir to the atmosphere) and 25% attributed to “soil disruption” (a carbon flux from the soil to the atmosphere). These are implemented by simply adding and subtracting fluxes from the appropriate reservoir in Bice’s simple global model. Further discussion of their implementation into the improved model is discussed below.

Fossil fuel emissions are much larger than the carbon flux due to land use changes. Bice’s simple global model does not have this carbon flow coming from a particular reservoir; a change is made following Ringer (2009) to have the fossil fuel emissions coming from the added earth reservoir. It is important to point out that both land use changes and fossil fuel emissions are estimates, and not actual measured data. The estimation methods involve using various figures for consumption of fossil fuels and assumptions in regard to land use changes. (Boden & Marland, 1996; Houghton, 2003)
1.11.1. Burning

In order to properly account for burning, the surface area of the land biota needs to be considered. Burning refers to a removal of the biomass from some part of the land biota reservoir, \( A_{\text{burn}} \). Carbon mass is assumed to be equally distributed over the area of the whole reservoir, \( A_{\text{LB}} \).

![Diagram of burning and photosynthesis](image)

**Figure 1.5: Area over which photosynthesis and burning effectively occur (own illustration)**

It has been explained that photosynthesis in this model can be dealt with as not dependent on changes in land biota carbon mass (obviously this does not cover extremes, where no biomass would certainly mean no photosynthesis). If it was, burning could be accounted for simply by making photosynthesis dependent on the land biota mass subtract the burning mass, as carbon is assumed to be equally distributed over \( A_{\text{LB}} \).

However, to account for the effect of burning, the flux of carbon due to photosynthesis (\( F_{\text{P, LB}} \)) over the whole area (\( A_{\text{LB}} \)) is calculated, to give a value in GtC/yr/m²:

\[
J_p = \frac{F_{\text{P, LB}}}{A_{\text{LB}}}
\]

**Eq. 1.36: Flux per unit area, dependent on calculated flux through the photosynthesis expression**
The carbon mass distribution across the reservoir is calculated by dividing mass of carbon (LB) by the area \( A_{LB} \) \((1.33 \times 10^{14} \text{ m}^2)\). The mass of burnt carbon per year is equal to area burnt per year multiplied by the carbon mass per unit area, and by rearranging this equation, \( A_{burn} \) can be found:

\[
F_{burn}(\text{GtC yr}^{-1}) = A_{burn}(\text{m}^2 \text{yr}^{-1}) \times \frac{LB \{ \text{GtC} \}}{A_{LB} \{ \text{m}^2 \}}
\]

\[
\therefore A_{burn}(\text{m}^2 \text{yr}^{-1}) = \frac{F_{burn}}{LB / A_{LB}}
\]

Eq. 1.37: Flux due to burning and carbon distribution used to calculate the area burnt.

Using the value for photosynthesis across the whole area \( A_{LB} \) \((F_{P,LB})\); the value for photosynthesis per unit area \((J_p)\); and assuming that \( J_p \) does not occur in \( A_{burn} \), it follows that in a particular year:

\[
F_p = F_{p,LB} - A_{burn} \times J_p
\]

Eq. 1.38: Calculating photosynthesis accounting for area burnt.

In this equation, \( F_{p,LB} \) is the value of photosynthesis calculated using Bice’s equation, as described earlier. Using Eq. 1.38 means that, for each time step of the model, photosynthesis flux is only calculated in the ‘non-burning’ area corresponding to the particular value of \( F_{burn} \) in GtC/yr.

However, using the equation in this way assumes that previously burnt areas regenerate immediately, as only the \( A_{burn} \) corresponding to the current \( F_{burn} \) is included – photosynthesis is assumed to be occurring over all other area, even if it had been burnt in the previous timestep. So, once an area has been burnt, photosynthesis will not occur for that year (or timestep), but will occur in the successive timestep, if Eq. 1.38 is used. In reality, it is not likely that a burnt area will regenerate immediately. To represent the fact that burnt areas will not regenerate immediately, burnt area can be integrated with time, and a proportion of this area considered having no photosynthesis. So, following this reasoning, the photosynthesis flux will be:

\[
F_p = F_{p,LB} - A_{burn}(t) \times J_p - RF \times J_p \int_0^{t-1} A_{burn}(t). dt
\]

Eq. 1.39: Calculating photosynthesis dependent on effects of current and past burning.
The factor ‘RF’ refers to the regrowth factor. A value of 0 indicates that all previously burnt areas are regenerated immediately in the next time step, and photosynthesis occurs as normal. A value of 1 indicates that no photosynthesis occurs on previously burnt areas. The ‘current’ $A_{\text{burn}}$ is included as a separate term, because the RF does not apply to this area. The graph below shows the model output compared to atmospheric CO$_2$ data for different RF values (the model used is a version of the improved final model).

![Atmospheric CO$_2$ data and model output for different RF](image)

**Figure 1.6: Model outputs compared with Mauna Loa/Law Dome CO$_2$ data for different RF values**

The curve for ‘area not included’ (ie. photosynthesis not accounting for any reduction due to $A_{\text{burn}}$) and for a RF of 0 are almost identical and overlap on the graph. This is because an RF of 0 refers to none of the ‘past’ $A_{\text{burn}}$ being considered, ie. that all 100% of all burnt area regrows, which is most similar to no $A_{\text{burn}}$ being considered at all. Higher RF values cause less photosynthesis, as there is less area available for this photosynthesis to occur. Less photosynthesis means that there is less carbon removed from the atmosphere, which can be seen with an RF of 1 having the highest atmospheric CO$_2$ concentration. The RF value of 0.24 seemed to fit to the data best and will be used in the improved model.

With including the alterations discussed into the improved model, burning is better represented, and results in the simulation are closer to measured data that those ignoring the area burnt when considering photosynthesis.
1.11.2. **Soil disruption**

Agriculture and land cultivation cause increased soil aeration and higher moisture content. These conditions result in an increased rate of soil respiration. The increased carbon flux due to this increased respiration is referred to as soil disruption. (Schlesinger & Andrews, 2000)

Estimates for carbon flux to the atmosphere from the soil reservoir due to soil disruption are available as global values in GtC/yr (a proportion of land use changes data). If it is assumed that the soil respiration in the equations is an estimate over the total global land surface area in which soil respiration could occur, then the reservoir surface area is not changed when additional disruption occurs. Soil disruption therefore refers to increased soil respiration in some proportion of the surface area already being considered, without the addition of any more surface area to the soil reservoir.

![Diagram showing soil disruption and soil respiration](image)

**Figure 1.7:** Diagram showing soil disruption and soil respiration (own illustration)
If it is assumed that soil respiration at the usual rate \( F_{SR} \) is an average for the entire area of the soil reservoir \( A_{SR} \), and increased respiration \( F_{SD} \) occurs in some area \( A_{SD} \) of the soil reservoir in addition to that which would naturally occur, we have:

\[
F_{SR} = D_{SR} \cdot A_{SR} \cdot \frac{C_{mass_{soil}}/A_{SR} \cdot \text{Depth}}{\Delta z} + F_{SD} \quad \text{[GtC/yr]}
\]

\[
F_{SD} = D_{SD} \cdot A_{SD} \cdot \frac{C_{mass_{soil}}/A_{SD} \cdot \text{Depth}}{\Delta z} \quad \text{[GtC/yr]}
\]

Eq. 1.40: Equations for soil disruption as a part of soil respiration

Using the estimates of flux due to soil disturbance (GtC/yr) for the term \( F_{SD} \) in the above equation, with the assumption that soil respiration at the usual rate is occurring across the entire surface area means that the estimates can be included in the model without any change in ‘natural’ soil respiration surface area. So, referring to the above equations, the effective parameters for \( F_{SD} \) are dependent on whatever the anthropogenic effects are – they are not known, but the result of the expression for \( F_{SD} \) as a time series in GtC/yr can be used without knowledge of these.

The mass balance for the soil reservoir accounts for losses due to soil disruption, as the soil disruption flux is a flow out of this reservoir in the same way as soil respiration. Runoff is the other flux out of the soil reservoir, and this is modelled by Bice as being dependent on the change in concentration in the soil reservoir. This is a very small flux, but is necessary in this model for the equilibrium case, although it is not included in other simple global carbon cycle models (Kwon & Schnoor, 1994; Lenton, 2000).

As runoff and soil respiration are dependent on the change in carbon mass in the soil reservoir, the inaccuracy of the integration method used in the modelling program, GoldSim (Euler integration, see Appendix F), could potentially increase when soil disruption is included. Soil disruption causes more change in the soil reservoir carbon mass, which the rates of soil respiration and runoff are dependent on. More rate change over a timestep means the model will be less accurate, as Euler integration assumes the rate of change to be constant over a timestep. With selection of a sufficiently small timestep, the inaccuracy in the context of this model will be negligible – other parameters in the model contribute much greater uncertainty and inaccuracy.
1.12. Comparison with Bice’s ‘complex’ model

Bice also presents a ‘complex’ version of his carbon cycle model, which includes more reservoirs and several additional fluxes between them. The main differences of Bice’s ‘simple’ model and ‘complex’ model are:

- The surface ocean is split into warm and cold surface ocean components. The fluxes corresponding to the surface ocean (biological pump, atmosphere-ocean transfer, runoff) are split into warm and cold components, with downwelling occurring from the cold ocean and upwelling occurring into the warm ocean. A flux of ‘advection’ is added, which describes the ocean surface currents transporting carbon from warm to cold parts of the surface ocean.

- The soil reservoir is split into the ‘litter’ and ‘soil’ reservoirs. There is corresponding respiration in both reservoirs, a burial flux from the litter reservoir to soil, and runoff coming from the litter reservoir. This aims to represent the different dynamics in the soil, where faster decomposition happens in the litter than in the soil.

![Figure 1.8: Comparing Bice's simple and complex models.](image)

The above figure compares model runs for Bice’s simple model and complex model using emissions data from 1850, with steady-states adjusted from those Bice uses (280ppm) so reflect those in 1850 (285ppm). The two curves are of similar shape, with the complex result always
above the simple model, thus using the more complex version does not really add a lot to the model (tuning parameters properly will be a better method of improving results). It was decided the simple model would suffice, and no further development on the 'complex' model was carried out.

Other simple models of the carbon cycle have some small differences with Bice's model. The model of Kwon & Schnoor (1994) splits the surface ocean into the warm and cold components. This was experimented with in Bice's “complex” model, which also separates this reservoir split but requires introduction of another carbon flow between them called advection, which is of quite uncertain magnitude. Thus it was decided not to include this in the simple model, as it does not have a huge effect on model output, and the idea is to keep the model as simple as possible.
2. Data evaluation

A summary of the required data for running the model and comparing results is as follows:

- Single pre-industrial steady state values of atmospheric CO$_2$ and global average surface temperature
- Time series to verify the model output in terms of atmospheric CO$_2$ and global average surface temperature
- Time series for the input of anthropogenic changes over the simulation period and for changes in solar irradiation affecting the global energy balance.

The validity of this data and selected datasets are discussed in the following sections.

2.1. Temperature

A significant amount of literature has been found regarding so called “average global surface temperature” measurements, and a number of datasets are available. Temperature measurements are necessary for calculating model parameters, and for model verification

An initial point to look at was the data used in the IPCC Assessment Report 4 (AR4), as this is an almost globally accepted standard. The AR4 uses several global average surface temperature datasets: CRUTEM3 (developed by the Climate Research Unit or CRU, UK, from the HadCRUT3 datasets), NCDC (National Climatic Data Centre, USA), GISS (Goddard Institute for Space Studies, USA) and Lugina et al. data.

There are also a number of satellite based datasets - UAH (University of Alabama) and RSS (Remote Sensing Systems, supported by NASA) are two of these. Satellite data uses a totally different set of measurements techniques (mainly radiation based) and differs from Earth based data. Satellite data is only available from 1979 onwards, so it not suitable for use in model verification over the planned simulation period.
A comparison of the mentioned global surface temperature anomaly datasets is shown below. CRU refers to the official WMO period 1961-1990 (i.e. anomaly relative to the 1961-1990 average), while NCDC and GISS use the periods 1901-2000 and 1951-1980 respectively.

![Comparison of temperature datasets](image1)

**Figure 2.1: Comparison of temperature datasets**

The CRUTEM3 dataset (referred to as “CRU data” in this report) is the only set that covers the period from 1850 to the present. This dataset will be used for model comparisons. For a look at global average temperatures in the last century, the NCDC dataset has been used. Global reference average temperature is given as 13.9°C. Recent average global surface temperature increases appear to be mainly attributed to land temperatures rising, as shown in Figure 2.2.

![Absolute NCDC temperatures for land, sea and global average](image2)

**Figure 2.2: Absolute temperature from NCDC LST, SST and GST datasets**
2.2. Atmospheric CO$_2$ data

As the model will be run from a pre-industrial semi-steady-state, considered to represent an equilibrium phase in the carbon cycle, it is important to have a figure for the atmospheric concentration of CO$_2$ at this time. It has been found that the generally accepted value is about 280 ppm – this is the value used in Bice’s model. However, the value of 280ppm was the atmospheric concentration around 1750 according to historical ice core CO$_2$ records, which is an issue when Bice considers 1890 to be the starting year for his simulations.

The Mauna Loa atmospheric CO$_2$ record (Keeling et al., 2009) is one of the longest time-series available with modern measurements, beginning in 1959. There are many other datasets of modern CO$_2$ measurements, but Mauna Loa is a non-disputed reference and thus will be used for comparisons with model data. Assuming that the atmosphere is a ‘well-mixed tank’ assumes that this measurement is representative of global atmospheric CO$_2$ concentration. In reality, there can be differences of up to 4 or 5ppm at different locations. This model can be assumed to be referenced to the Mauna Loa site in regard to atmospheric CO$_2$, so results are representative of this location.

Data prior to modern measurements is obtained through ice cores and tree-ring dating. The ice core data from Law Dome (Etheridge et al., 1998) covers the period from 1000 years ago to near present times, overlapping with the Mauna Loa record. Law Dome data overlayed with the Mauna Loa record is shown in Figure 2.3. This data series will be used to validate model output.
Figure 2.3: Graph showing CO₂ records from Law Dome and Mauna Loa with reference bands

The Law Dome atmospheric CO₂ data for the period 1800 to 1850 is around 285ppm, as illustrated in the graph zoomed in to this period (Figure 2.4), which therefore will be used as the initial (semi) steady state of the model. The reason for this is that temperature and emissions data are readily available from 1850 on. Using a value of 280ppm would mean that simulations should begin around 1750, but measured temperature data to verify model output is only available after 1850.

Figure 2.4: Illustrating “steady-state” of CO₂ concentration between 1800 and 1850 from the previous graph
It is important to stress the fact that the chosen “steady state” is not an absolute steady state. Anthropogenic effects through land use changes are thought to have been in effect since 1750 but will not be included in this model. Additionally, processes operating on longer timescales are not at steady state, but the longer time scales (i.e. multi-millennial) mean they are assumed to have a negligible effect over the simulation of the model.

2.3. Anthropogenic effects data

Bice uses a time-series of fossil fuel emissions, which directly feed into the model atmosphere reservoir, and land-use changes, of which 25% are classified as ‘soil disruption’ (carbon flow from soil reservoir to atmosphere) and 75% ‘burning’ (carbon flow from land biota to atmosphere).

Bice’s emissions and land use changes data covers the period 1890 to 1990 so additional data is required in order to have it as an input from 1850 onwards. Datasets compiled by Boden et al. (2009) for global annual fossil fuel emissions and by Houghton (2008) for annual carbon flux to the atmosphere through land use changes were obtained and cover the required range. These are the same sources as cited by Bice.

Bice’s model uses average emissions and land use changes for each ten year period since 1890. This was initially carried out for the data obtained (Table 2.1), but when testing use of 10-year averages (which are extrapolated by the modelling program) compared to using data points for each year, it was found that using the entire time series (i.e. 150 data points) produced a better result than using the 10-year averages.
Figure 2.5: Estimated flux to the atmosphere due to a) fossil fuel emissions and b) land use changes (Boden et al., 2009; Houghton, 2008)

Table 2.1: Atmospheric emissions used for 10 year averages (Boden et al., 2009; Houghton, 2008)

<table>
<thead>
<tr>
<th>10 year averages</th>
<th>Fossil Fuel Emissions GtC/yr</th>
<th>Land use changes GtC/yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1850</td>
<td>0.0678</td>
<td>0.53991</td>
</tr>
<tr>
<td>1860</td>
<td>0.1147</td>
<td>0.53146</td>
</tr>
<tr>
<td>1870</td>
<td>0.1813</td>
<td>0.62636</td>
</tr>
<tr>
<td>1880</td>
<td>0.2789</td>
<td>0.68799</td>
</tr>
<tr>
<td>1890</td>
<td>0.4092</td>
<td>0.70817</td>
</tr>
<tr>
<td>1900</td>
<td>0.6582</td>
<td>0.85646</td>
</tr>
<tr>
<td>1910</td>
<td>0.8763</td>
<td>0.82856</td>
</tr>
<tr>
<td>1920</td>
<td>0.9743</td>
<td>0.87331</td>
</tr>
<tr>
<td>1930</td>
<td>1.0406</td>
<td>0.93584</td>
</tr>
<tr>
<td>1940</td>
<td>1.3427</td>
<td>0.93483</td>
</tr>
<tr>
<td>1950</td>
<td>2.0183</td>
<td>1.34023</td>
</tr>
<tr>
<td>1960</td>
<td>3.0976</td>
<td>1.48226</td>
</tr>
<tr>
<td>1970</td>
<td>4.7004</td>
<td>1.29666</td>
</tr>
<tr>
<td>1980</td>
<td>5.4887</td>
<td>1.51202</td>
</tr>
<tr>
<td>1990</td>
<td>6.3521</td>
<td>1.56561</td>
</tr>
<tr>
<td>2000</td>
<td>7.2515</td>
<td>1.4714</td>
</tr>
</tbody>
</table>
Interestingly, using annual data points only produced a better result when surface area burned was accounted for in the amount of photosynthesis occurring (see 1.11.1). As described previously, area burnt needs to be accounted for in the flux of photosynthesis (and therefore respiration). When 10-year averages were used and this area was not accounted for, the 10-year averages produced a better result. When burning area was accounted for, the estimated annual average data values produced a better result. As burning area will be accounted for in the improved model, annual data will be used as opposed to 10-year averages as in Bice’s model.

Figure 2.6: 10 year average and annual data compared with burning area not included

Figure 2.7: 10 year average and annual data compared with burning area included
The assumptions in Bice’s model will continue to be used - all fossil fuel emissions go to the atmosphere reservoir, 75% of land use changes are represented by a flow between the land biota and atmosphere (burning), and 25% of land use changes are represented by a flow between the soil reservoir and the atmosphere (soil disruption). The fossil fuel emissions will be a flux from the earth reservoir to the atmosphere, which is a slight alteration as Bice’s model did not the earth reservoir.

2.4. Solar irradiation

Solar irradiation is the average solar flux from the sun (this is further explained in Appendix C). It has an average value of around 1366 W/m$^2$. It is generally referred to as Total Solar Irradiance (TSI).

While not part of the primary data evaluation, this will be described here as a time series of TSI and will be incorporated into the improved energy balance developed for the simple model. The reason for this is that changes of up to 2.5 W/m$^2$ in solar irradiance have been found in some datasets since pre-industrial times. As solar irradiance is a value used in calculating the global energy balance, this change could have some impact on global temperatures calculated in this way.

There are several methods for developing historical TSI records, which mainly involve analysing some aspect of solar variability. This has included the length and decay rate of the solar cycle, average level of sunspot number and the length and decay rate of individual sunspots. More accurate satellite data is only available from 1978 onwards.

A fairly recent reconstruction (Bard et al., 2000) uses ‘cosmogenic nuclides’. This method differs from others in that it measures a direct consequence, as opposed to a particular aspect, of solar irradiance (Sherwood & Idso, 2003). Bard et al. use $^{14}$C content in tree rings and $^{10}$Be in polar ice to construct the record. This record covers the period only up to 1961, so will not be suitable for use in the model. The work of Lean et al. (2000) is commonly referred to and often used when irradiance measurements are required. As this TSI dataset covers the period required this will be used.
Figure 2.8 shows a comparison of historical reconstructions with recent satellite actual measurements. All the historical reconstructions have good agreement with the recent measurements, but there is considerably variation in the datasets prior to this. Although oscillations appear to occur at relatively similar times, the magnitude varies greatly.

Figure 2.8: Historical solar irradiance reconstructions
3. Model improvements

Slight changes to the model in terms of general equation format and necessary assumptions/explanations have been described in the first chapter (Simple model evaluation).

This section proceeds to describe further changes to the simple model. The changes are slight alterations to the parameters used by Bice (in the case of photosynthesis CO₂ fertilisation and temperature related parameters), additions to the model (improved energy balance), or alteration of the temperatures used in the model. A summary of all changes made is provided at the end of this chapter, inclusive of the changes describes in the first chapter.

3.1. Steady-state average global surface temperature references

In the temperature relations for the original simple model, Bice specifies that temperature change (calculated from atmospheric CO₂ concentration using a single equation) is from a reference of 15°C, although this absolute temperature is not used anywhere in the model. The equations used to calculate the temperature sensitivity of photosynthesis and the compensation point for photosynthesis are both evaluated at 17°C, following the model developed by Gifford (1993), implying that Bice considers the temperature for photosynthesis to be 17°C.

Temperature data obtained has shown global average surface temperatures to be at around 14°C over the last century. (13.9°C for the NCDC data series, 14°C for the CRUTEM3 data series used in model comparisons). The absolute temperature, calculated using anomalies and this average figure, is 13.56°C in 1850 according to the CRU data.
The generally accepted average temperature at the Earth’s surface ($T_s$) as calculated using a global energy balance is 288.15 K or 15°C. The initial temperature for the energy balance developed will be this temperature. There is a small difference between this and the absolute temperature as taken from the measured average global surface temperature dataset (13.56°C in 1850 according to the CRU data). Changes in the temperature calculated through the energy balance will be linked to this measured average global surface temperature (GST). So, the temperature anomaly as calculated by the energy balance can be directly compared to the anomalies in the chosen dataset – calculated anomalies in the energy balance temperature are assumed to affect measured average GST by the same amount.

Bice uses a temperature of 17°C for photosynthesis parameters. It could be argued that this is adequate, considering that around 70% of photosynthesis is estimated to occur in the between 30°N and 30°S latitude (Potter et al., 1993), where temperatures are higher. Values from Melillo et al. (1993) are similar, estimating over half of NPP (29.6 GtC/yr of a total 53.2 GtC/yr) to occur between 22.5°N and 22.5°S latitude. As these regions have a much higher temperature (around 25°C), using 17°C could represent this to an extent. However, it was found that using the average land surface temperature (8°C) to calculate photosynthesis parameters produced better results. The reasoning for using the land surface temperature is that the photosynthesis considered in the model only occurs on land.

Sea-surface temperature is linked to the change in global average temperature calculated by the model. The steady state sea surface temperature is 16°C (289.15 K), and calculated changes in global temperature will be added to this. This makes the assumption that the SST is affected directly by global surface temperature. In reality, the relationship is not so direct and there may be some time lag, but this is assumed to be negligible and is not included in the model.

The diagram below shows the different temperatures used in the model. The global average surface temperature is calculated based on the global energy balance discussed in section 3.5. The measured global surface temperature (GST) as according to the temperature datasets (13.56°C in 1850, 14°C average 1961 to 1990) is slightly lower than the 15°C used as the steady
state temperature in the energy balance \( (T_s, \text{ Earth's surface blackbody emission temperature}) \). Anomalies in the energy balance surface temperature are assumed to translate to anomalies in the measured GST, so that the GST dataset can be used as a comparison for model output.

Figure 3.1: Temperature relationships in the improved model

3.2. Half-saturation value of photosynthesis

The half-saturation value in the photosynthesis flux, \( K_{hs} \), is an important parameter used to determine the response of photosynthesis to increasing atmospheric \( \text{CO}_2 \) concentrations (\( \text{CO}_2 \) fertilisation). The effect of changing \( K_{hs} \) on the resulting net primary production (photosynthesis – respiration) is shown in the graph below.

Figure 3.2: Effect of changing \( K_{hs} \) on NPP (NPP = photosynthesis – respiration)
The equations modelling photosynthesis flux in the model have been discussed in previous sections, and are again presented below. $K_{hs}$ is used in defining the ‘$P_{\text{max}}$’ parameter based on initial photosynthesis flux at initial conditions, and in the equation to calculate photosynthesis flux with changing atmospheric CO\(_2\) concentration:

$$P_{\text{max}} = \frac{\left( K_{hs} + (pCO_{2,\text{atm ini}} - pCO_{2,\text{atm min}}) \right) \ast F_{p,\text{ini}}}{(pCO_{2,\text{atm ini}} - pCO_{2,\text{atm min}})}$$

\textbf{Eq. 3.1: Equation for calculating $P_{\text{max}}$, GtC/yr (Bice, 2007)}

$$F_p = P_{\text{max}} \ast \frac{(pCO_{2,\text{atm}} - pCO_{2,\text{atm min}})}{(pCO_{2,\text{atm ini}} - pCO_{2,\text{atm min}})} \ast \left(1 + (\theta_p \ast \Delta T)\right)$$

\textbf{Eq. 3.2: Equation for calculating carbon flux through photosynthesis, GtC/yr (Bice, 2007)}

where: $pCO_{2,\text{atm ini}}$ = the initial concentration of atmospheric CO\(_2\) (steady state); $pCO_{2,\text{atm min}}$ = minimum concentration of CO\(_2\) for photosynthesis to occur; $\theta_p$ = temperature sensitivity of photosynthesis; $\Delta T$ = temperature change from reference; $F_{p,\text{ini}}$ = initial global photosynthesis used to set the initial conditions (at initial atmospheric CO\(_2\) concentration) through $P_{\text{max}}$.

For a constant temperature, the rate of photosynthesis ($F_p$) will be half of $P_{\text{max}}$ at an atmospheric CO\(_2\) concentration of $K_{hs}$ (plus the compensation point). Bice uses a value of 62.5 ppm for $K_{hs}$, so $P_{\text{max}}$ is calculated to be 125 GtC/yr based on the initial photosynthesis flux, $F_{p,\text{ini}}$, being 100 GtC/yr, for the initial atmospheric CO\(_2\) concentration of 280ppm. This means that at an atmospheric CO\(_2\) concentration of 92.5 ppm, $F_p$ will have a value of 62.5 GtC/yr. The $P_{\text{max}}$ value is only ever reached with infinite atmospheric CO\(_2\).

Bice follows the method of Gifford (1993) in the expression for photosynthesis flux, and the derivation of this is explained in Appendix A. However, Bice does not follow Gifford’s reasoning in determining the value $K_{hs}$ should take. Bice uses a $K_{hs}$ value that will yield a 25% increase in $F_{p,\text{ini}}$ at the limit of Eq. 3.2, whereas Gifford uses a $K_{hs}$ value that will cause a 25% increase in photosynthesis rate for a doubling of atmospheric CO\(_2\).

Estimated increase in photosynthesis rate or net primary production (NPP) for a doubling of atmospheric CO\(_2\) has been encountered numerous times in the literature reviewed. Considering rate increases to the limit of Eq. 3.2 (ie. $P_{\text{max}}$) in order to parameterise $K_{hs}$ is not as directly useful (although there is obviously an equivalent rate increase to $P_{\text{max}}$ for any rate increase caused by a
doubling of CO$_2$). (McGuire et al., 1997). The table below contains some estimates of the rate increase for a doubling of atmospheric CO$_2$.

Table 3.1: Estimates of increased NPP for a doubling of atmospheric CO$_2$

<table>
<thead>
<tr>
<th>Source</th>
<th>CO$_2$ doubling</th>
<th>Estimated Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Gifford, 1993)</td>
<td>340 ppm to 680 ppm</td>
<td>1.25 (1.1 min and 1.4 max credible values)</td>
</tr>
<tr>
<td>(McGuire et al., 1997)</td>
<td>340 ppm to 680 ppm</td>
<td>1.084</td>
</tr>
<tr>
<td>(Melillo et al., 1993)</td>
<td>340 ppm to 680 ppm</td>
<td>1.37</td>
</tr>
<tr>
<td>(Melillo et al., 1993)</td>
<td>312.5 ppm to 625 ppm</td>
<td>1.163</td>
</tr>
<tr>
<td>(Melillo et al., 1993)</td>
<td>Using model predicting for doubled CO$_2$ with no climate change</td>
<td>1.20 to 1.26</td>
</tr>
<tr>
<td>(Melillo et al., 1993)</td>
<td>Using model predicting for doubled CO$_2$ with climate change</td>
<td>1.20 to 1.26</td>
</tr>
</tbody>
</table>

Gifford (1993) calculates $K_{ns}$ by assuming that a doubling of atmospheric CO$_2$ from 340pppm (current conditions at the time of 1990) to 680ppm will increase the photosynthesis rate by a factor of 1.25, an estimate made after his evaluation of literature and models accounting for other factors, such as nitrogen and water availability. Melillo et al. (1993) estimates rates to increase between 1.16 and 1.26 for a doubling in CO$_2$ from 312.5ppm to 625ppm. The equivalent factor for Bice’s $K_{ns}$ value of 62.5ppm is 1.09.

Following the reasoning of Gifford, the $K_{ns}$ value in Bice’s photosynthesis rate equation would be 191.9ppm. Figure 3.3 shows the simulation results for the simple global carbon cycle model with varying $K_{ns}$ parameters. For this simulation, all other parameters remain the same as in Bice’s simple model (excepting the steady state used is 285ppm in 1850, and fossil fuel emission and land use changes data are used as described in 2.3- Anthropogenic effects data)
The $K_{hs}$ value of 191.9ppm provides a better fit with atmospheric CO$_2$ data from about 1960 onwards than the other values. The value used by Bice, 62.5ppm, does provide a better fit to the data until about 1920. However, the reasoning of Gifford (1993) appears to be credible, and the value of 191.9ppm provides a good fit to the data between 1850 to about 1910, with some deviation, then an excellent fit to the data from around 1950 to 2000. Thus, the value used for $K_{hs}$ in the improved model will be 191.9ppm. This corresponds to a rate increase in NPP of 1.25 for a doubling of atmospheric CO$_2$, which is within the range of estimates found.

### 3.3. Temperature sensitivity of photosynthesis

Temperature change and photosynthesis are related linearly in Bice’s model. In reality, photosynthesis has been shown to increase with temperature up to a point, but there will be an upper limit. It is assumed that the upper limit to the temperature effect will not come into play with the temperature change predicted over the time span of the model. There is also a lower limit to the temperature sensitivity, but again, this is not likely to be reached in this model, hence Bice justifies using a linear relationship as follows:

$$F_p = F_{c,0}(1 + (\theta_p \times \Delta T))$$

**Eq. 3.3: Photosynthesis temperature dependency used by Bice**

where: $\Delta T$ is temperature change from the reference temperature at initial conditions; $\theta_p$ is the temperature sensitivity ($^\circ$C$^{-1}$); and $F_{c,0}$ is the flux of photosynthesis at initial conditions with no increased CO$_2$ fertilisation.
The temperature sensitivity value used, $\theta_p = 0.04$, will increase photosynthesis by a factor of 1.4 if there is a 10 degree change in temperature. This parameter is referred to as the $Q_{10}$ factor, which is a commonly used value in describing the temperature effects on processes. Bice derives this temperature sensitivity from the terrestrial model developed by Gifford (1993). In this model, a sigmoidal function describing the relationship between NPP (Net Primary Production) and temperature is presented:

$$P = \frac{3000}{(1 + e^{1.315-0.119T})} \quad \{g/m^2/yr\}$$

Eq. 3.4: Sigmoidal function describing dependency of NPP on temperature (Gifford, 1993)

![Figure 3.4: Temperature dependence of NPP as in Eq. 3.4 (Gifford, 1993)](image)

This function is known as the ‘Miami Model’ (Leith, 1975), and is one of the grounding developments in NPP temperature sensitivity, although it is now considered outdated by some (Cramer et al., 1999). For the purposes of this simple model, using the ‘Miami Model’ to define temperature sensitivity will be satisfactory. Other relationships found are considerably more complex and dependent on factors that are not a part of this model.

Temperature sensitivity values found in literature are in the range of 1.03 to 2.0 for photosynthesis (Kwon & Schnoor, 1994). Bice’s $Q_{10}$ factor of 1.4 is therefore comparable to these ranges. As respiration is directly proportional to photosynthesis (a flux corresponding to 50% of photosynthesis...
flux) in the simple mode, respiration therefore has the same temperature sensitivity as photosynthesis. It then follows that the temperature sensitivity of NPP in the model (photosynthesis – respiration) is the same as for photosynthesis. This assumption is valid for the simple model, although reported Q_{10} values for respiration were found to be between 1.4 and 3 (Kwon & Schnoor, 1994), which is a significantly different range to photosynthesis. If photosynthesis and respiration have different temperature sensitivities, then the temperature sensitivity of NPP will not be the same as that for photosynthesis, and using a sensitivity derived from the ‘Miami Model’ relationship between NPP and temperature would not be valid. However, for simplicity, photosynthesis and respiration are assumed to have the same temperature sensitivity, thus validating this method of determining temperature sensitivity.

The function above (Eq. 3.4) is for NPP in gC/m$^2$/year, whereas photosynthesis in the model is in GtC/yr, with the surface area of the land mass included in the value. The temperature sensitivity per unit area or over the whole land biota area would be the same, so it is appropriate to just use the effective rate of NPP increase with temperature derived from the function above. This does not mean that the actual rate of photosynthesis is the same as NPP, photosynthesis happens at around twice the rate of NPP, it just means that the temperature dependencies will be the same.

The reference photosynthetic flux at initial conditions in the model is 100 GtC/yr for conditions with no change from the reference temperature, although of course the steady state reference temperature for photosynthesis will have some relation to this initial value. In Eq. 3.7, a temperature of 14°C (the global average surface temperature steady state for the model), corresponds to a NPP of 1760.58 g/m$^2$/year. This is considerably higher than global estimates (around 500 g/m$^2$/year) so this specific rate cannot be applied to the entire land surface area. However, this could be applied by assuming this rate to occur over some proportion of land surface area in which photosynthesis occurs, or by assuming some efficiency factor.

At T = 17°C Gifford states that Eq. 3.4 has a linear slope of 0.04/°C. That is, evaluating the NPP around 17°C results in a θ_p value of 0.04 for use in the temperature sensitivity equation. This value is used in Bice’s model, and calculation of this is illustrated in Example 3.1
Example 3.1: Calculating temperature sensitivity around 17°C

This approximation is effective around a fairly small range. However, $θ_p$, calculated according to the sigmoidal function in Eq. 3.4 changes considerably with absolute temperature. Figure 3.5 shows the relationship between temperature sensitivity, $θ_p$, and the absolute temperature it is calculated at.

$$P(16.5) = \frac{3000}{1 + e^{1.315 - 0.0119(16.5)}} = 1970 \text{ g.m}^{-2}\text{yr}^{-1}$$

$$P(17.5) = \frac{3000}{1 + e^{1.315 - 0.0119(17.5)}} = 2048.9 \text{ g.m}^{-2}\text{yr}^{-1}$$

$$F_p = F_{ce}\left(1 + (T_{senp} \cdot ΔGlobalTemp)\right)$$

$$2048.9 = 1970 \cdot (1 + (θ_p \cdot 1°C))$$

$$θ_p = \frac{2048.9}{1970} - 1 = 0.0401/°C$$

The options with temperature sensitivity are firstly to use Bice’s value, 0.04, which could be explained by assuming that the effective photosynthesis temperature, $T_p$, is higher than global average surface temperatures (0.04 is calculated by using the slope at $T = 17°C$, around 3 degrees higher than measured average global surface temperature). Secondly, $θ_p$ could be calculated at values of 14°C (to reflect measured average global surface temperature), or 8°C (to reflect global average land surface temperature).
For each degree that temperature changes, the linear approximation of temperature sensitivity will change by 0.00296/°C. The slope is negative, so for temperature increasing by 1°C, the temperature sensitivity will decrease by 0.00296. At 17°C, for example, an increase of 1°C will mean temperature sensitivity changes from 0.0451/°C to 0.0422/°C, or a reduction of 6.5%. Temperature changes in the model are unlikely to exceed more than about 2°C.

For photosynthesis, the compensation point (atmospheric CO₂ concentration below which temperature no photosynthesis can occur) is also affected by temperature. The temperature the compensation point is calculated for should be the same as that which temperature sensitivity is calculated at. The following section describes trial simulations with different methods used for evaluating compensation point and temperature sensitivity, as these should be considered together when deciding on the effective temperature for photosynthesis.

3.4. Compensation point

Following from the temperature used to calculate photosynthesis temperature sensitivity, the CO₂ compensation point (CO₂_min using the following calculation in Gifford(1993)) is also temperature dependent.

\[
CO₂_{min} = (0.511 \times 10^6) \cdot e^{\left(-\frac{2815.5}{T+273.2}\right)}
\]

Eq. 3.5: Expression for finding the CO₂ compensation point (Gifford, 1993)

The options for this improved model are to use the original value of Bice (30ppm evaluated at 17°C), to use a constant value at another temperature, or to calculate the value within the model, using this equation with temperature as an input parameter. The global average surface temperature in the model is T (average of around 14°C in recent times), and the temperature for photosynthesis is represented as Tₚ. As discussed previously, the first temperature that could be used is Tₚ = 14°C, to represent the average measured global surface temperature. The second temperature that could be used is Tₚ = 17°C, to reflect higher temperatures in the tropics where most NPP occurs. The third temperature trialled is Tₚ = 8°C (to reflect average LST of 8°C)
If a particular initial temperature is used in calculating the compensation point, it was decided that the temperature sensitivity, $\theta_p$, should also reflect this value. Figure 3.6 shows the results of:

- $pCO_{2,min}$ and $\theta_p$ constant at their respective values for $17^\circ$C, as in Bice's model
- $pCO_{2,min}$ and $\theta_p$ both a function of temperature, with the initial $T_p = 17^\circ$C
- $pCO_{2,min}$ a function of temperature, with $\theta_p$ at a constant value calculated for $T_p = 17^\circ$C
- $pCO_{2,min}$ constant, with $\theta_p$ calculated as a function of temperature, with initial $T_p = 17^\circ$C

Figure 3.7 shows the results obtained from the same method but at $14^\circ$C (the results are difficult to see as they are almost identical for constant $pCO_{2,min} & constant \theta_p$ as they are for $pCO_{2,min} = f(T)$ & constant $\theta_p$. As indicated in the graphs, the differences between model outputs for these changes are very small.

Figure 3.8 shows the same results but at $8^\circ$C. The differences are much greater in these simulations, with one very good fit from around 1960 onwards (for constant $pCO_{2,min}$ and $\theta_p$ evaluated at $8^\circ$C), but big differences between data and model output in the remaining simulations.

(Note: these simulations are using a version of the improved model.)
The best fits to the Mauna Loa/Law Dome data for each graph are:

- constant $p_{CO2_{min}}$ and constant $\theta_p$ evaluated for $T_p = 14^\circ$C
- $p_{CO2_{min}} = f(T)$ and constant $\theta_p$ evaluated at a temperature of $17^\circ$C.
- constant $p_{CO2_{min}}$ and constant $\theta_p$ evaluated for $T_p = 8^\circ$C

Comparison of these three options is shown in the graphs below. There is very little difference between the three methods.
When looking closely at the part of the results between 1980 and 2000, it can be seen that the model output using constant pCO2min and $\theta_p$ evaluated at $T_p = 8^\circ C$ is slightly better. As such, these values will be used in the improved model. This represents photosynthesis occurring at the average land surface temperature across the land surface.

As it was found that the best fit did not require any function of absolute temperature, the ‘$T_p$’ value will not actually be needed in the model, it will only be used to initially evaluate the parameters. As photosynthesis is dependent on change in temperature, calculating the temperature sensitivity using temperature change is the only temperature that is needed. Although absolute temperature for photosynthesis is not needed in the model, it is important to point out that the temperature for photosynthesis is at the land surface temperature, 8°C, and that this land surface temperature is assumed to change by the same amount as global surface temperature.
3.5. Energy balance

The sun supplies the energy for the Earth's heat budget. There are several major components that affect this budget: changes in the Earth's orbit and rotation, internal processes (such as cloudiness, ice cover etc.), and variations in solar activity are some examples. (Treut, 2007).

The energy transport model used in Bice’s simple model relates global average surface temperature directly to atmospheric CO$_2$ concentration in a single equation as shown in Eq. 3.6.

\[ \Delta T = 0.01^\circ C/ppm \]

Eq. 3.6: Temperature and CO$_2$ concentration relationship used in Bice’s model (Bice, 2007)

Incorporating the Earth’s global energy balance into the model will allow parameters such as surface albedo and solar irradiance to be included in the model, providing a more realistic approximation than the single equation above. An overview of the Earth’s energy balance is given in Appendix C. Essentially, the Earth’s atmosphere creates a warmer surface temperature than blackbody emission temperature at the edge of the atmosphere due to the ability of the atmosphere to absorb and re-radiate energy back to the surface. The composition of the atmosphere affects this process.

The approximation of the atmosphere using a ‘grey atmosphere’ model (Chamberlain, 1980; Lenton, 2000) is an option identified for the improved energy balance. The main assumptions of this method are that the opacity of the atmosphere is independent of the frequency of long-wave radiation and that it exists in radiative equilibrium. Only radiative energy transfer in the atmosphere is considered, ignoring the convective transfer (more complicated ‘radiative-convective’ models deal with both of these). The ‘two stream’ approximation is most often used, where there are vertical fluxes (up and down) of thermal infrared radiation. The grey atmosphere approximation is explained more thoroughly in Appendix C.

The grey atmosphere model used by Lenton (2000) models average global surface temperature using solar flux, albedo and opacity of the molecules CO$_2$, H$_2$O and CH$_4$ present in the atmosphere.
Earth’s surface emits radiation primarily in the infrared range, between 0.5µm and 30µm. H₂O is a strong absorber in the 0.5µm to 8µm band of radiation and CO₂ a strong absorber in the 12µm to 18µm band, whilst CH₄ absorbs at the lower end of Earth’s emission spectrum.

The dependence of energy flux at the surface on $\tau_g$ (total optical opacity of the atmosphere) in Lenton’s model is slightly different to the grey atmosphere approximation explained in Appendix C.3 ($\tau_g$ is multiplied by a factor of 0.75 instead of 0.5). The example in the appendix illustrates the concept; however Lenton’s model will be used because it has the associated expressions for calculating greenhouse gas opacities, as parameterised using a radiative-convective model. The opacity expressions will be used in the model, so using the energy balance they were developed with is appropriate.

Lenton (2000) describes net downward flux of radiation absorbed on Earth’s surface by the grey atmosphere approximation as:

$$F_d = \frac{(1 - \alpha_p)S_0}{4} \left(1 + 0.75\tau_g\right)$$

Eq. 3.7: Net downward flux of radiation absorbed on Earth’s surface, W/m² (Lenton, 2000)

where: $\alpha_p =$ surface albedo, $S_0 =$ solar flux at top of atmosphere for surfaces perpendicular to radiation (1368 W/m² in Lenton’s model); $\tau_g =$ (equivalent grey) vertical opacity of the greenhouse atmosphere

The opacity is dependent on the different gases in the atmosphere and is given by Lenton as based on CO₂, H₂O and CH₄ (this is an approximation, in reality there are more gases factors contributing):

$$\tau_g = \tau(CO_2) + \tau(H_2O) + \tau(CH_4)$$

Eq. 3.8: Expression for opacity of the atmosphere (dimension less) (Kasting et al., 1993; Lenton, 2000)

The values and expressions for the opacity of each gas are derived by Lenton from results of a radiative-convective model developed by Kasting et al. (1993). It was determined by this model that the opacity of CO₂ is a function of its mixing ratio (in ppm):

$$\tau(CO_2) = 1.73(CO_2)^{0.263}$$

Eq. 3.9: Equation for calculating opacity of CO₂ from atmospheric concentration (Lenton, 2000)
Methane (CH$_4$) in the atmosphere is assumed to be constant at the pre-industrial concentration of 650 ppbv which gives a constant opacity of 0.031. This assumption is not entirely correct, as atmospheric methane has changed over time, but the model does not account for this. Water vapour opacity is dependent on water vapour pressure ($P_{H_2O}$), and calculated using the following:

$$\tau(H_2O) = 0.0126(P_{H_2O})^{0.503}$$

Eq. 3.10: Expression for calculating opacity of H$_2$O (Lenton, 2000)

$$P_{H_2O} = H(P_0e^{-(\frac{L}{RT})})$$

Eq. 3.11: Expression for calculating water vapour pressure (Lenton, 2000)

where: $L$ = latent heat per mole of water; $R$ = gas constant 8.314 J/mol/K, $T$ = temperature (K) and $H$ = relative humidity; $P_0$ = constant for water saturation curve, $1.4 \times 10^{11}$ Pa

The water vapour pressure is dependent on temperature and relative humidity, related as shown in Eq. 3.11 (called the Clausius–Clapeyron equation). The equation for water vapour opacity is derived by Lenton from Eq. 3.11 and the results of a radiative-convective model. Eq. 3.10 is an approximation of the opacity of water vapour results for this radiative-convective model, valid for temperature ranges between 0°C and 40°C. (Lenton, 2000)

The outgoing radiation at the Earth’s surface is described by the blackbody emission. Balancing this with the downward flux (Eq. 3.8) and heat capacity of the system results in the following differential equation for temperature at the surface:

$$\frac{dT_s}{dt}.c = (F_d - \sigma T_s^4).A_E$$

Eq. 3.12: Energy balance at the Earth’s surface (Lenton, 2000)

where $A_E$ = surface area of the Earth ($5.101 \times 10^{14}$ m$^2$); $c$ = specific heat capacity of the Earth ($4.69 \times 10^{23}$ J.K$^{-1}$); and $T_s$ = temperature at Earth’s surface (initially 288.15 K).

This equation will be used in the improved model for the energy balance component, replacing the single equation with temperature change dependent on atmospheric CO$_2$. The specific heat capacity for the above equation is taken from Lenton (2000), who derives the value from a particular forcing of the HadCM3 GCM (General Circulation Model). This heat capacity is mainly due to oceans.
A constant value is used for relative opacity of methane (CH₄), assuming that it remains at its pre-industrial steady-state value. The function Lenton uses for this opacity is derived from the results of a radiative-convective model. In reality, methane concentration is estimated to have risen from the pre-industrial 650ppbv to around 1600ppbv in recent times (Etheridge et al., 1998). However, the model does not account for this.

### 3.6. Summary of changes

The following table contains a summary of the changes made to Bice’s simple model:

**Table 3.2: Summary of changes made to Bice’s original simple global model**

<table>
<thead>
<tr>
<th>Parameter/Change</th>
<th>Bice’s model value</th>
<th>Explanation</th>
<th>New value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial conditions</td>
<td>Initial CO₂ is 280ppm and the model is run using data from 1890 onwards.</td>
<td>280ppm is considered to be the pre-industrial CO₂ concentration</td>
<td>Initial CO₂ concentration of 285ppm with the model starting in 1850</td>
<td>Atmospheric CO₂ is 280ppm in 1750, but temperature datasets are only available after 1850. CO₂ is 285ppm between 1800 and 1850 so 285ppm will be used as the initial value for a starting year in 1850. (page 39)</td>
</tr>
<tr>
<td>Half saturation value</td>
<td>$K_{hs} = 62.5$ ppm</td>
<td>Bice uses this value as it causes a 25% increase from initial conditions to the maximum possible photosynthesis rate ($P_{max}$)</td>
<td>$K_{hs} = 191.9$ ppm</td>
<td>Following Gifford, 191.9ppm is the value that will cause a 25% increase in photosynthesis for a doubling of atmospheric CO₂ concentration (page 50)</td>
</tr>
<tr>
<td>Temperature sensitivity of photosynthesis</td>
<td>$\theta_p = 0.04 \degree$C</td>
<td>Constant evaluated using the NPP model of Leith at a temperature of 17°C</td>
<td>$\theta_p = 0.072 \degree$C</td>
<td>Evaluated at the average land surface temperature of 8°C (page 53)</td>
</tr>
<tr>
<td>CO₂ compensation point</td>
<td>$CO_{2,\text{min}} = 30$ ppm</td>
<td>Constant following the equation used in Gifford (1993), evaluated at a temperature of 17°C.</td>
<td>$CO_{2,\text{min}} = 23$ ppm</td>
<td>Evaluating the expression in Gifford for CO₂ compensation point at a temperature of 8°C (average LST) (page 57)</td>
</tr>
</tbody>
</table>
(Table 3.2 cont.)

<table>
<thead>
<tr>
<th>Parameter/Change</th>
<th>Bice’s model value</th>
<th>Explanation</th>
<th>New value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model temperatures</td>
<td>17°C</td>
<td>This temperature is used to evaluate temperature dependent constants, so it is assumed to be the implied steady state initial temperature.</td>
<td>Energy balance initial temperature $15^\circ C = 287.15$ K $T_p = 8^\circ C$</td>
<td>All other temperatures are assumed to be coupled to anomalies in this global energy balance temperature (page 39)</td>
</tr>
<tr>
<td>Photosynthesis and respiration flux accounting for burning</td>
<td>Photosynthesis and respiration does not account for lost area due to burning</td>
<td>Burning is simply removed from the reservoir, not taking into consideration the effects this may have for other fluxes.</td>
<td>Photosynthesis (and therefore respiration) are calculated based on the land biota area minus the effective area burnt.</td>
<td>Burning implies removal of a specific mass of carbon, which is calculated as being from a particular area ($A_{burn}$). Photosynthesis is then dependent on the land biota area ($A_{LB}$) subtract $A_{burn}$ (page 16 and 31)</td>
</tr>
<tr>
<td>Surface ocean average temperature</td>
<td>$T_{SO} = 288$ K</td>
<td>Constant value</td>
<td>$T_{SO} = 288.15$ K + $\Delta$Global temp</td>
<td>Assuming surface ocean temperature to change with global average surface temperature changes</td>
</tr>
<tr>
<td>Energy model</td>
<td>$\Delta T = 0.01$/ppm</td>
<td>Single equation directly relating temperature to atmospheric CO$_2$ concentration, following Kwon &amp; Schnoor (1994)</td>
<td>Using global energy balance to calculate temperature (detailed above)</td>
<td>Includes the various factors influencing the Earth’s heat budget to a better extent. (page 61)</td>
</tr>
</tbody>
</table>
4. Model results and discussion

The software used for modelling in this project was GoldSim. Implementation has not been discussed in any detail throughout the report and model improvements, as the nature of GoldSim is such that it is very straightforward and simple to construct a model of this kind. The final models are attached in Appendix E.

A key point in the Euler method used by GoldSim is that rates of change are considered constant across the time step used. Thus, for this approximation to produce acceptable results, the time step needs to be sufficiently small for there to be only a small rate change in reality. After experimentation with different time steps, it was decided that a step of 1 year was satisfactory (refer to Appendix F for results showing this).

This model can be divided into two separate sections. The ‘carbon cycle’ section describes the carbon fluxes between the various reservoirs, and is verified by comparing simulated atmospheric CO$_2$ concentration with the historical record from Mauna Loa/Law Dome. The second part is the energy balance, modelling average global surface temperature, for which the only available verification is measured average global surface temperature (GST) datasets. The links between these two ‘sections’ are the effect of atmospheric CO$_2$ concentration on the energy balance, and the effect of temperature change on the rates of photosynthesis (and plant respiration), soil respiration, and ocean-atmosphere CO$_2$ transfer.

4.1. Comparison to Bice’s simple model

As described in the body of this report, a number of alterations have been made to Bice’s original simple model. The steady-state of Bice’s model has atmospheric CO$_2$ concentration at 280ppm and is run from 1890 onwards, using fossil fuel emissions and land use changes data for a 100 year period. The steady state of the improved model is 285ppm and runs from 1850, using data over a 150 year period. Comparisons of the two models, each with their own input data, are shown below:
Figure 4.1: Comparison of simple model, improved model and Mauna Loa/Law Dome atmospheric CO₂ measurements

Figure 4.2: Comparison of temperature data, simple and improved models

For the temperatures, the CRU data has a slope of 0.57°C over the 150 years (0.0038°C/yr). The improved model has a slope of 0.735°C over 150 years (0.0049°C/yr), considerably higher than the data. The simple model slope is 0.65°C over the 100 years it runs for (0.0065°C/yr).

Comparing two models with different steady-states, simulation periods and input data means they will differ significantly. However, the purpose of this project was to make ameliorations to all
necessary aspects of Bice’s simple model. Comparing them in this sense is therefore necessary to show all the differences and improvements. Bice’s original model has a steady-state CO$_2$ concentration of 280ppm, but this is not consistent with the CO$_2$ concentration in 1890, when Bice’s model starts with input data for emissions and land use change. The idea of the project was to develop improvements on the simple model of Bice. ‘Improvements’ refer to changing element of the model based on the reasoning discussed, and these are verified as improvements when it can be seen that they provide a better model, as verified by measured data.

4.2. Effects of temperature change on the carbon cycle

It was found that the results for atmospheric CO$_2$ concentration with no temperature change were almost exactly the same as the results using the temperature change calculated using the energy balance. That is, the temperature change in the model (effecting photosynthesis, plant respiration, soil respiration and ocean-atmosphere transfer) is not necessary for atmospheric CO$_2$ to closely follow the measured atmospheric CO$_2$ data at Mauna Loa/Law Dome. The temperature effects on the relative carbon fluxes are described in more detail in Appendix D.1 and the essential result of this is shown below in Figure 4.3. The temperature effecting the ‘$T = T + \Delta T$’ simulation is shown in Figure 4.4.

![Figure 4.3: Model results for atmospheric CO$_2$ concentration, with and without the calculated $\Delta T$.](image)
The simulation where temperature change was zero is in fact slightly closer to the Mauna Loa/Law Dome data. A rather radical inference from this could be that there has been no temperature change; hence the simulation with no temperature change best fits the measured data. However, given the simplicity of the model, this is quite an extreme conclusion to reach and the results are far from conclusive on this point. The point remains that in this model, the temperature change calculated through the energy balance component is not necessary for the correctness of the carbon cycle component, when the carbon cycle is verified with atmospheric CO$_2$ measurements.

The ‘carbon cycle’ component to the model gives results fairly consistent with atmospheric CO$_2$ measurements, and the model structure is fairly representative of what is happening on a global scale. However, the energy model developed following Lenton has a number of parameters which are questionable. These are further discussed and commented on in Appendix D.2.
4.3. Simulation from 1850 to 2000 – CO₂ and temperature

The two key model outputs that can be verified with data are atmospheric CO₂ and temperature. Figure 4.5 and Figure 4.6 show the results of the improved model compared to these two data series. The model shows good agreement with the atmospheric CO₂ data from Mauna Loa/Law Dome. There is considerable deviation from the measured data for about 30 years, between 1920 and 1950, but the model is very close to the measured data from about 1950 onwards. A close up of the simulation compared to measured data over the period 1950 to 2000 is illustrated – there is extremely good agreement with measured data.

Figure 4.5: Model atmospheric CO₂ results compared to Mauna Loa/Law Dome data over entire simulation period 1850 to 2000.

Figure 4.6: Model temperature results compared to CRU temperature data, relative to 1961-1990 mean and adjusted to be zero in 1850.
The temperature calculated follows the same general trend as the CRU global average surface temperature dataset over the period 1850 to 2000. Temperatures anomalies as calculated by the model seem to be slightly higher than the CRU dataset. There is definitely not as good correlation as with atmospheric CO$_2$ model output and measured data. Temperature data shows considerable variation, with a low point in around 1910 and high point in around 1940 not represented by the model at all. The same general trend is exhibited from about 1960 onwards.

The difficulty and issues inherent in a so called ‘global average surface temperature’ are an issue when verifying the model with a temperature dataset. There are many factors playing a part in controlling temperature, and this result highlights that perhaps more work needs to be done on the energy balance model developed. Comparing the calculated temperature anomaly to the ‘measured’ average GST from CRU assumes that these are measurements of the same temperature, or at least, that the average temperature in the dataset would be effected by changes in the energy balance global average temperature.
4.4. Simulation from 1850 to 2000 – Reservoir size changes

Over the period 1850 to 2000 the total carbon flux to the atmosphere due to anthropogenic changes was 423.21 GtC. The atmosphere reservoir increased by 181.1 GtC over this time period, which leaves 242.1 GtC to have been taken into the other reservoirs. The table below shows the changes in reservoir sizes from 1850 to 2000 and the average uptake (or loss) of carbon in GtC/yr over this period. There is a large loss from the earth reservoir as fossil fuel emissions are taken from this reservoir.

Table 4.1: Reservoir changes over the whole simulation, values given in GtC

<table>
<thead>
<tr>
<th>Time (yr)</th>
<th>Atmosphere</th>
<th>Earth</th>
<th>Land Biota</th>
<th>Soil</th>
<th>Surface Ocean</th>
<th>Deep Ocean</th>
<th>Total land</th>
</tr>
</thead>
<tbody>
<tr>
<td>1850</td>
<td>600</td>
<td>40000</td>
<td>610</td>
<td>1580</td>
<td>891.63</td>
<td>38000</td>
<td>2190</td>
</tr>
<tr>
<td>2000</td>
<td>781.1</td>
<td>39724.3</td>
<td>657.04</td>
<td>1551.7</td>
<td>904.89</td>
<td>38060</td>
<td>2208.74</td>
</tr>
<tr>
<td>Change</td>
<td>181.1</td>
<td>-275.609</td>
<td>47.04</td>
<td>-28.3</td>
<td>13.26</td>
<td>60</td>
<td>18.74</td>
</tr>
<tr>
<td>Av. GtC/yr</td>
<td>1.207333</td>
<td>-1.8374</td>
<td>0.3136</td>
<td>-0.1886</td>
<td>0.0884</td>
<td>0.4</td>
<td>0.12493</td>
</tr>
</tbody>
</table>

Between 1980 and 2000, on the other hand, all reservoirs are net sinks for the increased carbon in the atmosphere.

Table 4.2: Reservoir changes from 1980 to 2000, values given in GtC

<table>
<thead>
<tr>
<th>Time (yr)</th>
<th>Atmosphere</th>
<th>Earth</th>
<th>Land Biota</th>
<th>Soil</th>
<th>Surface Ocean</th>
<th>Deep Ocean</th>
<th>Total land</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>717.26</td>
<td>39843</td>
<td>634.25</td>
<td>1548.8</td>
<td>900.66</td>
<td>38038</td>
<td>2183.05</td>
</tr>
<tr>
<td>2000</td>
<td>781.1</td>
<td>39724.39</td>
<td>657.04</td>
<td>1551.7</td>
<td>904.89</td>
<td>38060</td>
<td>2208.74</td>
</tr>
<tr>
<td>Change</td>
<td>63.84</td>
<td>-118.609</td>
<td>22.79</td>
<td>2.9</td>
<td>4.23</td>
<td>22</td>
<td>25.69</td>
</tr>
<tr>
<td>Av. GtC/yr</td>
<td>3.192</td>
<td>-5.93047</td>
<td>1.1395</td>
<td>0.145</td>
<td>0.2115</td>
<td>1.1</td>
<td>1.2845</td>
</tr>
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</table>
4.5. Predicted future scenarios

Model results can only be totally validated when measured data is available to verify outputs. However, with a model that is shown to accurately reflect historical changes, it follows that this model may also be able to make predictions of the future. Ideally, a model should not be parameterised using the data it is predicting, which has been the case in this model to an extent, but the lack of measured data to verify parameters with is a problem.

Assuming that other parameters remain the same in the future, the inputs required for the model are predicted emissions through fossil fuel burning and land use changes. The IPCC has several ‘emissions scenarios’ which are often used in model predictions. These emissions scenarios are for the more complex GCMs (General Circulation Models) used by the IPCC in future climate predictions, but the fossil fuel and land use changes data required for this simple model can be extracted from these.

Predictions will be made for 40 years into the future and compared with predictions made using other models. Although the data has not been used in model testing and development, land use change and fossil fuel emission data estimates are available up until 2005. Following this, estimates of future use are required. The prediction scenario data is shown in the table below. This has been obtained from the IPCC data website and literature regarding their Special Report on Emissions Scenarios (SRES). There are six groups of SRES scenarios, and two of these have been selected to test with the model. The ‘A1F1’ scenario has high estimates for emission predictions, whereas AT1 has lower scenarios. In addition to these, one scenario from the 1992 IPCC report has been used, which represents stabilising of emissions, more so than the two SRES scenarios used. Zero emissions have also been considered to observe the response of the system.

Table 4.3: Future emissions scenarios used (IPCC, 2009)
*FF refers to fossil fuel emission, LU refers to land use changes

<table>
<thead>
<tr>
<th>Year (values in GtC/yr)</th>
<th>SRES A1F1</th>
<th>SRES AT1</th>
<th>IS92c</th>
<th>Zero emissions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FF</td>
<td>LU</td>
<td>FF</td>
<td>LU</td>
</tr>
<tr>
<td>2010</td>
<td>8.65</td>
<td>1.08</td>
<td>8.33</td>
<td>1.04</td>
</tr>
<tr>
<td>2020</td>
<td>11.19</td>
<td>1.55</td>
<td>10</td>
<td>0.26</td>
</tr>
<tr>
<td>2030</td>
<td>14.61</td>
<td>1.57</td>
<td>12.26</td>
<td>0.12</td>
</tr>
<tr>
<td>2040</td>
<td>18.66</td>
<td>1.51</td>
<td>12.6</td>
<td>0.05</td>
</tr>
</tbody>
</table>
For all IPCC SRES scenarios, the predicted temperature rise for the next two decades is around 0.2°C (Centre, 2009). For the results predicted by the improved model, the A1F1 scenario predicts temperature increases of 0.244°C/decade over the next two decades. AT1 predicts 0.19°C/decade, and IS92c 0.17°C/decade. This indicates that the simple model is estimating temperatures in the same range as the more complicated IPCC models.
The results of the simulation with zero emissions are shown below. Of course, these are not realistic, but this shows that even if emissions did stop, CO$_2$ concentrations would not reach their pre-industrial levels anytime in the near future.

![Figure 4.10: Results for zero emissions after 2005](image)

### 4.6. Factors effecting energy balance and ‘climate’

Anthropogenic effects are thought to potentially impact the climate in a number of ways. The only factor representing climate in this model is the global average surface temperature, which is affected by parameters in the grey atmosphere approximation and energy balance. Atmospheric CO$_2$, H$_2$O and CH$_4$ are included in the model, but there are a number of other effects not considered. For example, aerosols (air-borne particles) are an oft-discussed factor in more complex climate models.

The energy balance model of Lenton (2000) has been incorporated into the improved model in its original form, and results do show reasonable agreement with the temperature trend in the CRU average GST dataset. This ignores a number of other climatic effects, but inclusion of a complex energy model such as those used in GCMs is beyond the scope of this project. However, the lack of confidence in the energy balance parameters (refer to Appendix D.2 for results) may also translate to more complex models.

They grey atmosphere approximation of Lenton, using relative opacities, is assumed not to be effected by changes in planetary albedo or solar irradiation. The relative humidity in this model also remains constant. Changes in atmospheric methane concentration are not included either, which have been considerable. The model is developed assuming a strong effect of atmospheric CO$_2$ and
associated feedbacks on temperature, and although the theory of it is based on accepted principles, the fact remains that this model has been designed (and parameterised) with this assumption. It is unlikely that Lenton would have used this model if it did not increase temperature due to atmospheric CO$_2$ concentration, given that this is the generally accepted hypothesis. A vital point is that the atmospheric CO$_2$ increase by itself is not the sole cause of the temperature increase; water vapour feedback plays a huge role. The factors affecting the global energy balance with the other temperature relationships in the model are illustrated in Figure 4.11.

![Figure 4.11](image)

**Figure 4.11: Factors effecting global energy balance and model temperatures**

The values used in the model are 1368 W/m$^2$ for solar irradiation, ~0.225 for planetary albedo and relative humidity at 0.62. The albedo is based on the necessary value to produce a temperature of 15°C with 1368 W/m$^2$ irradiation in a radiative-convective model developed by Kasting (1993) which Lenton uses for parameterising his model. Planetary albedo is generally around 0.3. Lenton’s much smaller value parameterises the effect of clouds as modelled in the radiative convective model, and using the smaller value in the improved model (as Lenton does) therefore also parameterises this effect to an extent.

The relative humidity (H) is a factor used to calculate the opacity of the H$_2$O in the atmosphere. Relative humidity describes the amount of water vapour in the atmosphere relative to the saturated
amount at a particular temperature. This varies widely on a global scale, with the surface average being around ~0.77. Lenton (2000) uses a value of 0.62, which is less than the surface average, but more representative of the whole atmosphere according to Lenton’s reasoning. The opacity of H$_2$O in the atmosphere is dependent on the water vapour partial pressure, which is effected by temperature. Alterations to this part of the energy balance were found to have a large effect on temperature, with slight reductions of relative humidity over the simulation causing much lower temperature outputs. The results for this are again in Appendix D.2.

There are two important feedback effects on temperature in this model. The opacity of H$_2$O is dependent on temperature and provides a positive feedback effect. An increase in temperature increases opacity, thus feeding back to cause higher temperatures. The negative feedback effect is that of the dependence of surface temperature, $T_s$, on the blackbody emission term ($T_s^4$), so there is a negative feedback on surface temperature related to the fourth power of the temperature (increased $T_s$ increases this long-wave radiation, thus lowering surface temperature). (Lenton, 2000)

It was found that making relatively small changes to planetary albedo, while removing the effect of increasing CO$_2$, could produce temperature results similar to those in the control case using Lenton’s model (see Appendix D.2). The outcome of this investigation is the finding that relatively small changes to energy balance parameters can have a large effect on simulated temperature changes. The fact that these parameters are not certain, but have often been chosen so that output reflects temperature anomalies in measured GST datasets, brings a lot of uncertainty into the model. The reliability of the model in regard to calculated temperature is greatly reduced, as the changes in parameters experimented with are not unrealistic.
4.7. Feedback effects

In this model, the forcing parameter causing change is essentially the increased atmospheric CO$_2$ due to fossil fuel emissions and land use changes (removal of land biota area and carbon content from the soil reservoir are also anthropogenic change). Increased atmospheric CO$_2$ affects the model’s environmental processes in various ways, and the ‘feedback effects’ refer to the effect that changes in these environmental processes have on the initial forcing parameters. The system, although modelled in a relatively simple fashion, has complex and interacting processes.

Increased atmospheric CO$_2$ in this model increases the rate of photosynthesis (and therefore respiration) because of the CO$_2$ fertilisation effect. This provides a negative feedback effect on atmospheric CO$_2$ concentration, by the increased rate of removing carbon from the atmosphere. The average uptake of carbon by the land biota in the model was 1.2 GtC/yr between 1980 and 2000, compared uptake to the land biota was 0.31 GtC/yr over the timespan of the whole model. Other studies assuming a strong fertilisation effect have estimated net carbon uptake to be between about 1.2 and 2.6 GtC/yr in recent times. (Gifford, 1993)

This model does not include any specific parameters restricting the CO$_2$ fertilisation effect. In reality, the fertilisation may not be as large. For example, the availability of nutrients such as nitrogen may constraint the effect. (Gifford et al., 2000).

Increased atmospheric CO$_2$ causes an increase in average surface temperature when using the ‘grey atmosphere approximation’ to model the energy balance. Increased temperature has a number of effects. Photosynthesis, plant respiration and soil respiration are all temperature dependent, increasing with temperature. The net uptake of carbon from the atmosphere to the land biota (net primary production) will therefore increase with increasing temperature, providing a indirect negative feedback effect on atmospheric CO$_2$. As temperature is affected by atmospheric CO$_2$, there is an indirect negative feedback on temperature caused by net primary production. The feedback effect of soil respiration is positive – an increasing temperature will increase soil
respiration, therefore increasing CO$_2$ flux to the atmosphere, providing an indirect positive feedback.

The value of the ocean sink is reduced by increasing temperature, as CO$_2$ uptake is reduced due to the decreasing solubility. This provides a weak positive feedback. There is also a (larger) negative feedback in that increasing atmospheric CO$_2$ increases the concentration gradient for ocean-atmosphere transfer, resulting in more transfer to the ocean, reducing CO$_2$ in the atmosphere.

**4.8. Issues with verifying temperature results**

There is some ambiguity as to what the ‘measured’ temperature data is actually measuring, and whether is it an acceptable verification for the temperature as calculated in this global energy balance. Comparing anomalies in measured data and model output takes away some of the differences in comparing these two different values, but this assumes that whatever temperature is being calculated by the energy balance effects whatever temperature it is that is ‘measured’ in the average GST datasets.

These datasets are the primary ‘proof’ that average global temperature has been rising since the onset of the industrial era and its associated anthropogenic perturbations. Yet, the temperature that they are actually measuring is not exactly relevant – a global average of temperatures does not have a great deal of meaning and is not necessarily representative of ‘Earth’s temperature’. Temperature anomalies are somewhat more relevant, but there are still a number of issues in the temperature datasets such as urban heating and insufficient distribution of historical temperature records meaning that extrapolation is necessary.

Satellite temperature anomalies have been shown to reflect the effects of recent climatic events, namely the 1998 El Nino Southern Oscillation (ENSO) on the anomalies and the cooling effect following the Mt. Pinatubo volcanic eruption due to aerosols, as indicated in Figure 4.12. These anomalies are of course not subject to the same issues as the datasets based on historical
measurements from weather stations, but the relevance between these temperature measurements and the temperature being calculated in the energy balance is still not a certainty.

![UAH Globally Averaged Satellite-Based Temperature of the Lower Atmosphere (Jan. 1979 thru Nov. 2009)](image)

Given that point, there is not really another means to determine if temperature is actually changing with a more long term trend (of course it is always changing in the short term, the Earth does not exist in perfect equilibrium). So, if we accept that the ‘anomalies’ in these so called average global surface temperature datasets are an indicator of long term trends, and that these global average anomalies are effected in the same way by the kind of temperature that can be calculated from a global energy balance such as the one described here, yet more problems occur due to the uncertainties in the factors effecting this energy balance.
5. Conclusion

Global average surface temperatures are said to have risen by 0.6°C over the last century or so, and this is reflected in the model. The atmospheric CO\textsubscript{2} in the model output has a very close fit with measured data, especially in the period from 1950 to the present.

Whilst the ‘carbon cycle’ component to the model produced similar results to Mauna Loa/Law Dome, it was noticed that these results were also similar to the data if no temperature change was assumed. The energy balance model also produced results in line with the anomalies of the historical CRU dataset, but not as closely fitting the data in recent times as the atmospheric CO\textsubscript{2} results did. The temperature predictions using the improved model were found to be in line with IPCC predictions using several different emissions scenarios.

The energy balance model was used as a tool to indicate how small changes in the model parameters could have large effects due to the climatic feedback systems in operation. It was shown that selection of a series of small planetary albedo changes could produce anomalies similar to the CRU dataset, when the effect of increased CO\textsubscript{2} was ignored. As albedo could easily be wrong by a small amount, parameterising albedo for use in an energy model meant the reliability of that model in reproducing actual temperature changes was not high. Changes in relative humidity were another factor that had a strong effect on temperature output, causing lower anomalies over the simulation for small percentage changes. Consequently, the uncertainty of parameters in the energy balance, and the strong effect of small (and not improbable) changes to parameters, made the temperature predicted by the model fairly unreliable.

The model showed good agreement with the CRU temperature anomalies. However, the question of how calculated model temperature relates to the temperature these anomalies represent is an issue.

The modelling of atmospheric CO\textsubscript{2} concentration is a lot simpler and produced much better results than the temperature changes calculated by the energy balance component. Atmospheric CO\textsubscript{2}
concentration can easily be measured and the approximation of a well-mixed tank in the context of the atmosphere reservoir is valid. By using the Mauna Loa record for recent times, it is assumed that this model is parameterised to model (and predict, in the projections carried out) the atmospheric $\text{CO}_2$ concentration at Mauna Loa.

Parameterising a simple model with a more complex model helps to account for factors not included in a simple system. The energy balance developed with an approximation for a grey atmosphere is parameterised at steady state using results of a more complex radiative-convective model. Soil respiration temperature sensitivity is parameterised by matching outputs of a simple model to those of a more complex model. Although validating and finding parameter values using another model is obviously not as desirable as using measured data, it can assist with accounting for factors not specifically included in a simple model. As measured data for many variables is simply not available, estimates from other models are the next best choice.

Use of future emissions scenarios for model prediction showed some agreement with the results of more complex models. The predicted temperature increase over the next two decades from IPCC models is around $0.2^\circ\text{C}/\text{decade}$. The temperature increase for the emissions scenarios used was between $0.19^\circ\text{C}/\text{decade}$ and $0.244^\circ\text{C}/\text{decade}$. However, as previously presented, the energy balance model is shown to be sensitive to uncertainties for these results. Agreement of this model with the projections of more complex models does not provide any verification that projections are correct. It merely shows that they are in agreement with other models, which are also affected by the uncertain values discussed.

The intention of the project was to make improvements to the initial simple global carbon cycle model of Bice, and this has been successfully carried out. Comparisons between the initial model and improved model indicate that the improved model better reproduces the atmospheric $\text{CO}_2$ concentration and temperature anomalies used for verification. Inclusion of an improved energy balance was part of the task, and this too has been carried out, using a grey atmosphere approximation. This energy balance models the measured data fairly well.
The ‘control’ case energy balance included in the improved model is reliant on a relationship between atmospheric CO$_2$ concentration and temperature. However, it has been shown that temperature can be reproduced in similar ranges by ignoring CO$_2$ concentration and changing other parameters. Therefore, this model does not definitively indicate that atmospheric CO$_2$ increases are necessary for the observed temperature change, as is the generally accepted hypothesis. Nor does it indicate or ‘prove’ that CO$_2$ does not affect temperature, but it does successfully demonstrate the uncertainties in the causes of temperature change. This is important, given that uncertain parameters in this model are also uncertain parameters in more complex General Circulation Models used by bodies such as the IPPC in climate modelling and projections.

5.1. Further work

The dynamic network of carbon transfer throughout the Earth is not independent of other material flows. Nitrogen, oxygen and water are three key material cycles that are related to some processes in the carbon cycle. These materials are included in the complex models of particular aspects of the carbon cycle, such as photosynthesis models, but simple global carbon models found in literature do not appear to deal with these material flows. Further investigation and inclusion of these flows is an area identified for further work on this simple global carbon model.

The energy balance used in the improved model models radiative transfer only. This approximation is fairly removed from the reality, and not a suitable approximation for energy transfer in the opinion of some authors (Huben, 1997; Shepherd, 2002), whereas Lenton (2000) finds the results of his radiative-only energy transfer model to be in agreement with results of more complex radiative-convective models. The reason for this is that Lenton parameterises the radiative model using results of a radiative-convective model. This is a key point in the use of simple models – they can describe processes to a fairly good degree as they can be parameterised to include the more complex effects that are not directly included in the model. However, an obvious improvement would be improving the energy balance used in the model to include convective as well as radiative transfer.


Data on Global Change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy.


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Appendix B  Further explanation on selected carbon cycle processes
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Appendix D  Energy balance and temperature effects
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Appendix E  GoldSim models

The model has been developed using the simulation program ‘GoldSim’. In order to view the improved model, a version compatible with ‘GoldSim Player’ is attached electronically in the ‘Appendix E’ folder. To view this model, GoldSim Player can be downloaded from http://www.goldsim.com/.

Appendix F  GoldSim model implementation

Appendix G  Abbreviations and parameters