Using Multivariate Time Series to Estimate Location and Climate Change Effects on Historical Temperatures Employed in Future Electricity Demand Simulation

Ross S. Bowden and Brenton R. Clarke

Long-term historical daily temperatures are used in electricity forecasting to simulate the probability distribution of future demand but can be affected by changes in recording site and climate. This paper presents a method of adjusting for the effect of these changes on daily maximum and minimum temperatures. The adjustment technique accommodates the autocorrelated and bivariate nature of the temperature data which has not previously been employed. The data is from Perth, Western Australia, the main electricity demand centre for the South-West (of Western Australia) Interconnected System. The statistical modelling involves a multivariate extension of the univariate time series “interleaving method”, which allows fully efficient simultaneous estimation of the parameters of replicated VARMA processes.

Temperatures at the most recent weather recording location in Perth are shown to

Ross S. Bowden is a consulting statistician in Perth, Western Australia, and is a PhD candidate in Mathematics and Statistics, School of Engineering and Information Technology, Murdoch University, Murdoch, Western Australia, 6150 (E-mail: ross.bowden@iinet.net.au). Brenton R. Clarke is Senior Lecturer, Mathematics and Statistics, School of Engineering and Information Technology, Murdoch University, Murdoch, Western Australia, 6150 (E-mail: B.Clarke@murdoch.edu.au).
be significantly lower compared to previous sites. There is also evidence of long-term heating due to climate change especially for minimum temperatures.

**Key Words:** Temperatures; location change; climate change; electricity peak demand; VARMA.

1. **Introduction**

This paper uses time series methods to estimate and adjust for the effect of changes of location and climate on daily maximum and minimum temperature data for Perth, Western Australia, as used in electricity demand simulation for the South-West of Western Australia. In undertaking the estimation, this paper also explores the temperature data’s stochastic generating mechanism which is one of the primary determinants of daily electricity demand.

The daily maximum and minimum temperatures are modelled as a bivariate time series while accommodating and estimating the effect of location movements and climate change on the daily values. The model used also incorporates the annually replicated nature of the time series process. To accommodate this replication, the current research analyses the daily temperature data by week-in-the-year with fully efficient use of all available historical data, simultaneously for all years.

To date, none of the published temperature adjustment methods (as used for example in climate change studies) appear to have employed time series or multivariate models (see Section 1.2). In particular, the authors could identify no specific published work on the adjustment of the historical temperature record used for estimating future electricity demand.

Electricity systems are required to supply the daily and (resulting) annual maximum demand on the power system (as well as providing for aggregate energy needs). Hence there is a major focus in power supply companies on servicing the annual peak demand
from customers. To this end, $479 US billion was invested per year on average by the world’s electricity companies from 2011 to 2013 [25].

These investment decisions are critically determined by long-term (5 to 20 year) forecasts of electricity maximum demand. Any improvement in the accuracy of demand forecasting can result in substantial savings in capital expenditure. Moreover an appreciation of the probability of certain future demand outcomes allows the electricity planning staff to more exactly match the risk of plant availability with the variability of customer demand. This results in an optimal balance of capital expenditure versus system reliability.

The underlying growth in annual electricity maximum demand is primarily driven by economic and social factors. However the annual maximum demand is simply the largest of the daily maximum demands which are further determined by season in the year, day-of-the-week and public holiday effects, daily current and lagged weather variables, plus autocorrelated (but otherwise unexplained) random influences.

The primary weather variables which influence daily maximum demand and which are readily available for most locations are the daily maximum and minimum temperatures. As a point of reference, high temperatures in Western Australia in summer have strong effects on peak demand whereas low temperatures in winter have a less pronounced effect; in countries in higher latitudes the influence of winter temperatures is stronger. Either way, stable historical series of daily maximum and minimum temperatures are required for demand simulation and these depend on adjustments making allowance for location and climate change in recordings of past data. Methods for making such adjustments are discussed below.

1.1 Electricity Demand Forecasting and Weather Recordings

The methods for predicting long-term maximum demand growth typically encompasses a wide variety of approaches. These include regression and econometric models, time series
analysis (including exponential smoothing), neural networks, support vector machines and knowledge-based expert systems (see [2], [11] and [7]).

To some extent all these models use an estimate of the pattern of weather on the day of the annual maximum demand. This is typically the mean of the relevant weather variables on the maximum demand day even though there is usually little actual data on which to base a direct estimate (with only one value per year of strictly relevant historical daily temperatures). Also the weather variables of interest may include lagged and transformed values of readily available weather readings. For example variables can be constructed representing the effect of runs of days of relatively unchanged (but hot) weather which can see a gradual increase in electricity demand due to the so-called heat bank effect in homes and businesses. Typically some form of running total of weighted daily degree-days over say 35 degrees Celsius is employed here. Hence not only will there typically be a very limited number of days of annual maximum demand on which to base an assessment of the typical peak day weather but there are also subtle relationships with the weather occurring on peak and past days.

A recent approach to addressing these issues [12] is illustrated in Figure 1. This method forecasts both the mean and the distribution of annual maximum demands in any future year by using a detailed and sophisticated regression model between historical daily maximum electricity demand and weather (as well as other variables discussed earlier such as day-of-the-week). The daily regression model is typically fitted using daily demand and weather data for the past 5 to 10 years. This approach effectively allows the estimation of the (daily) maximum demand on any day given the determinants listed above. The model is then used with a forecast of growth (discussed previously) and say sixty years of year-by-year historical daily weather data (as well as simulated daily autocorrelated error terms) to repeatedly generate daily electricity demands for a future year. This then results in replications of the forecast annual maximum demand for that same future year. There will be one simulated annual maximum demand for each year of available historical
weather data. Empirical distributions can be created of those simulated future annual electricity demands (which, of course, take as given the underlying forecast of growth).

This is particularly useful for modeling the balance of possible electricity demand versus possible available power plant capacity because it creates an empirical distribution of forecast annual maximum demand. Amongst other metrics, this allows for the estimation of the so-called once-in-ten-year demand forecast (which, given an accurate forecast of growth, has a 50% chance of being exceeded once every ten years).

A critical assumption here is that the historical weather record is stable, reflecting at least current patterns. However this is unlikely to be true for two reasons. It is now well established that global temperatures are increasing due to climate change. The Australian Commonwealth Scientific and Industrial Research Organisation (CSIRO) conclude that “Australia’s climate has warmed by 0.9°C since 1910” [6] with similar changes for Perth, Western Australia, of between 1.0°C and 1.5°C. This will likely have influenced the historical temperature record used in demand simulation. Also recording stations often change location over time for various reasons such as alterations in the use of the site. It is possible that this has also lead to changes in the level of recorded temperatures (see the next section).

Both of these effects will influence the long-term historical weather record. It is estimated that a one degree Celcius increase in daily maximum temperatures in summer can increase maximum electricity demand in the South-West Interconnected System (SWIS) by up to 2% [4] (The annual system maximum demand in the SWIS occurs in summer when the main drivers of demand are airconditioning and refrigeration). This temperature relationship (with appropriate seasonal variation) is employed in electricity demand simulation.

Moreover the time series generating mechanism of the daily temperature data should be consistent (that is, replicated) from year-to-year within the expected seasonal cycles. This would be expected from the seasonally recurring nature of the local climate system.
Figure 1: Forecasting the distribution of annual electricity maximum demand using simulation.
Therefore, whilst accounting for the replicated bivariate autocorrelated nature of the data, the historical readings for Perth should be adjusted for location and climate change. This paper estimates the effect of these changes on daily temperature readings for Perth, Western Australia, which is the main centre of electricity demand for the SWIS.

1.2 Adjustment of Historical Weather Records for Extraneous Effects

The long-term daily weather data available in Europe and the USA are reviewed in [24] and [16] respectively. They conclude that adjustment of the data is required because of the number of sites where disturbance of the historical record has resulted in substantial data contamination. A similar conclusion for a group of US sites is reached by [19].

To adjust for these issues, the World Meteorological Organization (WMO) has published guidelines for undertaking so-called homogenisation of weather records for (amongst other effects) location changes [1]. The WMO guidelines focus initially on identifying so-called break-points which are suspected of initiating a change in (at least) the level of the time series process but are of unknown date. Once these dates are identified (which is already the case with the Perth data), the time series is then adjusted for these interventions to match the level of the most recent recording station so that on-going adjustment is not required (at least in the medium term). The adjustment methods which include averaging, site differencing and regression typically do not account for the autocorrelated multivariate nature of the data.

The published methods up to 1998 are summarised in [18]. As with the WMO guidelines the authors firstly discuss detecting a change in homogeneity and then review adjustment methods. The authors discuss approaches by country and region. In common with the WMO methods, there is little accommodation for the autocorrelated or multivariate nature of the daily readings. Reviews of methods for identifying change points with respect to homogeneity and for subsequently adjusting the data series are contained in [21] and [10]. The adjustment methods focus on simple average differences with re-
spect to a relatively stable reference site. The actual site adjustments for a New Zealand dataset from [21] were from 0.02 to 1.58°C. Similar methods to estimate the location and urbanisation effects of recording stations around Beijing are employed by [26] with site effects of between 0.43 and 0.95°C.

1.3 Statistical Modelling of Replicated Bivariate Temperature Time Series

The time series modelling of daily temperatures used in this paper employs the interleaving method (see [3]) extended herein to the multivariate case to model replicated realisations of a multiple time series process (that is, the Perth daily maximum and minimum temperature records since 1943). This extension allows fully efficient estimation of locational and climate change effects as well as of the time series coefficients. The time series analysis uses Vector Autoregressive Moving Average (VARMA) models which are a multivariate extension of the univariate ARMA models (see Section 2).

Section 2 in this paper provides a brief overview of VARMA models and Section 3 extends the univariate interleaving method to multivariate processes. Section 4 uses the interleaving methodology to estimate the effect of location and climate change by week and season on over sixty years of daily maximum and minimum temperatures for Perth. It also explores the time series structure which is the generating mechanism behind one of the major day-to-day determinants of electricity demand.

In this paper the terms VARMA and VARMAX (VARMA with extraneous inputs) are used interchangeably. The term VARMA is employed in general to refer to multivariate extensions of autoregressive moving average models. However, when considered necessary to be explicit concerning extraneous inputs, the term VARMAX will be employed.
2. VARMAX Models

A Vector Autoregressive Moving Average process of order \(p\) and \(q\) with extraneous inputs (VARMAX(\(p,q\))), \(\{x_t\}_{t=1}^n\), is a \(k\)-dimensional multiple time series generated by the model,

\[
\Phi(B)(x_t - \mu_{x_t}(z_t)) = \Theta(B)a_t
\]

(2.1)

where \(\{a_t\}_{t=1}^n\) is a series of \(k\)-dimensional independent identically distributed random error vectors with constant variance matrix, \(\Sigma_a\), \(E(a_t) = 0\) for all \(t\) and \(E(a_t a_u') = 0\) for all \(t \neq u\). Also \(\Phi(B)\) and \(\Theta(B)\) are matrix polynomials in \(B\), the backshift operator, of order \(p\) and \(q\) respectively. The roots of \(\det(\Phi(B)) = 0\) and \(\det(\Theta(B)) = 0\) all lie outside the unit circle ensuring stationarity and invertibility respectively.

Typically the zeroth order matrix in the polynomial, \(\Phi(B)\), is an identity matrix and similarly for \(\Theta(B)\). In this case, \(\Sigma_a\) is of general symmetric positive-definite form and this specification results in a canonical formulation for the VARMAX model which allows for unique identification. It is often assumed that the innovations are multivariate normal (This is not required for Theorem 1 in Section 3 although it is needed for our model fitting).

We will assume that \(\mu_{x_t}(z_t) = E(x_t|z_t) = \Psi z_t\) where \(\{z_t\}_{t=1}^n\) is a series of explanatory (input) vectors and \(\Psi\) is the matrix of regression parameters.

The above is one form of standard VARMAX specification. The VARMAX process also could be described as being generated by a seemingly-unrelated regression model (see [14]) with VARMA errors in that the series mean vector corrects the series mean level to zero before application of the VARMA filter. However if the extraneous variables are introduced on the right hand-side of (2.1) their influence on the time series vector can only be assessed with knowledge of the VAR filter. This alternative expression of the VARMAX model is,
\[ \phi(B)x_t = \Upsilon z_t + \theta(B)a_t, \]  

(2.2) 

where \( \Upsilon = \phi(1)\psi \), that is, 

\[ \psi = \phi(1)^{-1}\Upsilon. \]  

(2.3) 

This specification (2.2) is used by the software employed in this paper (the R package, dse) to fit the replicated VARMAX model to daily temperatures.

An alternative specification of \( \phi(B) \), \( \theta(B) \) and \( \Sigma_a \) is possible which allows unique identification (See [15] pp. 447ff). This uses the unique Cholesky LDL decomposition of the innovations covariance matrix, that is, \( \Sigma_a = LDL' \) where \( L \) is upper triangular with a unit diagonal (so-called "unitriangular") and \( D \) is a diagonal matrix.

This alternate specification to (2.1) is,

\[
(L^{-1} + L^{-1}\phi_1 B + L^{-1}\phi_2 B^2 + \ldots + L^{-1}\phi_p B^p)(x_t - \mu_x(z_t)) \\
= (I + L^{-1}\theta_1 L B + L^{-1}\theta_2 L B^2 + \ldots + L^{-1}\theta_q L B^q)u_t. \quad (2.4)
\]

where \( u_t = L^{-1}a_t \) and hence \( V(u_t) = D \).

This now provides a representation with a diagonal innovations covariance matrix but where the zeroth order MA matrix is an identity matrix and the zeroth order AR matrix is upper unitriangular because \( L^{-1} \) is upper unitriangular. This AR formulation explicitly makes the first element of \( x_t \) (that is, \( x_{1,t} \)) a linear function of elements \( (x_{2,t}, \ldots, x_{k,t}) \) as well as other elements of \( x_t \) at non-zero lags. Similarly \( x_{2,t} \) is a linear function of
(x_{3,t}, \ldots, x_{k,t}) as well as other elements of x_t at non-zero lags, and so on.

In the context of the bivariate application in Section 4, this formulation which is used in this paper appears more natural as the x_{1,t} and x_{2,t} are recorded sequentially on the same day and is similar to that of a periodically correlated ARMA model (see [17]).

3. The Interleaving Method

In this section, we prove that replicated independent VARMAX processes can be represented as a single VARMAX process with the same dimension as each of the replicated series. This result allows model fitting using existing VARMAX software.

3.1 Replicated VARMAX Process

Let the i^{th} replicated k-dimensional vector series over the time span, t = 1,\ldots,n, be \{x_{i,t}\}_{t=1}^n, i = 1, \ldots, m and assume each series is generated by the following VARMAX(p,q) model,

\[ \phi(B)(x_{i,t} - \mu_{x_{i,t}}(z_{i,t})) = \theta(B)a_{i,t} \]  

(3.1)

where \{a_{i,t}\}_{i=1}^n is a series of independent zero-mean identically-distributed random error vectors with \(E(a_i a_j') = \Sigma_a\) for i = j and t = u and 0 otherwise. Hence the error vectors have a variance (matrix) of general form but otherwise the vectors are assumed to be independent between realisations at all lags and within realisations at all non-zero lags. Also \(\phi(B)\) and \(\theta(B)\) are matrix polynomials in B, the backshift operator, of order p and q respectively.

The (conditional) mean of \(x_{i,t}\) is,

\[ E(x_{i,t}|z_{i,t}) = \mu_{x_{i,t}}(z_{i,t}) = \Psi z_{i,t} , \]

where \(\{z_{i,t}\}_{t=1}^n, i=1,\ldots,m\) are m series of explanatory vectors. It is possible to effectively
have a unique parameterisation of $\psi$ (say $\psi_i$) for each realisation, $i$, through simply expanding the dimension of the input vector, $z_{i,t}$, by a factor of $m$.

We will call the time series (3.1) a RVARMA (replicated VARMA) process, that is, RVARMA($p, q, m$). It has a mean which can vary with each series realisation but it otherwise maintains a consistent generating mechanism between realisations. In fact the mean can be any linear combination of the extraneous vector variables, $z_{i,t}$ (and, in general, can be a non-linear function of the $z_{i,t}$).

3.2 Equivalent Replicated VARMAX Representation

We now state a theorem that reduces the apparent dimensionality of the replicated process by a factor of $m$. The proof is contained in an appendix.

**Theorem 1.** Let the replicated $k$-dimensional series $\{x_{i,t}\}_{t=1}^n$, $i = 1, ..., m$, be generated by the above RVARMA($p,q,m$) process (see (3.1)), and let,

$$
\begin{align*}
    y_{m(t-1)+i} &= x_{i,t}, \\
    w_{m(t-1)+i} &= z_{i,t} \text{ and} \\
    \epsilon_{m(t-1)+i} &= a_{i,t}.
\end{align*}
$$

Then,

$$
\phi(B^m)(y_s - \mu_{y_s}(w_s)) = \theta(B^m)\epsilon_s
$$

where $E(\epsilon_s) = 0$, $V(\epsilon_s) = \Sigma_a$ and $E(\epsilon_s\epsilon_r') = 0$, $s \neq r$. That is, the interleaved series, $\{y_s\}_{s=1}^{mn}$, is a $k$-dimensional VARMA process of order $(mp, mq)$.

To paraphrase Theorem 1 (herein called the Multivariate Interleaving Theorem), any $m$ replicated independent $k$-dimensional VARMA($p,q$) time series, each of length $n$, can be represented by one $k$-dimensional VARMA($mp, mq$) process of length $mn$. This equivalence is achieved by interleaving the $m$ series and by ensuring that AR and MA parameters
are only non-zero at orders that are multiples of $m$. The equivalence uses an interleaving which is illustrated in Figure 2 for two artificial bivariate series, each of length seven.

![Interleaving of an artificial bivariate series](image)

**Figure 2:** Multivariate interleaving of an artificial bivariate series.

By using VARMA software such as R’s dse and MTS packages [20], Scilab’s Grocer [22] and Gauss’s Time Series MT [8] (which all allow subsets of the VAR and VMA matrix parameters to be set to zero) the interleaving method can be employed in RVARMA model fitting without preparing purpose-built computer programs. These packages use maximum likelihood estimation. Of course, the interleaving method can also be applied to estimation approaches other than those employed in this paper including robust methods and least squares.
4. Effects of Location and Climate Change on Daily Maximum and Minimum Temperatures for Perth, Australia

This section estimates the effect of location and climate change on the daily maximum and minimum temperature readings from 1943 to 2009 for Perth, Western Australia. The results can be used to adjust the historical record employed in electricity demand simulation. As mentioned previously, the dse package from R is used to fit the associated RVARMA model via (conditional) maximum likelihood. The modelling also provides an understanding of the relationship between maximum and minimum temperatures which is informative in forecasting daily electricity demand up to a week ahead.

Figure 3 plots the daily maximum and minimum values for three years and it is clear that there is a strong relationship between values on the same day. The expected seasonal cycle is also evident as is an increase in the variability of the maximum temperatures over summer.

Given the increased variability in summer and the known changes in weather patterns between summer and winter it is likely that VARMA models of the bivariate daily temperature data vary over the year. However the models are likely to remain unchanged for any particular part of the year. Hence the modelling in Sections 4.1 to 4.3 was undertaken separately for each week-in-the-year of daily data but simultaneously for all years. This provides estimates of the effect of location (using binary intervention variables which are similar to dummy variables in regression analysis (see [4] p. 259)) and climate change (via a trend term) by week-in-the-year and ultimately by month and season. The five locations where the temperature data was collected are listed in Table 1.

4.1 Model Identification with Interleaving

To begin the RVARMA model fitting, an interleaved bivariate series by each week-in-the-year was created using the daily maximum and minimum temperatures from 1943 to
Table 1: Change of location for Perth’s temperature recording device. The readings for King’s Park begin in the current analysis on 1st January 1943.

<table>
<thead>
<tr>
<th>Location</th>
<th>Last Recording Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>King’s Park</td>
<td>August 1963</td>
</tr>
<tr>
<td>Old Hale School</td>
<td>June 1967</td>
</tr>
<tr>
<td>Wellington St</td>
<td>May 1992</td>
</tr>
<tr>
<td>Perth Airport</td>
<td>November 1993</td>
</tr>
<tr>
<td>Mt Lawley</td>
<td>December 2009</td>
</tr>
</tbody>
</table>

2009. The RVARMA model order was determined by examining the raw sample cross-correlations (see [4]), the prewhitened sample cross-correlation function (see [13] and [9] pp. 237ff) and the sample partial lag autocorrelation function [23] after correction for all intervention effects (herein called detrending) and adjusting for interleaving. For the first week of interleaved data ($n = 67 \times 7$) these correlations are plotted in Figures 4, 5 and 6 respectively with the 95 percent confidence intervals shown as dashed lines. The full (and hollow - see below) points indicate the interleaved lags ($-3 \times 67, \ldots, 3 \times 67$) that correspond to lags -3 to 3 in the original series.

Detrending involved identifying and fitting a univariate interleaved AR(2) model [3] separately to both daily maximum and minimum temperatures with associated intervention terms for the change in location and for the long-term trend in temperature. The resulting intervention terms (without the autoregressive filters) were then used to adjust the two univariate time series for the change in location and for the long-term increase in temperatures. The resultant zero-mean series were then prewhitened.

Prewhitening applied the AR(2) filter from the AR detrending model for daily minimum temperatures to both the detrended daily maximum and minimum temperatures. The sample cross-correlations were then calculated for the two filtered series. This prewhitening applied the constraints imposed by interleaving, that is, only parameters for AR orders corresponding to multiples of the number of replicated series (years) were
Figure 3: Three years of daily maximum and minimum temperatures (in degrees Celsius). The x-axis values are the week-in-the-year and the plot reveals the seasonality in mean and variance.

As mentioned above, the sample partial lag correlation matrix was also employed in model selection [23]. The sample partial lag correlation matrix is the sample cross correlation matrix at a lag of $k$ time intervals after removing the (linear) influence of the intervening lags. For an AR($p$) process the correlations cut off at lag $p$ as with the multivariate partial autoregression matrix.

To accommodate the constraints imposed by interleaving, the partial lag correlations were derived by only fitting autoregressions to multiples of the number of replicated series (that is, of years). In the approach of [23] for estimating the partial lag correlations, this implies fixing the (detrended) sample cross-correlations to zero for the other lags before...
4.2 Identification Results

In Figure 4, the sample detrended cross-correlations between maximum and minimum temperatures for week one (as indicated by the black dots at lags that are multiples of 67) show a range of significant values. However the sample prewhitened detrended cross-correlations (in Figure 5) show significant values at lags 0 and -1 only (that is, lags 0 and -67 with interleaving). This demonstrates the ability of prewhitening to substantially simplify the model selection process within an interleaving paradigm.

Sample partial lag correlations between maximum and (lagged) minimum temperatures are plotted in Figure 6. As with cross-correlations the values corresponding to lags $-3$ to 3 in the original replicated series are marked by full dots. The values reveal significant correlations at lags $\pm1$ and arguably at $\pm2$. The hollow dots indicate partial lag correlations calculated without setting relevant intermediate correlations to zero.

Given that these correlation results were similar for all weeks, it was decided to fit a RVAR model of order two (that is, a RVAR(2,0,67) model) for each week. This was undertaken using the interleaving method from Section 3 (with $m=67$, that is, a total sample per week-in-the-year of $67 \times 7 = 469$) and employing the method of conditional maximum likelihood via the R package, dse.

The dse package uses the VARMAX representation (2.2) but the results in this paper employ the representation (2.4), derived by applying the Cholesky LDL transformation from Section 2 and the transformation of the process mean from (2.3). Hence the VAR(2) models in this paper utilise an intervention vector that is the (conditional) mean of the process, a zeroth order VAR matrix that is upper unitriangular and a variance matrix of the innovations vector that is diagonal.

This formulation permits what is arguably a simpler interpretation of the estimates whereby the mean correction due to the intervention terms is applied directly to the vector
Figure 4: Detrended cross-correlations using interleaving (week 1) with 95% confidence intervals. The black dots indicate the sample correlations at lags -3 to 3 in the original series. The lack of prewhitening makes order selection difficult.

Figure 5: Prewhitened detrended cross-correlations using interleaving (week 1) with 95% confidence intervals. The black dots indicate the sample correlations at lags -3 to 3 in the original series. Prewhitening is effective in simplifying the cross-correlations.

Figure 6: Wei’s partial lag detrended correlations using interleaving (week 1) with 95% confidence intervals. The black dots indicate the sample correlations at lags -3 to 3 in the original series. The circles are the same values uncorrected for assumed zero correlations at lags that are not multiples of the number of replicated series (67 in this case). The significant correlations (across all weeks-in-the-year) suggest an AR(2) process.
time series before application of the AR filter, the daily maximum temperature is related
to the (earlier) minimum temperature on the same day and the elements of the innovation
vector are independent.

The standard errors of the sample parameter estimates for model (2.4) were not imme-
diately available from the model fits and had to be derived after transformation from the
fitted model (2.2) using simulation (10,000 simulations). To achieve this, the estimated
innovations variance matrix for (2.2) is assumed to be independent of the other sample
parameter estimates for (2.2). Repeated realisations of the sample variance matrix of
the innovations from the fitted model (2.2) were simulated using bootstrapping on the
model’s residual vectors. The other parameter estimates from (2.2) were randomly sim-
ulated using an assumption of multivariate normality where the mean vector is the VAR
and intervention parameter estimates (in (2.2)) and the variance matrix is the associated
Hessian-derived variance matrix.

Each set of combined simulated parameter estimates for (2.2) was transformed using
the LDL transformation from Section 2 used in model (2.4) and the mean transformation
(2.3). The empirical distribution of the resulting simulated parameters was then used to
calculate the sample variance matrix of the parameter estimates for (2.4). The square roots
of the diagonal elements of the matrix were used as standard errors of the transformed
parameters estimates shown in Figures 7, 8, 9 and 10.

4.3 Estimation of the Location and Climate Change Effects and of the VARMA
Generating Mechanism

The VAR parameters are plotted by week in Figure 7. Note that these are the nega-
tive of the VAR parameters from representation (2.4) to reflect their use in a predictive
formulation.

It is clear that the parameters change substantially and relatively smoothly over the
year with the strongest relationship between maximum temperatures and past maximum
and minimum temperatures in summer with little relationship in winter. The minimum
temperatures show a much weaker set of relationships although the VAR parameters are
now generally strongest in winter.

![Graphs showing relationships between maximum and minimum temperatures](image)

Figure 7: VAR(2) parameters by week with 95% confidence intervals. There is evidence
of seasonality in the AR parameters for the maximum and, at a lesser extent, minimum
temperature models.

The variances of the (independent) innovations by week are shown in Figure 8 and
indicate that the variation of the maximum temperatures changes substantially over the
year with the greatest variance in summer. The minimum temperatures show a relatively
unchanged variance.

The RVARMA modelling by week produced estimates of the effect of location and
climate change. However the week-by-week results were busier than the VAR parameters
already discussed and made application difficult. Hence the results were averaged by
Figure 8: Innovation variance by week with 95% confidence intervals. Again there is evidence of seasonality, at least for maximum temperature innovations.

month (that is, by 28 days) with associated calculation of standard errors (see below) and these were used in the effect plots in this paper.

Given the autocorrelated nature of the data, it is likely that the weekly intervention estimates are autocorrelated. Hence the standard errors of the monthly mean effect estimates were derived using the well-known result for the variance of the mean of correlated random variables (See [5] pp. 15f.). In this it was assumed that only the (auto-)correlation between adjacent week’s intervention terms is non-zero. This correlation was estimated using simulation which employed the estimated VARMAX models to repeatedly generate new bivariate input series. The models used in this paper were then fitted to the simulated data and the sample week-to-week correlation of the resulting parameter estimates was derived from the simulated estimates. These correlations increased the standard errors of
the estimated effects by approximately 10 percent compared to an independence model.

The top left plot in Figure 9 shows the mean daily maximum temperatures by month for the base site used to 1963 and exhibit the expected seasonal cycle. The next four plots show the mean difference by month between the data recorded at the site for the period in question and the base site. The sixth plot shows the annual trend in maximum temperatures (by month) to 2009.

Over and above climate change, the results suggest colder temperatures in summer for the Wellington St site (1967-92) and the current Mt Lawley site (1993-09). This implies that maximum temperatures recorded at Mt Lawley in summer are likely to underestimate the true maximum temperature compared to the historical record from King’s Park (pre-1963).

To accommodate the apparent variation in effects by season, Table 2 shows the mean maximum temperature effects by season (summer = months 1 to 3, autumn = 4 to 5, winter = 6 to 8, spring = 9 to 13 which were chosen to group together months showing similar VARMA parameter estimates and mean temperatures) with standard errors and t statistics (significant effects are indicated by“*”). The seasonal standard errors used the same approach as for the monthly standard errors.

The estimated annual (positive) trend in maximum temperatures from combining the fifty-two weeks’ results is 0.0147°C (±0.0108 being 95% confidence limits). This equates to 0.98°C (±0.72) as the total increase over 67 years. There appears to be a higher rate of increase for summer compared to other seasons.

For minimum temperatures (see Figure 10) the monthly results for locality are similar to those for maximum temperatures although there is additional evidence of lower minimum temperatures over the whole year at the current Mt Lawley site. To again accommodate the apparent variation in effects by season, Table 2 shows the mean effects by season. Over the fifty-two weeks the mean difference for the Mt Lawley site compared to the King’s Park site is −1.84°C (±0.40). The (positive) annual trend in minimum
temperatures is 0.0179°C (±0.0078) or 1.20°C (±0.52) as a total over 67 years. Again as with the maximum temperatures, there appears to be a higher rate of increase for summer compared to other seasons.

Table 2: Seasonal effect of change of location and of climate (summer = months 1 to 3, autumn = 4 to 5, winter = 6 to 8, spring = 9 to 13).

<table>
<thead>
<tr>
<th>Season</th>
<th>Old Hale School</th>
<th>Wellington Street</th>
<th>Perth Airport</th>
<th>Mt Lawley</th>
<th>Annual Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Effect</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summer</td>
<td>-0.19</td>
<td>-1.33</td>
<td>-1.14</td>
<td>-1.25</td>
<td>0.0270</td>
</tr>
<tr>
<td>Autumn</td>
<td>-0.44</td>
<td>-0.24</td>
<td>-0.62</td>
<td>-0.01</td>
<td>0.0291</td>
</tr>
<tr>
<td>Winter</td>
<td>0.52</td>
<td>0.16</td>
<td>-0.42</td>
<td>0.40</td>
<td>0.0031</td>
</tr>
<tr>
<td>Spring</td>
<td>0.04</td>
<td>0.07</td>
<td>-0.24</td>
<td>0.66</td>
<td>0.0085</td>
</tr>
<tr>
<td>SE</td>
<td>0.43</td>
<td>0.45</td>
<td>0.94</td>
<td>0.75</td>
<td>0.0146</td>
</tr>
<tr>
<td>t Value</td>
<td>-0.44</td>
<td>*-2.97</td>
<td>-1.21</td>
<td>-1.67</td>
<td>1.85</td>
</tr>
<tr>
<td>Summer</td>
<td>-1.26</td>
<td>-0.66</td>
<td>-0.81</td>
<td>-0.01</td>
<td>*2.47</td>
</tr>
<tr>
<td>Autumn</td>
<td>*2.96</td>
<td>0.90</td>
<td>-1.31</td>
<td>1.32</td>
<td>0.53</td>
</tr>
<tr>
<td>Winter</td>
<td>0.13</td>
<td>0.24</td>
<td>-0.45</td>
<td>1.39</td>
<td>0.92</td>
</tr>
<tr>
<td>Spring</td>
<td>0.27</td>
<td>0.28</td>
<td>0.53</td>
<td>0.47</td>
<td>0.0092</td>
</tr>
<tr>
<td><strong>Minimum Temperature</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Effect</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summer</td>
<td>-0.83</td>
<td>-1.38</td>
<td>-3.83</td>
<td>-3.34</td>
<td>0.0346</td>
</tr>
<tr>
<td>Autumn</td>
<td>0.05</td>
<td>0.26</td>
<td>-1.37</td>
<td>-1.66</td>
<td>0.0175</td>
</tr>
<tr>
<td>Winter</td>
<td>0.04</td>
<td>0.61</td>
<td>-0.38</td>
<td>-1.40</td>
<td>0.0029</td>
</tr>
<tr>
<td>Spring</td>
<td>-0.06</td>
<td>0.26</td>
<td>-1.94</td>
<td>-1.29</td>
<td>0.0170</td>
</tr>
<tr>
<td>SE</td>
<td>0.20</td>
<td>0.20</td>
<td>0.43</td>
<td>0.34</td>
<td>0.0066</td>
</tr>
<tr>
<td>t Value</td>
<td>-4.21</td>
<td>*-6.79</td>
<td>*-8.97</td>
<td>*-9.81</td>
<td>*5.22</td>
</tr>
<tr>
<td>Summer</td>
<td>0.19</td>
<td>0.88</td>
<td>*-2.25</td>
<td>*-3.41</td>
<td>1.84</td>
</tr>
<tr>
<td>Autumn</td>
<td>0.12</td>
<td>1.88</td>
<td>-0.64</td>
<td>*-2.51</td>
<td>0.27</td>
</tr>
<tr>
<td>Winter</td>
<td>-0.38</td>
<td>1.63</td>
<td>*-6.74</td>
<td>*-4.81</td>
<td>*3.26</td>
</tr>
</tbody>
</table>


Figure 9: Estimates by month for maximum temperatures of the effect of change of location and of trend over time with 95% confidence intervals. The plot of annual trend also shows the overall mean annual change as a horizontal full line. The results suggest colder temperatures in summer for the Wellington St site (1967-92) and the current Mt Lawley site (1993-09). Also there is a total increase over 67 years of 0.98 °C (±0.72).

5. Conclusions

In this paper the effects of location and climate change on historical temperature recordings for Perth, Western Australia, (as employed in electricity demand simulation) have been estimated. The analysis used multivariate time series models and incorporated a multivariate extension of the univariate interleaving method [3]. The interleaving approach allows replicated realisations of the same VARMA process to be modelled as a single VARMA series of the same dimension as each of the original series but with extended length. The analysis show that the current weather recording site for Perth has colder
temperatures than the original location and there was also evidence of rising temperatures due to climate change.

Accordingly, in order to prepare a stable time series of daily readings for electricity demand simulation, it is recommended that the daily historical values for maximum and minimum temperatures be adjusted for location effects using the mean seasonal effects detailed above. The effects of climate change should be incorporated by using the associated annual growth factors by season. The effects of climate and site change will adjust the temperature readings for certain locations by over two degrees Celsius. Hence the
associated simulated electricity demand forecasts could change by up to 4% (see Section 1). It is further recommended that any long-term time series of temperature data used in electricity demand simulation be examined and adjusted for the effects of climate and location change.

Acknowledgements

The authors would like to acknowledge the work and advice of Dr Paul Gilbert, the author of the R multiple time series package, dse, used in the final model fitting. We also acknowledge the assistance of Mr Éric Dubois, the author of the Scilab econometrics package, Grocer, and Professor Ruey Tsay, the author of the r package, MTS, both used at an earlier stage of the current research. Finally we thank Professor Adrian Bowman for his insightful comments on a draft of the paper.

References


Appendix: Proof of the Multivariate Interleaving Theorem

Proof. Define \{y_s\}, \{w_s\} and \{\epsilon_s\} using (3.2). For a given \(i = 1, ..., m\), let \(s = m(t-1)+i, t = 1, ..., n\) and the difference equation (3.1) can be expressed as,

\[
\Phi(B^m)(y_s - \mu_{y_s}(w_s)) = \Theta(B^m)\epsilon_s
\]

where \(s = i, m + i, 2m + i, ....\). It is also known that, for \(s = i, m + i, 2m + i, ....\), \(E(\epsilon_s) = E(a_{i,t}) = 0, V(\epsilon_s) = V(a_{i,t}) = \Sigma_a\) and \(E(\epsilon_s\epsilon_r') = E(a_{i,t}a'_{i,u}) = 0, s \neq r,\) where \(r = i, m + i, 2m + i, ....\). We finally note that the replicated multivariate time series are independent so \(E(\epsilon_s\epsilon_{r^*}) = 0\) for all \(s \neq r^*\) where \(s = 1, ..., mn\) and \(r^* = 1, ..., mn\). Hence the replicated process (3.1) leads to the interleaved process (3.3).