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Risk preferences of Australian academics: where retirement funds are invested tells the story

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Abstract: Risk preferences of Australian academics are elicited by analyzing the aggregate distribution of their retirement funds (superannuation) across available investment options. Not more than 10% of retirement funds are invested as if their owners maximize expected utility under the assumption of constant relative risk aversion with an empirically plausible level of risk aversion. An implausibly high level of risk aversion is required to rationalize any investment into bonds when stocks are available. Not more than 36.54% of all investments can be rationalized by a model of loss averse preferences. Moreover, the levels of loss aversion typically reported in the experimental studies imply overinvestment in bonds, which is not observed in the data. Up to 67.18% of all investments can be rationalized by rank-dependent utility or Yaari's (1987) dual model with empirically plausible parameters. A median Australian academic behaves as if maximizing rank-dependent utility with parameter $\gamma \in [0.76, 0.79]$ in a Tversky and Kahneman (1992) probability weighting function.

Keywords: Risk preference; Expected utility theory; Loss aversion; Rank-dependent utility; Optimal portfolio investment; Superannuation

JEL Classification codes: D81

Risk preferences of Australian academics: where retirement funds are invested tells the story

In any choice situation under risk, the observed decisions reveal some information about risk preferences of a decision maker. The attitudes towards risk inferred from one choice situation can be used for predicting the decision maker's behavior in another choice situation. Thus, an elicitation of risk preferences becomes a valuable microeconomic tool. Yet, few decision problems present themselves as choices under risk. Most involve subjective probabilities (making the problem of a choice under uncertainty or ambiguity) and imprecise payoffs. Such additional layers of complexity interfere with the elicitation of risk attitudes, often making the task difficult or even impossible.

To overcome such difficulties, risk attitudes are often elicited in a controlled laboratory experiment. Yet, the elicitation of risk attitudes by experimental methods typically relies on hypothetical or small real incentives. For instance, Wakker and Deneffe (1996) designed a tradeoff method for eliciting a (Bernoulli) utility function over money.[1] Under expected utility theory, risk preferences of a decision maker are completely captured by the curvature of his/her utility function. Yet, for observing any significant change in the curvature, the utility function is elicited over a range of large outcomes that are often not feasible in a laboratory experiment. Consequently, the tradeoff method is typically used with hypothetical incentives (*e.g.*, Abdellaoui, 2000; Bleichrodt and Pinto, 2000; see, however, also Johnson *et al*. (2015)).

An incentive-compatible method of preference elicitation often relies on binary choices. Yet, one binary choice reveals little information about risk preferences and a decision maker is usually asked to make several binary choices. For example, Holt and Laury (2002) designed a popular method of eliciting risk preferences where a decision maker faces ten binary choices. Yet, decisions observed in several binary choices may not always "pin down" the risk attitude of a decision maker. A preference revealed in one binary choice problem may contradict to the preference revealed in another binary choice problem (*e.g.*, Blavatskyy, 2009, figure 2, p. 46).

A major alternative to the series of binary choices is a choice among several risky prospects. A decision maker who selects one choice option while several other options are available (but not chosen) may reveal a lot of information about his/her risk attitudes. For instance, Binswanger (1981) elicited risk preferences by asking subjects to choose one out of eight available binary lotteries (played out on a toss of a fair coin). In a similar vein, Eckel and Grossman (2002) designed a method of eliciting risk preferences where a decision maker is asked to choose the most preferred alternative out of five available lotteries.[2] These lotteries offer a tradeoff between risk and expected

[1] Abdellaoui (2000) subsequently extended the method to eliciting a probability weighting function. Blavatskyy (2006) optimized the method to address the problem of error propagation in chained elicitation questions.
[2] Dave et al. (2010) and Reynaud and Couture (2012) compare this method with multiple price list format of Holt and Laury (2002).

value (so that a lottery with a higher expected payoff also has a higher variance of payoffs). A decision maker who selects a lottery with a lower expected payoff (and a lower variance of payoffs) reveals relatively risk averse preferences.

The method of eliciting risk preferences from choice among several risky prospects remarkably resembles the problem faced by many Australian employees. Specifically, Australian employees can invest their retirement funds (superannuation) into several investment options with varying expected returns and risk. For most popular investment options, the distribution of past returns is a common knowledge. Thus, the investment problem can be reduced to a standard choice among risky lotteries (under the assumption that past returns accurately predict uncertain future returns). People who choose to invest in bonds (stocks) reveal relatively risk averse (loving) preferences. Yet, unlike with laboratory methods, these preferences are revealed under high real incentives.

Watson and McNaughton (2007) were the first and only study to analyse (gender differences in) risk preferences of Australian academics using the data on their superannuation choices. In particular, Watson and McNaughton (2007, p. 57) used a linear regression of a revealed investment choice on explanatory variables (gender, age, income). Watson and McNaughton (2007, p. 57) modelled the revealed choice as a categorical variable (with one denoting investment into the safest option and six denoting investment into the riskiest option). A simple linear regression of a categorical variable could be a misspecified econometric model and a multinomial probit or logistic regression might be a more appropriate specification.

In contrast to Watson and McNaughton (2007), we consider a structural model of optimal portfolio investment. Specifically, a decision maker selects a share of retirement funds to be invested in each available investment option in order to maximize the utility of an overall portfolio. Watson and McNaughton (2007) did not consider the distribution of investment options' past returns in their analysis. In contrast, we assume that investors use this information to evaluate the probability distribution associated with different investment options.

The remainder of the paper is organized as follows. Section 1 describes the aggregate distribution of retirement funds of Australian academics across various investment options. It also describes the distribution of investment options' past returns. In section 2 we find the optimal portfolio for an expected utility maximizer with a constant relative risk aversion utility function. This optimal portfolio is compared to the actual aggregate distribution of retirement funds and the descriptive validity of expected utility is called into question. In section 3 (4) we repeat the same exercise for a model of loss averse preferences (rank-dependent utility). Section 5 concludes.

1. Data

We elicit risk preferences of Australian academics by analyzing the distribution of their retirement funds across available investment options. Current and former employees of Australia's higher education and research sector (as well as their family members) often invest their retirement funds through the sector investment fund UniSuper. The fund is one of the largest superannuation funds in Australia with more than 450 000 member accounts as per 31 December 2013. The fund offers its customers 15 investment options where they can invest their retirement funds.

Figure 1 shows the aggregate distribution of retirement funds across 15 available options. As per 31 January 2014, "Balanced" was, by far, the most popular investment option with AUD 9 832.2 million (or 48.07%) invested in it. The second most popular option was "Growth" with AUD 3 564.8 million (or 17.43%) invested in it. The third most popular option was "High Growth" with AUD 2 003.2 million (or 9.79%) invested in it. Altogether these three options attracted just over three quarters of all retirement funds. Another four options ("Conservative Balanced", "Cash", "Capital Stable" and "Socially Responsible High Growth") attracted AUD 3 699.1 million (or 18.08%) of retirement funds. The remaining eight options attracted less than 6.63% of retirement funds. These investment options were relatively recently introduced to the customers and no long-term data are yet available for assessing their performance. We do not analyze these eight options in this paper.

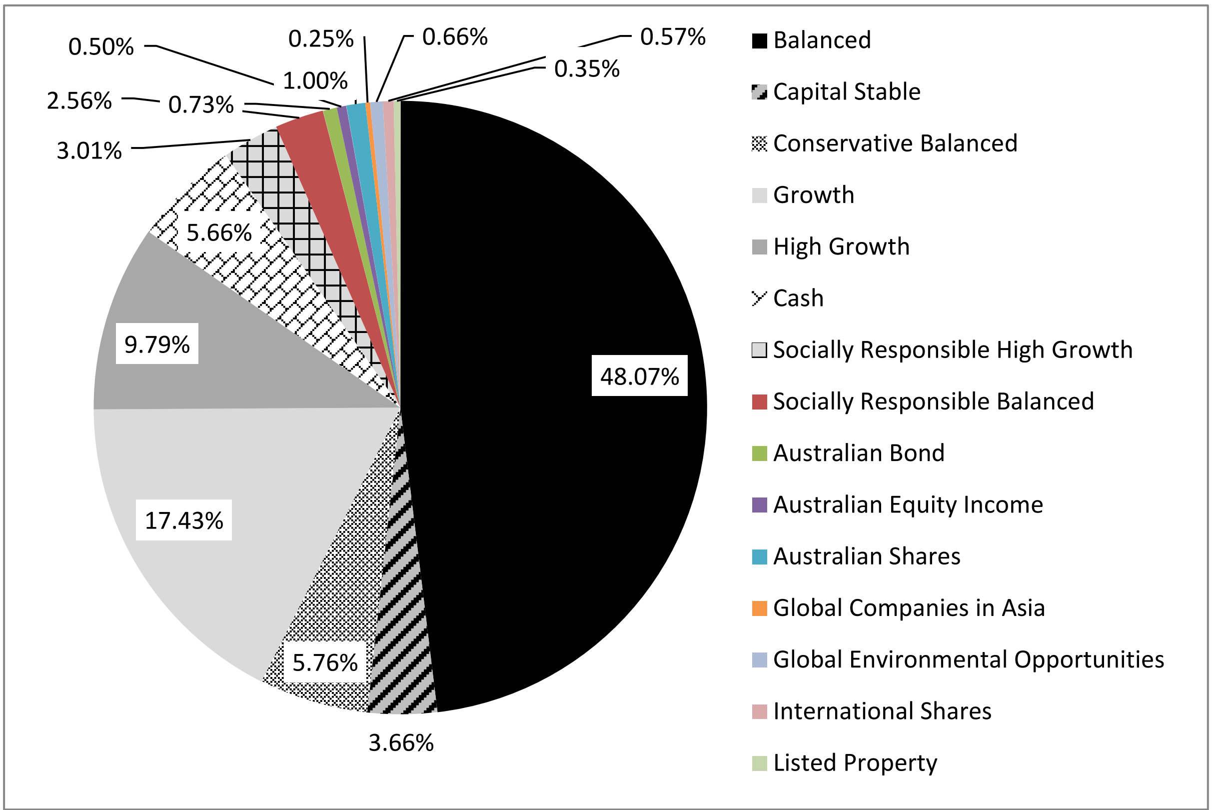

Figure 1 The aggregate distribution of retirement funds across 15 available investment options

Option / Period	Balanced	Capital Stable	Conservative Balanced	Growth	High Growth	Cash	Socially Responsible High Growth
Q1 2004	0.0368	0.0247	0.0313	0.0415	0.0446	0.0126	0.0318
Q2 2004	0.0263	0.0155	0.0197	0.0325	0.0302	0.0122	0.0310
Q3 2004	0.0389	0.0300	0.0317	0.0432	0.0372	0.0122	0.0242
Q4 2004	0.0536	0.0293	0.0411	0.0621	0.0699	0.0121	0.0657
Q1 2005	0.0211	0.0081	0.0128	0.0279	0.0248	0.0126	0.0208
Q2 2005	0.0395	0.0315	0.0352	0.0419	0.0453	0.0132	0.0517
Q3 2005	0.0442	0.0205	0.0328	0.0543	0.0694	0.0128	0.0618
Q4 2005	0.0402	0.0266	0.0354	0.0457	0.0590	0.0132	0.0515
Q1 2006	0.0466	0.0241	0.0368	0.0594	0.0714	0.0131	0.0766
Q2 2006	-0.0125	-0.0028	-0.0110	-0.0183	-0.0361	0.0135	-0.0220
Q3 2006	0.0406	0.0274	0.0330	0.0450	0.0446	0.0142	0.0379
Q4 2006	0.0492	0.0229	0.0372	0.0594	0.0793	0.0167	0.0713
Q1 2007	0.0319	0.0207	0.0265	0.0365	0.0485	0.0142	0.0416
Q2 2007	0.0315	0.0132	0.0222	0.0393	0.0497	0.0150	0.0363
Q3 2007	0.0259	0.0229	0.0250	0.0261	0.0181	0.0115	0.0213
Q4 2007	-0.0127	-0.0022	-0.0094	-0.0155	-0.0193	0.0120	-0.0434
Q1 2008	-0.0557	-0.0111	-0.0376	-0.0732	-0.0895	0.0063	-0.0964
Q2 2008	-0.0170	-0.0048	-0.0120	-0.0192	-0.0226	0.0182	-0.0216
Q3 2008	0.0014	0.0173	0.0053	-0.0030	-0.0093	0.0159	-0.0434
Q4 2008	-0.0926	-0.0304	-0.0739	-0.1237	-0.1388	0.0081	-0.1577
Q1 2009	-0.0155	-0.0042	-0.0079	-0.0156	-0.0188	0.0059	-0.0251
Q2 2009	0.0140	0.0081	0.0239	0.0144	0.0217	0.0075	0.0342
Q3 2009	0.0955	0.0581	0.0852	0.1082	0.1192	0.0089	0.1362
Q4 2009	0.0291	0.0101	0.0203	0.0312	0.0369	0.0094	0.0402
Q1 2010	0.0164	0.0183	0.0182	0.0155	0.0127	0.0082	0.0119
Q2 2010	-0.0365	0.0007	-0.0225	-0.0469	-0.0549	0.0097	-0.0860
Q3 2010	0.0493	0.0283	0.0394	0.0563	0.0619	0.0109	0.0543
Q4 2010	0.0261	0.0142	0.0202	0.0302	0.0330	0.0120	0.0270
Q1 2011	0.0170	0.0154	0.0160	0.0183	0.0196	0.0105	0.0259
Q2 2011	-0.0176	0.0039	-0.0060	-0.0260	-0.0317	0.0110	-0.0376
Q3 2011	-0.0464	-0.0038	-0.0249	-0.0604	-0.0693	0.0112	-0.0877
Q4 2011	0.0258	0.0230	0.0247	0.0263	0.0278	0.0116	0.0287
Q1 2012	0.0589	0.0294	0.0432	0.0688	0.0747	0.0095	0.0656
Q2 2012	-0.0134	0.0195	0.0025	-0.0242	-0.0347	0.0098	-0.0451
Q3 2012	0.0552	0.0252	0.0396	0.0638	0.0702	0.0085	0.0678
Q4 2012	0.0294	0.0167	0.0268	0.0506	0.0380	0.0087	0.0687
Q1 2013	0.0529	0.0239	0.0334	0.0485	0.0704	0.0066	0.0615
Q2 2013	0.0195	0.0044	0.0199	0.0216	0.0233	0.0067	0.0074
Q3 2013	0.0600	0.0356	0.0398	0.0739	0.0834	0.0066	0.1050
Q4 2013	0.0320	0.0155	0.0216	0.0395	0.0430	0.0066	0.0383

Table 1 Quarterly returns on investment options for time period 2004-2013

Table 1 shows quarterly returns on seven most popular investment options over the last ten years (2004-2013). We shall assume that decision makers take a discrete probability distribution presented in Table 1 as an accurate approximation of the true (unknown) joint distribution of assets' returns. This information is publicly available for customers on the website of UniSuper.

Figure 2 plots the expected quarterly return and its standard deviation for seven investment options. If Australian academics cared only about the expected return and its standard deviation (or variance) then they would have never invested into "Socially Responsible High Growth". This investment option is dominated by three other available investment opportunities (offering a higher return and a lower risk). Yet, as per 31 January 2014, "Socially Responsible High Growth" attracted AUD 615.6 million (or 3.01%) of retirement funds. Similarly, if Australian academics cared only about the expected return and its standard deviation (or variance) then they would have never invested into "Conservative Balanced". For example, this investment option is dominated by a 50%-50% mixture of investment opportunities "Capital Stable" and "Balanced". Yet, as per 31 January 2014, "Conservative Balanced" attracted AUD 1 177.2 million (or 5.76%) of retirement funds. In fact, holding money in "Socially Responsible High Growth" and "Capital Stable" is never optimal under all preference specifications considered in this paper.

The remaining five investment options offer a classical trade-off between risk and return. In terms of an increasing risk (and an increasing expected return) the sequence of investment options can be arranged as: "Cash", "Capital Stable", "Balanced", "Growth" and "High Growth".

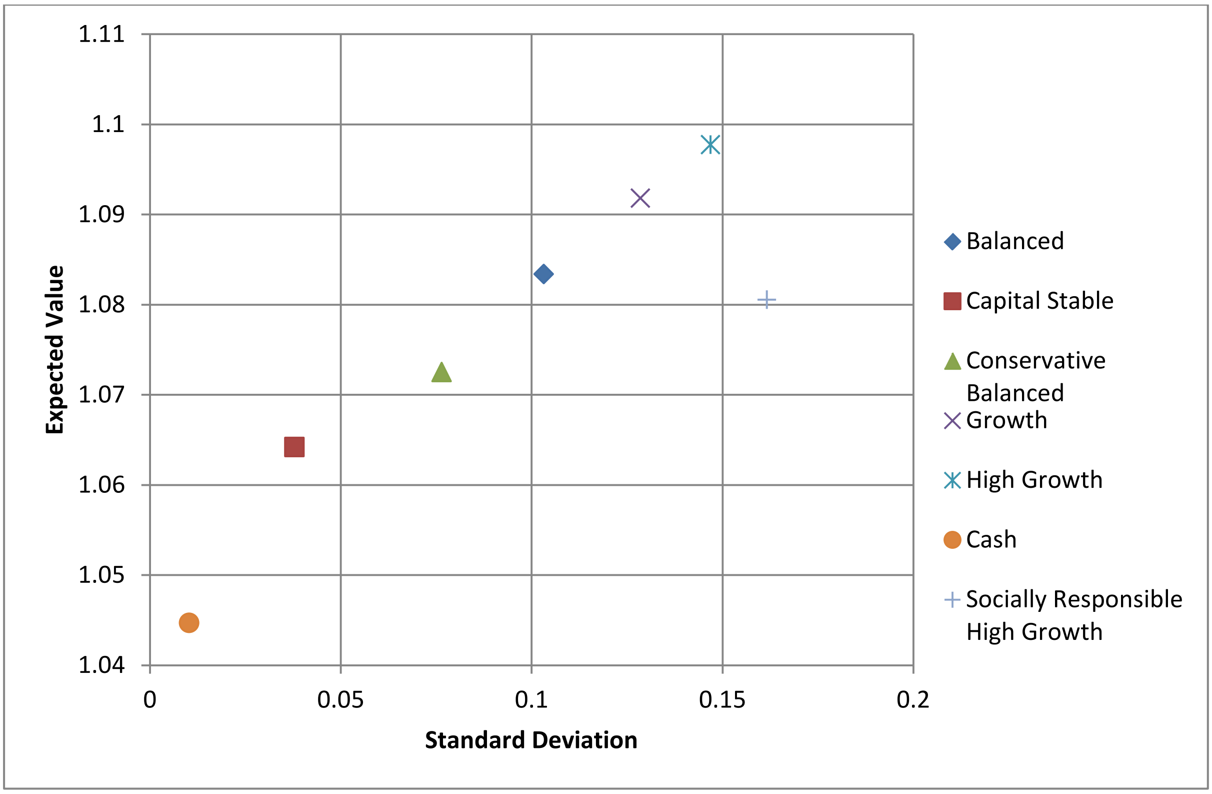

Figure 2 Expected return and its standard deviation for seven investment options

Figure 3 plots cumulative distribution functions of investment options. The cumulative distribution function of "Socially Responsible High Growth" lies nearly always above that of "High Growth", i.e. the former option is nearly first-order stochastically dominated by the latter option. In general, however, the cumulative distribution functions exhibit a single-crossing property: an investment option with a larger probability of high returns also yields a larger probability of low returns.

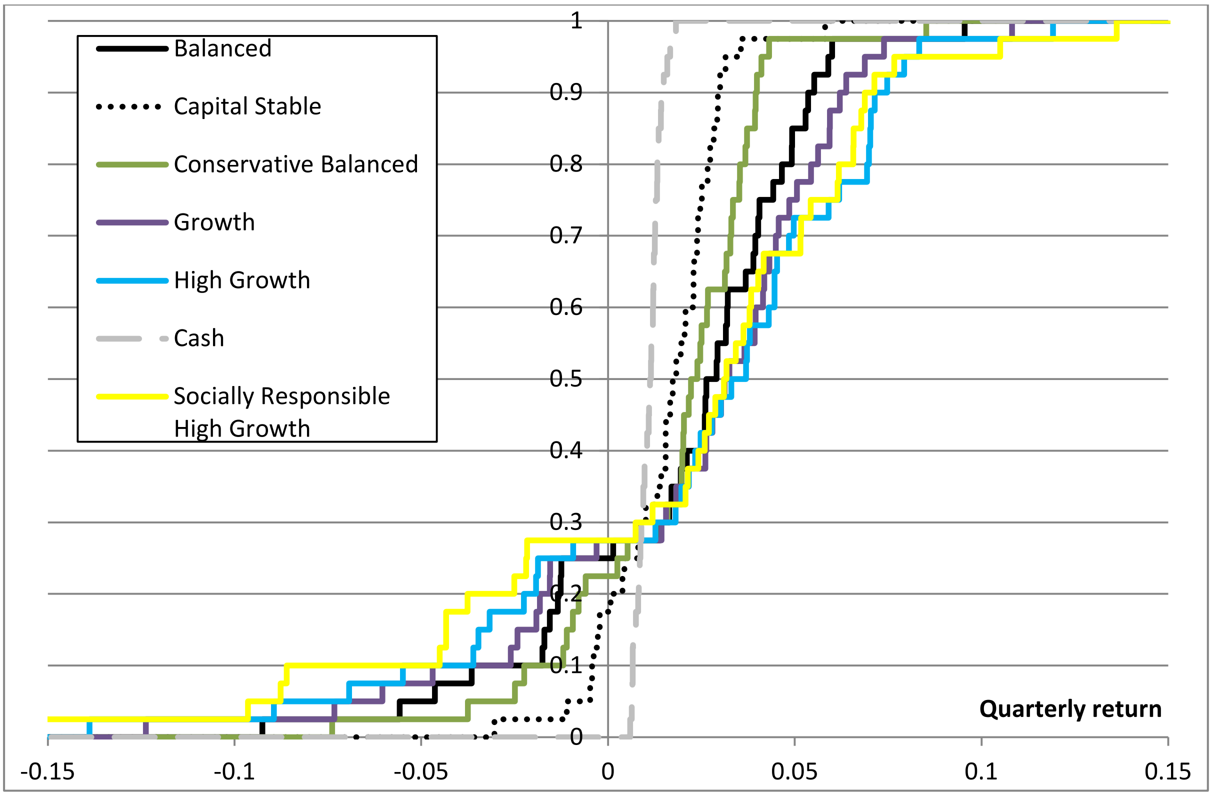

Figure 3 Cumulative distribution functions for seven investment options

2. Optimal portfolio for an expected utility maximizer with constant relative risk aversion

Consider an investor who has retirement funds $w \geq 0$ and maximizes expected utility with a (Bernoulli) utility function $u(.)$. The investor decides on how to allocate funds w across available investment options. Let X_i denote a (random) return on $1 invested into an i-th investment option, $i \in \{1,\ldots,7\}$. The investor's problem can be written as (1), where E denotes mathematical expectation.

$$\text{(1)} \quad \begin{gathered} \max_{a_1,a_2,\ldots,a_7} Eu(w[a_1X_1 + a_2X_2 + \cdots + a_7X_7]) \\ a_1, a_2, \ldots, a_7 \geq 0 \\ a_1 + a_2 + \cdots + a_7 = 1 \end{gathered}$$

We consider a constant relative risk aversion utility function (2) that has one subjective parameter—a coefficient of relative risk aversion r. The main advantage of this functional form is that the optimal portfolio becomes independent of the stock of retirement funds w in investor's possession.

$$\text{(2)} \quad u(x) = \begin{cases} \frac{x^{1-r}}{1-r}, & r \neq 1 \\ \ln x, & r = 1 \end{cases}$$

Solving problem (1) for the investment options described in Table 1 we obtain the following results. For a coefficient of relative risk aversion $r \leq 2.81$ the optimal portfolio is to invest all funds into one option—"High Growth". For a coefficient of relative risk aversion in the range $r \in [2.82,3.55]$ the optimal portfolio is to split funds between "High Growth" and "Growth" as shown on Figure 4. For a coefficient of relative risk aversion in the range $r \in [3.56,3.75]$ the optimal portfolio is again degenerate—to invest all funds into one option—"Growth".

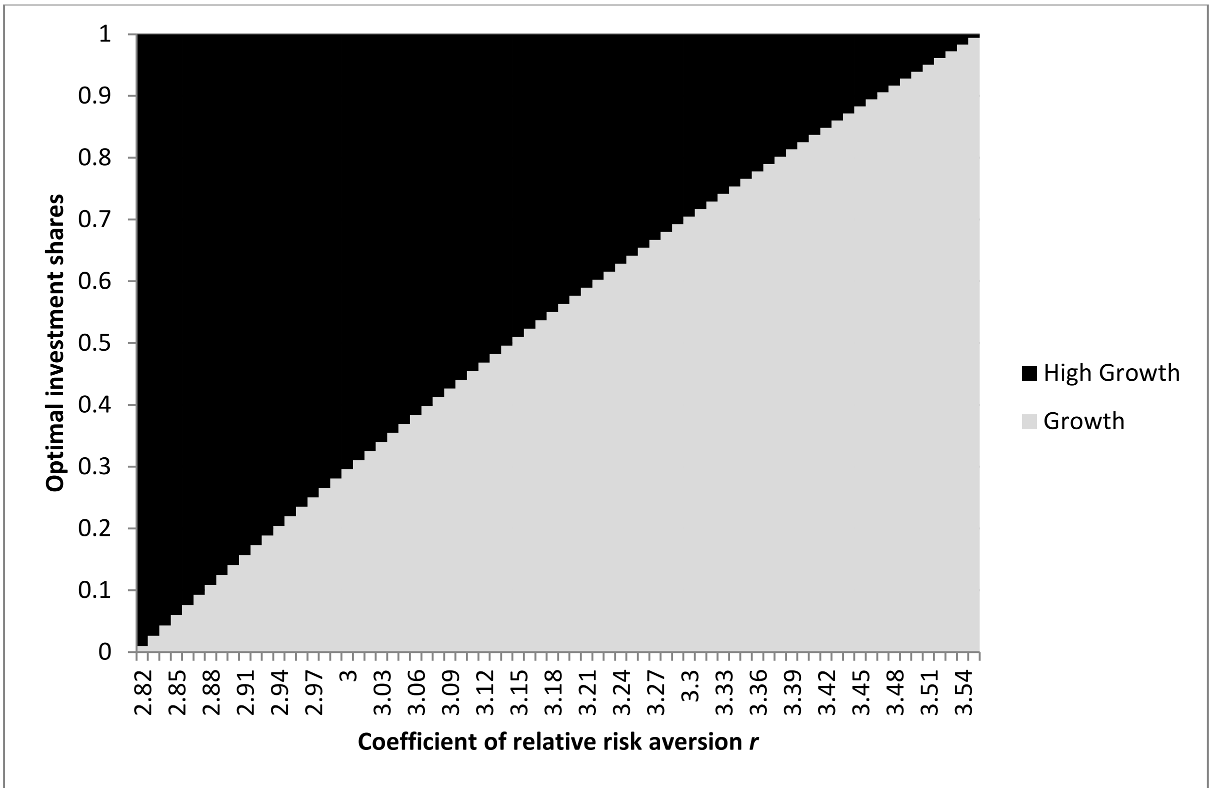

Figure 4 Optimal portfolio for a coefficient of relative risk aversion *r*∈[2.82,3.55]

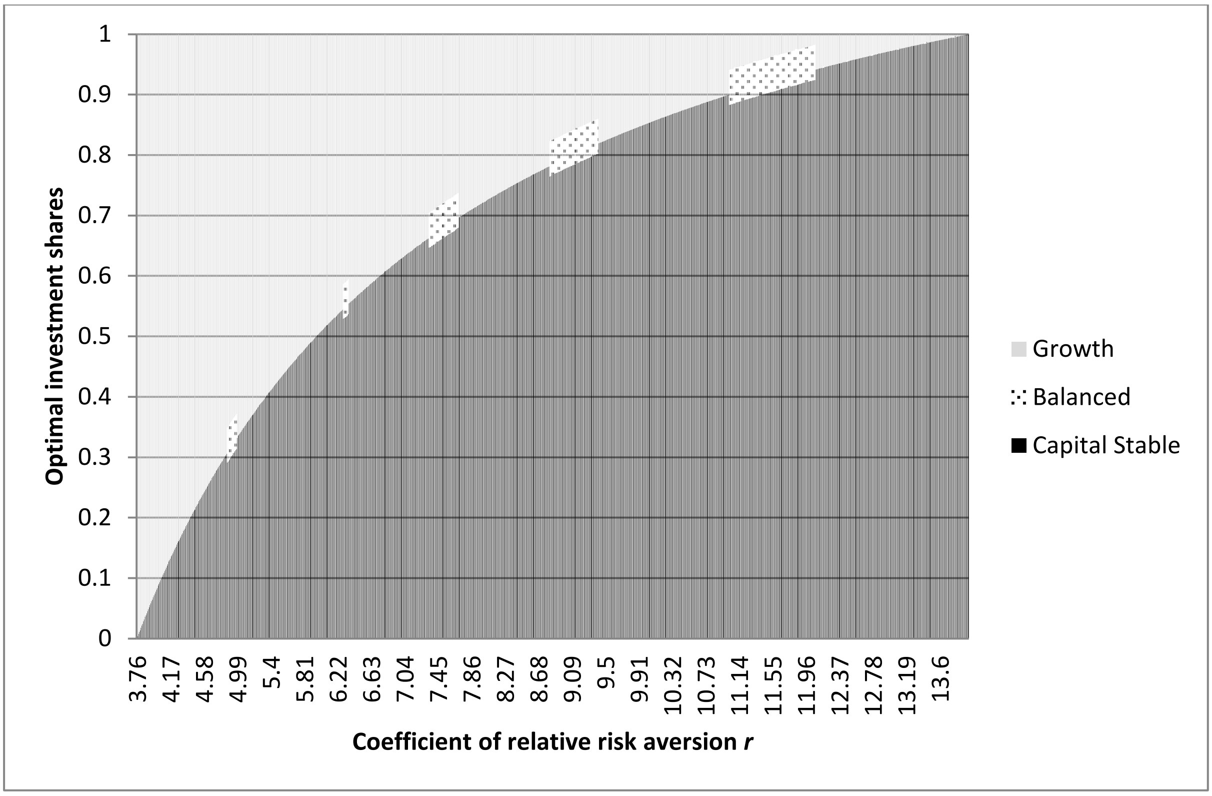

Figure 5 Optimal portfolio for a coefficient of relative risk aversion *r*∈[3.76,13.95]

For *r*∈[3.76,13.95] the optimal portfolio is to diversify between "Growth", "Capital Stable" and "Balanced" as shown on Figure 5. An investor mostly diversifies between two options: "Growth" and "Capital Stable". Only for some levels of risk aversion a small stake in "Balanced" is optimal.

For *r*∈[13.96,19.35] the optimal portfolio is to invest all funds into one option—"Capital Stable". Finally, for a coefficient of relative risk aversion greater than 19.36 the optimal portfolio is to split funds between "Capital Stable" and "Cash". This result is consistent with findings in the existing literature: only implausibly high levels of risk aversion could rationalize investing in bonds when stocks are available (*cf*. Mehra and Prescott, 1985; Blavatskyy and Pogrebna, 2010, p. 160).

In sum, investing a significant portion of retirement funds into the investment option "Balanced" is never an optimal strategy for an expected utility maximizer (irrespective of his or her coefficient of relative risk aversion). Thus, a model of a representative Australian academic as an expected utility maximizer with a constant relative risk aversion utility function does not match revealed investment decisions (*cf*. figure 1).

An empirically plausible level of risk aversion is *r*≤2.81 (*e.g*., Blavatskyy and Pogrebna, 2010a, Table III, p.973 and Table V, p. 979). In this case, the optimal investment strategy is to invest fully into one option "High Growth". Yet, only 9.79% of retirement funds are invested in this option. Thus, not more than 10% of Australian academics make investment decisions that are consistent with expected utility maximization (assuming constant relative risk aversion).

3. Optimal portfolio for an investor with loss averse preferences

Kahneman and Tversky (1979, p. 279) argued that people derive utility from relative rather than absolute wealth levels and the disutility from a negative change in wealth outweights the utility from a positive change of the same magnitude. This assumption became known as loss aversion. Benartzi and Thaler (1995) showed that a diversified portfolio that includes both high-yielding stocks and low-yielding bonds can be rationalized by loss averse preferences (rather than by a neoclassical concave utility function). In this section we investigate the optimal portfolio of retirement funds for an investor with loss averse preferences.

Formally, the investor's problem is (1) with a new utility function (3) that captures loss averse preferences. Utility function (3) has two subjective parameters: a power coefficient *r* and a coefficient of loss aversion $\lambda \geq 1$.[3] Function *sign*(*a*) is equal to minus one if $a<0$, equal to zero if $a=0$ and equal to one if $a>0$. For non-negative returns $x \geq w$ utility function (3) becomes a constant relative risk aversion utility function (2) with utility being measured on the relative rather than on the absolute wealth level. The same holds for negative returns $x<w$ except that utility function is also multiplied by $-\lambda$ in this case. Thus, a decision maker derives a greater disutility from the negative returns if the coefficient of loss aversion λ is greater than 1.

(3) $$u(x) = \left[\frac{1+\lambda}{2} sign(x-w) + \frac{1-\lambda}{2}\right] \frac{|x-w|^{1-r}}{1-r}, \quad r \neq 1$$

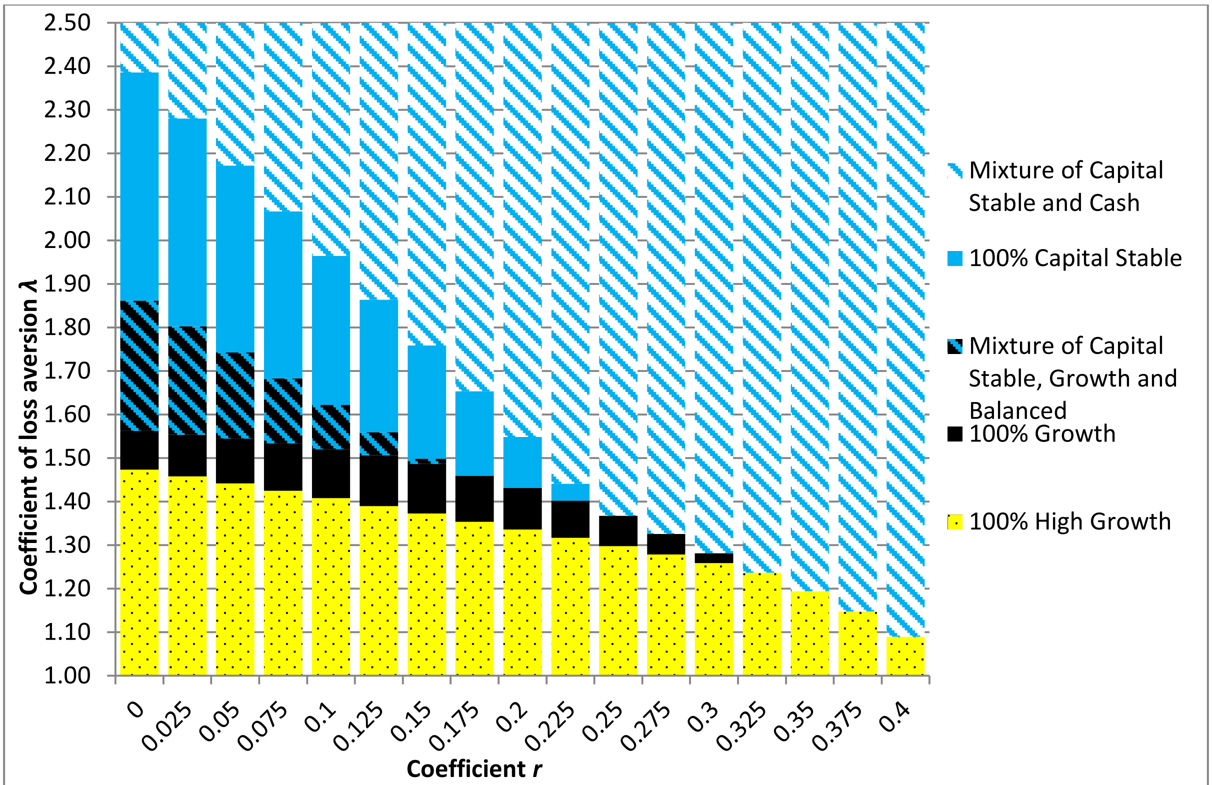

Figure 6 Optimal portfolio for a coefficient of loss aversion $\lambda \in [1,2.5]$ and coefficient $0 \leq r \leq 0.4$

[3] A parametric form of prospect theory often uses a power coefficient $\alpha=1-r$. We use coefficient $r=1-\alpha$ to facilitate the comparison of results with the previous section.

Note that zero return on an investment portfolio (*i.e*. a situation where an investor ends up with the same stock of retirement funds *w* that were initially invested) serves as a reference point in utility (3).

Figure 6 presents the solution to problem (1) with utility function (3) for a range of coefficients $\lambda \in [1,2.5]$ and $0 \leq r \leq 0.4$. When a coefficient of loss aversion is close to 1, the optimal portfolio is to invest all funds into investment option "High Growth" (for $0 \leq r \leq 0.4$). This reconfirms our results from the previous section. For all coefficients *r*, when a coefficient of loss aversion becomes sufficiently high (*e.g*., $\lambda > 2.386$) the optimal portfolio is to diversify between "Capital Stable" and "Cash".

For a relatively high coefficient *r*, these two optimal portfolios exhaust the set of optimal solutions for all levels of loss aversion. For example, when $r=0.325$ the optimal portfolio is either to invest all funds into "High Growth"(when $\lambda \leq 1.237$) or to diversify between "Capital Stable" and "Cash" (when $\lambda \geq 1.238$). For a relatively low coefficient *r*, the set of optimal solutions includes more possibilities. For example, when $r=0.175$ the optimal portfolio is either to invest all funds into "High Growth"(when $\lambda \leq 1.354$); or to invest all funds into "Growth"(when $1.355 \leq \lambda \leq 1.460$); or to invest all funds into "Capital Stable"(when $1.461 \leq \lambda \leq 1.654$); or to diversify between "Capital Stable" and "Cash" (when $\lambda \geq 1.655$).

For very low values of coefficient *r* there is a range of loss aversion parameters for which it is optimal to invest a small stake at the most popular investment option "Balanced" (*cf*. figure 1). For example, consider the case when $r=0$. In this case, utility function is piecewise linear with a kink at zero (where the slope of the utility function changes from λ to 1). In this case, the optimal portfolio is to invest all funds into "High Growth"when $\lambda \leq 1.474$ and to invest all funds into "Growth" when $1.475 \leq \lambda \leq 1.563$. Yet, when the coefficient of loss aversion is in the range $1.564 \leq \lambda \leq 1.862$ the optimal portfolio is to diversify between "Growth", "Capital Stable" and "Balanced" as shown on figure 7. Figure 7 is qualitatively similar to figure 5 in the previous section. The optimal solution is to diversify mostly between two options: "Growth" and "Capital Stable". Only for some levels of loss aversion a small stake in "Balanced" is optimal. When $1.863 \leq \lambda \leq 2.386$ the optimal portfolio is to invest all funds into "Capital Stable". Lastly, when $\lambda \geq 2.386$ the optimal portfolio is to diversify between "Capital Stable" and "Cash".

In sum, the predictions from the model of loss averse preferences are qualitatively similar to those for expected utility with constant relative risk aversion. However, in the latter model an implausibly high level of risk aversion is required for rationalizing any investment that is made not into "High Growth". In the former model, on the other hand, such investments are consistent with an empirically plausible level of loss aversion. In fact, loss aversion may even appear to be a "too strong" force—for the levels of loss aversion typically reported in experimental studies (*e.g*., $\lambda=2.25$

found in Tversky and Kahneman, 1992) the optimal portfolio involves investing into "Capital Stable" and "Cash—the two options that do not attract much retirement funds (*cf*. figure 1).

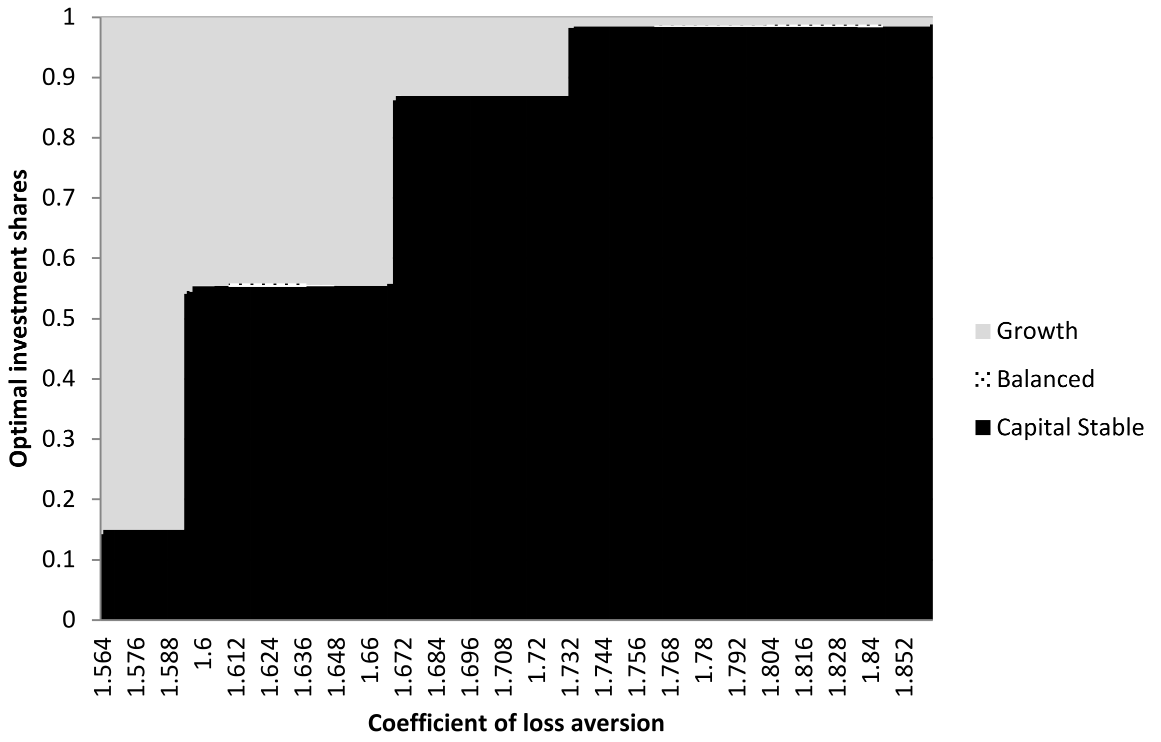

Figure 7 Optimal portfolio for a coefficient of loss aversion $\lambda\in[1.564,1.862]$ and $r=0$

For an empirically plausible range of coefficients λ and r the model of loss averse preferences can rationalize investing into four investment options: "High Growth", "Growth", "Capital Stable" and "Cash". These four options attract 36.54% of all retirement funds. Thus, not more than 36.54% of all retirement funds are invested as if their owners are prone to loss aversion. This is in line with findings of Blavatskyy and Pogrebna (2009, 2010) who report that loss aversion cannot rationalize individual choice patterns in three experiments on optimal portfolio investment.

4. Optimal portfolio under rank-dependent utility

Quiggin (1981) rank-dependent utility is a special case of Tversky and Kahneman (1992) cumulative prospect theory. A characteristic feature of rank-dependent utility is a non-linear transformation of (de-)cumulative probabilities by means of a probability weighting function. In this paper we consider a popular functional form (4) for a probability weighting function proposed by Tversky and Kahneman (1992).[4] The probability weighting function (4) has one subjective parameter $\gamma>0$. When $\gamma=1$ function (4) is linear and rank-dependent utility coincides with expected utility.

(4) $$\pi(q)=\frac{q^{\gamma}}{(q^{\gamma}+(1-q)^{\gamma})^{\frac{1}{\gamma}}},\quad \text{for all } q\in[0,1]$$

[4] Alternative functional forms were proposed by Goldstein and Einhorn (1987), Prelec (1998) and Blavatskyy (2014).

Consider an investor who invests a share α_i of his/her retirement funds *w* into an *i*-th investment option, *i*∈{1,…,7}. Let $\boldsymbol{x}_i=(x_{i1}, x_{i2}, \ldots, x_{iN})'$ denote a vector of possible returns on one dollar invested in an *i*-th investment option, *i*∈{1,…,7}, *N*≥1. Note that we assumed that these vectors are given in the corresponding columns of Table 1. Furthermore, let us denote the row of non-linear decision weights by (5).

$$\boldsymbol{p} = \left(\pi\left(\frac{1}{N}\right), \quad \pi\left(\frac{2}{N}\right) - \pi\left(\frac{1}{N}\right), \quad \cdots, \quad \pi\left(\frac{N-1}{N}\right) - \pi\left(\frac{N-2}{N}\right), \quad 1 - \pi\left(\frac{N-1}{N}\right)\right) \tag{5}$$

The problem of an investor who maximizes rank-dependent utility can be then written as (6)

$$\begin{gathered}\max_{a_1,a_2,\ldots,a_7} \boldsymbol{p} * sort[u(w[a_1\boldsymbol{x_1} + a_2\boldsymbol{x_2} + \cdots + a_7\boldsymbol{x_7}])] \\ a_1, a_2, \ldots, a_7 \geq 0 \\ a_1 + a_2 + \cdots + a_7 = 1\end{gathered} \tag{6}$$

where function *sort*[.] arranges the elements of a vector in a descending order and utility function *u*(.) is applied element-wise to a vector of portfolio returns. We consider a constant relative risk aversion utility function (2).

The solution to problem (6) can be summarized as follows. First, when coefficient *γ* is sufficiently high, the optimal portfolio for a rank-dependent utility maximizer is the same as for an expected utility maximizer. For example, when *γ*≥0.83 it is optimal to invest all retirement funds into "High Growth" for an empirically plausible range of coefficients of relative risk aversion *r*∈[0, 0.9]. This confirms our findings in section 2 (when coefficient *γ* is sufficiently close to 1, a probability weighting function is almost linear and rank-dependent utility nearly coincides with expected utility).

Second, when coefficient *γ* is sufficiently low the optimal portfolio for a rank-dependent utility maximizer is to invest all retirement funds into "Cash". For example, when *γ*≤0.57 it is optimal to invest 100% into "Cash" for a range of coefficients of relative risk aversion *r*∈[0, 0.9]. Intuitively, when coefficient *γ* is low, a decision maker behaves as if he/she severely overestimates the likelihood of the lowest ranked outcomes. Hence, it becomes optimal to invest in bonds.

Third, for an intermediate range of coefficients *γ*∈[0.62, 0.75] it is optimal to invest all retirement funds into "Capital Stable" for a range of coefficients of relative risk aversion *r*∈[0, 0.9]. Again, this result is qualitatively similar to our previous results for expected utility and the model of loss averse preferences. Yet, rank-dependent utility also has a comparative advantage over the previously considered models, as described in the next result.

For each value of a coefficient of relative risk aversion *r*∈[0, 0.9] there is a narrow range of values for parameter *γ*∈[0.76, 0.79] when it becomes optimal to invest a significant share of retirement funds into "Balanced". A black (yellow) area on figure 8 shows all parameters *r* and *γ* when the share of retirement funds invested into "Balanced" is at least 95% (50%) in the optimal portfolio. For example, when utility function is linear (*r*=0)[5] and *γ*∈[0.763, 0.764] then a rank-dependent utility maximizer prefers to invest more than 95% of retirement funds into "Balanced". The same behavior is observed when *r*=0.5 and *γ*∈[0.784, 0.786].

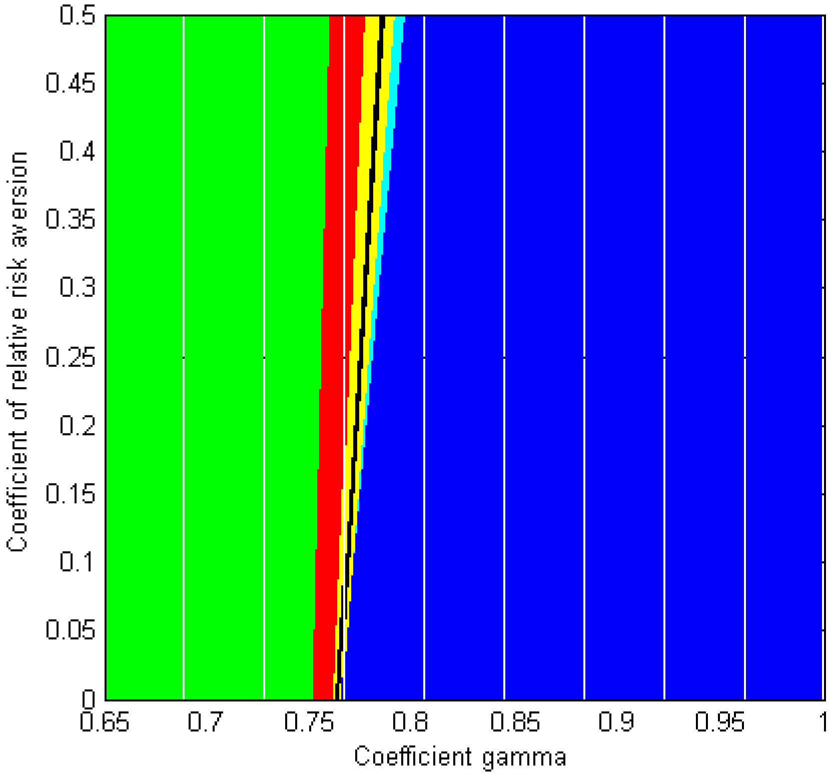

Figure 8 Optimal portfolio under rank-dependent utility with parameters *γ*∈[0.65,1] and *r*∈[0,0.9]: green area—100% "Capital Stable"; blue area—100% "High Growth"; red area—a mixed portfolio with at least 50% in "Capital Stable"; black area—a mixed portfolio with at least 95% in "Balanced"; yellow area —a mixed portfolio with at least 50% in "Balanced"; cyan area—a mixed portfolio with at least 50% in "High Growth"

Investment option "Balanced" is, by far, the most popular choice attracting nearly one half of all retirement plans. Thus, a median Australian academic behaves as if maximizing rank-dependent utility with parameter *γ*∈[0.76, 0.79] in a Tversky and Kahneman (1992) probability weighting function. The range *γ*∈[0.76, 0.79] is somewhat higher compared to the values typically elicited in laboratory experiments with hypothetical incentives (*cf*. Table 1 in Blavatskyy, 2005, p. 678). For example, Tversky and Kahneman (1992, p.312) report a median value *γ*=0.61 in their experiment. Abdellaoui (2000, p. 1509, Table 9) reports a median value *γ*=0.60. Bleichrodt and Pinto (2000, p. 1494, Table 4) report a median value *γ*=0.674.

When it comes to rationalizing the aggregate distribution of retirement funds, the descriptive success of rank-dependent utility comes mostly from its inverse S-shaped probability weighting function (*cf*. Fehr-Duda and Epper, 2012, p. 589). The curvature of utility function appears to be of a

[5] This special case is known as Yaari's (1987) dual model.

minor importance (as manifested by a steep slope of black and yellow regions in figure 8). In fact, a simplified version of rank-dependent utility with a linear utility function, known as Yaari's (1987) dual model, has the same descriptive success. Thus, while it is possible to pin down a narrow range of parameters $\gamma \in [0.76, 0.79]$, little can be inferred about coefficients of relative risk aversion.

5. Conclusion

Expected utility is the most popular model of decision making under risk. In many macroeconomic applications a representative agent is assumed to maximize expected utility with a constant coefficient of relative risk aversion. Yet, we find that such a model can rationalize not more than 10% of investment decisions of Australian academics (with regard to their retirement funds). In contrast, rank-dependent utility or Yaari's (1987) dual model with an empirically plausible range of coefficients can rationalize up to 67.18% of all investments. This result calls for a wider application of rank-dependent utility when it comes to modelling a representative agent in an economy.

Financial economics traditionally employed a linear utility function over money (*e.g.*, Markowitz, 1952). In contrast, microeconomic theory typically relied on a non-linear utility function to capture the risk attitude of a decision maker (e.g., Bernoulli, 1738). When it comes to investment decisions of Australian academics, the curvature of utility function does not appear to make a significant contribution to the descriptive accuracy of a model. In contrast, a non-linear transformation of (de-)cumulative probabilities appears to be crucial for a good descriptive model.

The model of loss averse preferences significantly improves upon classical expected utility. Nonetheless, it can rationalize not more than 36.54% of all investments. While expected utility with a plausible level of risk aversion implies overinvestment in stocks, the model of loss averse preferences goes to the other extreme—conventional levels of loss aversion imply overinvestment in bonds. This result confirms the findings of Blavatskyy and Pogrebna (2009, 2010) who report that loss aversion cannot rationalize individual portfolio allocations in laboratory experiments.

A median Australian academic behaves as if maximizing rank-dependent utility with an inverse S-shaped probability weighting function. Such a decision maker generally dislikes risk with one exception—he/she is attracted to positively skewed distributions (offering a small chance of a significant gain). One must be aware of such a gambling bias when inviting academics to advise or to take decisions in government, grant-awarding agencies and university administration. A decision maker with a preference for gambling might not be always socially optimal.

Even though expected utility fails to describe decision making under risk, it is still widely regarded as a normative model. Somewhat surprisingly, investment choices are rather limited for a decision maker willing to apply expected utility as a prescriptive model. For a wide range of coefficients of relative risk aversion that are empirically plausible ($r \leq 2.81$) the optimal portfolio is to invest all retirement funds into one option—"High Growth". Here lies a clear policy recommendation for financial institutions to introduce more investment options that allow separation between groups of expected utility maximizers with different degrees of risk aversion. Currently, there is only an illusion of choice for expected utility maximizers: one investment option is strictly preferred to the remaining six options for all levels of risk aversion that are empirically plausible.

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