Investigating the effects of Unbalanced Voltages and Voltage Harmonics on a Three-Phase Induction Motors Performance

School of Engineering and Energy

This report is submitted to the School of Engineering and Information Technology, Murdoch University in partial fulfillment of the requirements for the degree of Bachelor of Engineering

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Abstract

This investigation aims to examine and demonstrate the problems associated with unbalanced voltages and voltage harmonics during the operation of a three-phase induction motor. The application of an induction motor under balanced voltages is used as a comparison tool to explore in more detail the effects of the motors process under unbalanced voltages and voltage harmonics.

In implementing this project, MATLAB was used as the simulation software. Through MATLAB, three separate programs were created that calculate and plot the torque-speed characteristics, line currents and the input and output power, which allowed for direct comparisons of each condition. However, to obtain such results, analysis of the motor operations must be considered for balanced and unbalanced voltages, and voltage harmonics, that involves parameter testing.

This project was completed to a satisfactory level, however due to the parameter testing resulting in incorrect values, this affected the direct comparison of the final simulation programs of the motor under each case.

This investigation provides a basis of further work that may be used to help reduce unbalanced voltages and voltage harmonics. It also allows for greater expansion into the realm of unbalanced voltage harmonics, which are ever present in real life three-phase induction motors.
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1. Introduction

1.1 Project Scope

Three-phase induction motors are widely used in industrial drives due to being rugged, reliable and economical. They are used with variable speed drives (VSDs), which offer energy savings in variable-torque centrifugal fan, pump and compressor load applications. However, induction motors are also susceptible to various problems due to an unbalanced supply and harmonics.

This project was proposed to analyse the operation of a three-phase induction motor under these conditions, and to understand the problems that may be caused in operating under these conditions. Though, to determine the true nature that these effects will have on the induction motor, direct comparisons have to be made with an identical motor operating balanced under conditions as well. This will allow for a proper determination of the problems associated with the electromagnetic torque, torque-speed characteristics, line currents and output power.

In conducting this investigation, parameter testing was required to determine the electrical parameter of the induction motor. There are a variety of tests to use in determining these parameters. These include the blocked-rotor test, the no load test and the DC test were used. These tests are conducted in a specific order as some tests require electrical parameters calculated in the previous test. The order these tests were conducted are:

- DC Test;
- Blocked-Rotor Test;
- No Load Test.
The DC test is required to calculate the stator resistance, whilst the blocked-rotor test determined the stator reactance and the rotor resistance and reactance. The no load test estimates the magnetizing reactance. The parameter tests are paramount in obtaining results for the simulation.

The investigation analyses the time harmonics rather than the space harmonics. Space harmonics depend on the design of an induction motor whereas time harmonics depend on the applied three phase voltages to the motor. In the case of voltage harmonics, both balanced harmonics and unbalanced harmonics were investigated. The simulation of unbalanced harmonics will take into account unbalanced voltages as well as time harmonics.

For this investigation all testing of induction motor operation under balanced and unbalanced voltages and voltage harmonics are conducted only through simulation, with live testing only used for parameter estimation.

1.2 Investigation Requirements

The investigation for this thesis included creating a simulation program that allowed for a comparison of motor characteristics for a three phase induction motor operating under a balanced three phase power supply, unbalanced three phase power supply and voltage harmonics.

The requirement of the simulation program included the ability to display and plot various selected variables. These plots were then used to compare, the motor operation under unbalanced voltages and voltage harmonics.
1.3 Review – MATLAB

Throughout this investigation, MATLAB was used to produce results and plots that allowed for a comparison of the three-phase induction motor under balanced three phase voltages, unbalanced voltages and voltage harmonics. Each case will require more than one MATLAB program to calculate and plot the results.

The first program for each case was dedicated to determining and plotting the torque-speed characteristics. The electromagnetic torque will require the use of calculated slip values and Thevenin equivalents of the impedance and voltage. The electromagnetic torque equation was placed within a loop that will continuously calculate the torque under that specific condition. This loop will then allow for the generation of torque-speed characteristics. The torque-speed characteristics are based on plotting the electromagnetic torque against the mechanical speed of the motor. In the case of voltage harmonics the first program will calculate and plot both the electromagnetic torque and the torque-speed characteristics, as well as the line current. This is due to the electromagnetic torque requiring the line current calculation to determine its final result.

The second program for each case is required to calculate the line current and real and reactive power. Excluding the voltage harmonics condition, both unbalanced voltages and the balanced system will require the determination of the impedance going into the motor. The line current is then determined through division of the line voltage by the impedance. The line current allows for the final values to be produced for the output power. When
obtaining those answers several variables need to be determined beforehand. These variables include the phase angle of the power, input power, stator copper losses, air gap power, rotor copper losses, power converted and the output power.

It must be noted, that due to problems encountered during the investigation, the simulation programs were reduced to one solitary program. This program determined and plotted only the torque-speed characteristics, line currents and output power of the three-phase motor. The input power which was required to be used as a comparison tool against the output power, was not calculated due to a recurring problem.

1.4 Document Overview
This report contains a total of nine chapters with each topic described below:

Chapter 1: Introduction
This chapter presents the aim and requirements of the investigation and a review of the simulation software used.

Chapter 2: Three-Phase Induction Motor Review
This section gives an overview of the construction and operational characteristics of a three-phase induction motor.

Chapter 3: Motor Design
This chapter illustrates the design classes of an induction motor and their operational characteristics according to set standards.
Chapter 4: Unbalanced Voltages

This chapter provides an overview of the causes of unbalanced voltage and the effects it has on an induction motors operation.

Chapter 5: Variable Speed Drives

This section examines the problems associated with variable speed drives output voltage and current harmonics.

Chapter 6: Voltage Harmonics

This section provides an overview of the causes of harmonics and the effect it has on an induction motors operation.

Chapter 7: Determination of Motor Parameters

This chapter involves the determination of motor parameters for a three-phase induction motor as well as examining a laboratory set-up for future use.

Chapter 8: Simulation Results

This chapter displays the final simulated results of the investigation and compares them to determine any problems that occur due to unbalanced voltages and voltage harmonics.

Chapter 9: Conclusion

This final section gives an overall summary of the work conducted in the investigation. It also explains further work that can be conducted and improved upon for future needs.
2. Three-Phase Induction Motor Review

Three phase induction motors are the most commonly used type of motor in industrial applications. These applications range from conveyers, pumps, air conditioning and electrical substations. The squirrel cage motor design is used more often than its wound rotor counter-part due to wound rotor induction motors requiring much more maintenance because of the wear associated with its brushes and slip rings (Circuits, 2014). For this investigation the three-phase induction motor, which has a wye formation, will be based upon a Leroy Somer Australia LSMV motor. The motor used has a squirrel caged rotor design.

2.1 Construction of the Motor
2.1.1 Construction of the Stator

The stator windings are located inside of the stationary housing of the induction motor (Mirza, 2010). The stator windings can be constructed for single phase or three phase motors. This investigation looks at only a three-phase motor. The stator section of the motor is a laminated iron core with slots, where the three phase winding coils are placed. Each of the three windings are individually overlapping each other, whereby they are electrically and mechanically 120° out of phase. The connection of these windings are either in a wye or delta connection depending on the requirement of the motor. In this project, the stator windings are assumed to be connected in wye. The stator windings have a low resistance. Varnish or oxide coating is used as insulation between the windings of the stator (Automation, 1996).
2.1.2 Construction of the Rotor

A squirrel-cage rotor design consists of a shaft with bearings, a laminated iron core and rotor conductors (Jenneson, 1985). Rather than a winding, rotor bars are slotted into the laminated iron core. These rotor bars are short-circuited at each end by a solid ring. Due to this no extra external resistance in series with the rotor can be added. They are often made of copper strips welded to copper rings however, for small and medium size motors aluminium may be used instead. Varying the physical design features of the rotor bars affects the performance of the motor. By having the bars deeper in the rotor their inductance will increase, which gives a lower starting current and creates a lower pull-out torque. The rotor conducting bars are not parallel to the shaft of the motor, but they are slightly skewed (Jenneson, 1985).

2.2 Operating Principles

2.2.1 Stator

During operation, the stator windings are magnetised by the current flowing through it, which creates a magnetisation current. The current generates a rotating magnetic field, which turns with a synchronous speed $n_{sync}$ (Automation, 1996). The synchronous speed also helps determine the slip speed and slip of the induction motor. The magnetic field rotates in a direction, clockwise or counter-clockwise, depending on the order of the stator windings. The speed of the magnetic fields rotation is given by:

$$n_{sync} = \frac{120 \times f_e}{p}$$

Equation 1: Synchronous Speed Equation
The rotating magnetic field, $\beta$, from the stator passes through the rotor and it induces a voltage, $e_{ind}$, in the rotor bars. The induced voltage is given by:

$$e_{ind} = (v \times \beta) \times l$$

*Equation 2: Induced Voltage Equation*

2.2.2 Rotor

The induced voltage produced by the magnetic field from the stator produces a current flow within the rotor, see Equation 3. The peak current lags behind the peak voltage due to the inductance of the rotor. The current then produces a magnetic field in the rotor, $\beta_R$, which allows for the generation of torque, $\tau_{ind}$. The torque is generated by the induced voltage causing the current in the rotor to flow in a direction that is opposite to that of the magnetic field in the stator, $\beta_S$. Equation 4, demonstrates how this leads to a twisting motion in the motor, which generates torque in the rotor (James, 2012).

$$I_\phi = \frac{V_\phi}{Z_{eq}}$$

*Equation 3: Line Current of the Motor for one Phase*

$$\tau_{ind} = k \times \beta_R \times \beta_S$$

*Equation 4: Induced Torque of the Motor created from the rotating Magnetic Fields*

2.2.3 Slip Speed and Slip

An induction motors speed depend on the rotor’s voltage and current. Any voltage that is induced on the bars of the rotor, depends on the rotor speed relative to the magnetic field (Chapman S. J., Electric Machinery Fundamentals 4th Ed., 2005). The use of the terms slip speed and slip allow for a simpler
definition of the relative motion of the rotor and the magnetic fields. The slip speed, $n_{\text{slip}}$, of the motor is determined by the difference of the synchronous speed, $n_{\text{sync}}$, and the mechanical shaft speed, $n_m$, as seen in Equation 5.

$$n_{\text{slip}} = n_{\text{sync}} - n_m$$

*Equation 5: Slip Speed Equation*

The slip is determined on a per-unit or a percentage basis. The slip is the difference between the synchronous speed, $n_{\text{sync}}$, and mechanical shaft speed, $n_m$, over the synchronous speed. The slip, $s$, is defined as:

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \times 100\%$$

*Equation 6: The Definition of the Slip*

When the motor operates at synchronous speed, $s = 0$, the mechanical shaft speed is equivalent to the slip speed. Though, when the rotor is idle, voltage and current are induced in the rotor. The rotor may not rotate because of the load torque exceeding the induced torque. The rotor does not rotate at this point and the slip becomes, $s = 1$. The mechanical shaft speed is also zero at this point in time. Between these two slip points is where the other motor speeds occur.
The mechanical speed of the rotor can also be expressed in terms of the slip and synchronous speed.

\[ n_m = (1 - s) \times n_{sync} \]

*Equation 7: Mechanical Speed of the Motor*

\[ \omega_m = (1 - s) \times \omega_{sync} \]

*Equation 8: Angular Speed of the Motor*

2.3 Torque-Speed Characteristics

The torque-speed characteristic of a three-phase induction motor can be divided into three sections: The low slip region, moderate slip region and the high slip region. Each section covers various regions of operation of the motor.

The pull-out torque for an induction motor is approximately 200% of the rated full load torque of the machine as seen in Figure 4. The starting torque on the other hand, is only 150% of the full load or operating torque (Chapman S. J., Electric Machinery Fundamentals 4th Ed., 2005).

2.3.1 Low-Slip Region

In the low slip region, the slip of the motor increases approximately linearly with an increased load. However, the mechanical speed decreases at the same rate with the increased load. The reactance in the rotor, \( X_2 \), is negligible in the low-slip region. This leads to the rotor power factor being almost unity, whilst the current in the rotor increases linearly with slip.
The steady state operating range of the motor is included within this region. Whilst under normal operation, an induction motor will have a linear speed drop as torque increases.

2.3.2 Moderate-Slip Region
The frequency within the moderate-slip region is higher than in the low-slip region, meaning the rotor reactance is of the same order of magnitude as the rotor resistance (Chapman S. J., Electric Machinery Fundamentals 4th Ed., 2005). The rotor current in this region does not increase as quickly as that in the low slip region. The maximum torque the motor can operate at, the pull-out torque, occurs when the increase in the rotor current is then balanced out by a decrease in the rotor’s power factor. The pull-out torque in this region cannot be exceeded.

2.3.3 High-Slip Region
The high slip region displays a decrease in the induced torque as the load increases. This is due to the reduction in the rotor power factor being greater than the increase in the rotor current of the motor. If the rotor is driven faster than the synchronous speed, then the direction of the torque reverses and the motor becomes a generator, which in turn converts mechanical power to electrical power.
2.4 Power and Power Losses in Three-Phase Induction Motors

2.4.1 Power Losses

In a three-phase induction motor there are five sets of power losses that occur during its operation in outputting power. These losses in order include:

- Stator Copper Losses, $P_{SCL}$
- Core Losses, $P_{core}$
- Rotor Copper Losses, $P_{RCL}$
- Friction and Winding Losses
- Stray Losses or Miscellaneous Losses

\[
P_{SCL} = 3 \times I_1^2 R_1
\]

Equation 9: Stator Copper Losses

\[
P_{core} = 3 \times E_1^2 G_C
\]

Equation 10: Core Losses in a Motor

\[
P_{RCL} = 3 \times I_2^2 R_2
\]

Equation 11: Rotor Copper Losses

It must be noted that the power loss equations are based on a three-phase motor. However, the equivalent circuit of the motor demonstrates only one phase so the equations must be adjusted accordingly to represent three phases rather than one.

The first losses encountered in a three-phase induction motor are the stator copper losses. This occurs when the stator windings are energised. Friction and winding losses are due to the friction caused by components in the
motor, such as bearing wear and the rotating friction caused by elements in the
tool. Miscellaneous losses are combined from several other minor losses such
as flux leakage, that are induced by the motor current and the air gap power.
Both the friction and winding losses as well as the miscellaneous losses are
commonly grouped together as rotational losses, $P_{Rot}$, due to the difficulty in
determining them individually (Chow, 1997).

For this investigation the friction and winding losses, core losses and
miscellaneous losses are neglected.

### 2.4.2 Power in Induction Motors

The input power to a three-phase induction motor is in the form of three
phase electric voltages and currents. This input power encounters many losses
along the way before finally generating the required output power of the motor.

$$P_{in} = \sqrt{3} \times V_T I_L \cos \theta$$

*Equation 12: Input Power for a Three-phase Induction Motor*

![Power-flow diagram of an Induction Motor](Eqbal, n.d.)
After the input power has encountered both the stator copper losses and core losses, the remaining power is transferred to the rotor by the rotating magnetic field. This power is known as the air gap power, $P_{AG}$.

$$P_{AG} = P_{in} - P_{SCL} - P_{core}$$

Equation 13: Air Gap Power through the Motor

The air gap power incurs the rotor copper losses, where the remaining power is converted from electrical to mechanical energy. However, considering that both the friction and winding losses and miscellaneous losses are negligible, the output power of the motor will be equal to the power converted. If the negligible losses were included then the output power would be the difference between the power converted and the rotational losses.

$$P_{conv} = P_{AG} - P_{RCL}$$

Equation 14: Power Converted through the Motor

$$P_{out} = P_{conv} - P_{F&W} - P_{misc}$$

Equation 15: Real Output Power of an Induction Motor

$$P_{out} = P_{conv}$$

Equation 16: Output Power of an Induction Motor

2.5 Equivalent Circuit

The equivalent per-phase circuit of a three-phase induction motor is required to derive equations such as the Thevenin equivalents, induced torque, line current and power. However, due to the equivalent circuit of the motor only
valid for a single phase rather than three-phases any determinations of equations must be adjusted to fit a three-phase model as to avoid producing results only for a single phase.

The three phase induction motor equivalent circuit can be derived from the transformer model equivalent circuit. This is due to an induction motor essentially behaving as a transformer in its operation. The main difference between the final equivalent circuits of a motor compared to the transformer model is the motor’s equivalent circuit does not take into account the turn’s ratio that a transformer does as seen in Figure 4.

The stator is supplied by a three-phase voltage, which is balanced, that drives a three-phase current through the winding of the stator, as illustrated in Figure 4. The applied voltage, $E_1$, across the stator side is equivalent to the summation of the induced voltage and the voltage drops across both the stator

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1. *This investigation does not examine capacitance, $R_C$, in the final per-phase equivalent circuit as seen in Figure 4.*
resistance and reactance, \( R_1 \) and \( X_1 \). This provides the stator side of the equivalent circuit.

In producing the final equivalent circuit the rotor circuit model of an induction motor must be considered, see Figure 6. The transformer model of the circuit can be represented with the rotor circuit model. Using the effective turns ratio, see Appendix 11.1, the transformer model circuit can be whittled down to the final induction motor per-phase equivalent (Shahl, 2014), as shown by Figure 7.

Figure 3: Rotor Circuit Model (McFayden, 2014)

Figure 4: Final Per-Phase Equivalent Circuit (McFayden, 2014)
3 Motor Design

Three-phase induction motors are designed according to specific standards. There are four main designs for an induction motor. Depending on the origin on the standards the name of the designs will be different. For this investigation both the National Electrical Manufacturers Association, NEMA, and Standards Australia are used.

Under Australian standards, (60034.12, 2009), three-phase induction motors can be designed from any of the four categories:

Design N: Normal starting torque three-phase cage induction motors intended for direct-on-line starting, having 2, 4, 6 or 8 poles and rated from 0.4 kW to 1600 kW.

Design NY: Motors similar to design N, but intended for star-delta starting. For these motors in star-connection, minimum values for $T_1$ and $T_U$ are 25 percent of the values of design N, see tables in (60034.12, 2009).

Design H: High starting torque three-phase cage induction motors with 4, 6 or 8 poles, intended for direct-on-line starting, and rated from 0.4 kW to 160 kW.

Design HY: Motors similar to design H but intended for star-delta starting. For these motors in a star-connection, minimum values for $T_1$ and $T_U$ are 25 percent of the values of design H, see tables in (60034.12, 2009).
Based on the NEMA standards, three-phase induction motors can be designed from the following categories:

- Design A;
- Design B;
- Design C;
- Design D.

Even though the design values are different when comparing the Australian and NEMA standards\(^2\), they are actually one in the same as indicated below:

- Design N is equivalent to Design A;
- Design NY is equivalent to Design B;
- Design H is equivalent to Design C;
- Design HY is equivalent to Design D.

The design standard for the three-phase induction motor used for this investigation is the design N from Standards Australia. The design N class was chosen as the motor was designed with a normal starting torque, a normal starting current and a low slip. Under NEMA standards this is equivalent to design A (NEMA A, B, C and D Design).

\(^{2}\text{NEMA standards could not be properly obtained to compare design classes directly.}\)
3.1 Design Requirements

Each design has a set of rules relating to the motors torque characteristics, locked rotor apparent power and starting requirements.

3.1.1 Torque Characteristics

The starting torque is represented by three characteristic features. These features shall be in accordance with the appropriate values given in (60034.12, 2009). The values in (60034.12, 2009) are minimum values at rated voltage. Higher values are allowed.

The motor torque at any speed between zero and that at which breakdown torque occurs shall be not less than 1.3 times the torque obtained from a curve varying as the square of the speed and being equal to rated torque at rated speed. However, for two pole motors with type of protection ‘e-increased safety’ having a rated output greater than 100 kW, the motor torque at any speed between zero and that at which breakdown torque occurs shall not be less than 1.3\(^3\) times the torque obtained from a curve varying as the square of the speed and being equal to 70 percent rated torque at rated speed. For motors with type of protection ‘e’, the three characteristic torques shall be in accordance with appropriate values given in (60034.12, 2009).

3.1.2 Locked Rotor Apparent Power

The locked rotor apparent power shall be not greater than the appropriate value given in (60034.12, 2009). The values given in (60034.12, 2009) are independent of the number of poles and are maximum values at rated voltage.

\(^3\) The factor 1.3 has been chosen with regard to an under voltage of 10 percent in relation to the rated voltage at the motor terminals during the acceleration period.
voltage. For motors with type of protection ‘e’, locked rotor apparent power shall be in accordance with the appropriate values given in (60034.12, 2009)

3.1.3 Starting Requirements

Motors shall be capable of withstanding two starts in succession (coasting to rest between starts) from cold conditions and one start from hot after running at rated conditions. The retarding torque due to the driven load will be in each case proportional to the square of the speed and equal to the rated torque at rated speed with the external inertia given in (60034.12, 2009).

In each case, a further start is permissible only if the motor temperature before starting does not exceed the steady temperature at rated load. However, for two pole motors with type of protection ‘e-increased safety’ having a rated output greater than 100 kW, the retarding torque due to the driven load is proportional to the square of the speed and equal to 70 percent rated torque at rated speed, with the external inertia given in (60034.12, 2009). After this starting, load with rated torque is possible.

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4 It should be recognised that the number of starts should be minimised since these affect the life of the motor.

5 AS 60034.12 displays all tables relating to the starting requirements of Design Class N.
4 Unbalanced Voltages

Three-phase induction motors are balanced when the three phase voltages are of equal magnitude and they are displaced by a phase of 120° from each other. However, induction motors encounter various unbalances during operation. The nature of these unbalances can be in the form of unequal voltage magnitudes from a power source, under-voltages and over-voltages, and phase angle displacement from 120° (Jouanne, 2001). Harmonic distortion can cause unbalanced voltages, but that is examined later through voltage harmonics. This investigation only examines phase deviations against the motor.

Three-phase induction motors are designed to handle small amounts of unbalanced voltage. To mitigate the effects of imbalance on a three-phase induction motor, standards have been put in place to tackle the imbalance and reduce its effects through various means such as derating.

4.1 Operating Performance

The main problem of unbalanced voltages in a motor is the significant increase of heat. The voltage imbalance creates a current unbalance that is six to ten times the magnitude of the voltage imbalance. This extreme current unbalance creates heat in some of the motors windings that breaks down the motor insulation which causes cumulative and permanent damage to the motor (Company, 2009).
4.1.1 Sequences
An unbalanced three-phase voltage can be represented by the summation of three sequences:

- Positive Sequence;
- Negative Sequence;
- Zero Sequence.

Only the positive and negative sequence are considered here, as the zero sequence does not produce current or a rotating magnetic field.

In the case of unbalanced voltages, the three-phase induction motor can be considered as being equivalent to two identical induction motors that are mounted on a single shaft. The positive and negative sequences represent the ‘two different’ motors in question. The positive sequence voltage produces torque in the direction of rotation, clockwise, as if the motor were balanced. Though, the negative sequence on the other hand, produces a torque that is in the opposite direction of rotation, anti-clockwise, to the positive sequence (H. R. Reed, 2009). The negative sequence voltage produces an air gap flux that rotates against the rotation of the motor, which produces larger currents if the unbalance is significant. These negative sequence currents are primarily large due to the negative sequence having a reduced impedance.

4.1.2 Effects on Operation Characteristics
The effect of excessive heat that is produced by unbalanced voltages is in part due to the increased losses of the motor due to the negative sequence currents and voltages. The rotor losses are increased due to the current displacement. The positive sequence plays a part in the extra heat in that due to
the unbalance the positive sequence voltage drops, causing an increase in the positive sequence currents within the stator and the rotor (60034.26-2009, 2009).

The torque of the three-phase induction motor is affected in that the locked-rotor, pull-up and breakdown torques are decreased due to the voltage imbalance. If the imbalance were to be extreme, causing significantly larger current unbalances than the motor can handle, the torques may not be adequate for the application. The full-load speed of the motor also sees a slight decrease when the motor operates with an imbalance (Enrique Quispe).

The interaction of both the positive and negative sequences opposing the rotating magnetic fields produces pulsating electromagnetic torque and velocity disturbances. Aside from excessive heat and increased motor losses, the effects of large negative sequence currents exasperates problems such as vibrations, acoustic noises, shortened life span and a decrease in the rotating torque (Davar Mirabbasi).

Due to the negative sequence effect on the induction motor, the excessive heat that is produced has quite a profound effect, as an increase of voltage unbalance exponentially increases the temperature of the motor (Figure 5). Large unbalances are therefore a must to avoid or risk destroying the motor.
4.2 Equivalent Circuit

Under unbalanced voltages, the induction motor is equivalent to ‘two different’ motors acting on the shaft. This means that there are two equivalent circuits to deal with in regards to the induction motor.

The two equivalent circuits will be based on both the positive and negative sequence components of the motor. The zero sequence is not considered as there is no zero sequence current due to the absence of a neutral wire. Considering that the positive sequence component operates as an induction motor that is balanced, its equivalent circuit is identical to the circuit given in Figure 6.

However, the negative sequence component has an almost identical equivalent circuit; the only difference being is that the slip value changes on the rotor side, as seen in Figure 6. The slip change is due to the rotating magnetic field created by the negative sequence current of the motor operating in the
opposite direction to the positive sequence. This slip for the negative sequence is determined from the slip in the positive sequence (see Appendix 11.2).

![Negative Sequence Voltage Equivalent Circuit](image)

Figure 6: Negative Sequence Voltage Equivalent Circuit

4.2.1 Symmetrical Components

Under the final equivalent circuit for both the positive and negative sequence of a three-phase induction motor, it is possible for symmetrical components to be used. These components are an integral part of determining the electromagnetic torque, line currents and power through the motor for their respective sequence. However, the use of symmetrical components allow for the calculation of the unbalanced phase currents, which provide the overall line current for the motor under unbalanced voltages.

The phase currents of the motor give an unnecessarily large input power for the motor, but in turn cause a significant amount of losses, both stator and rotor copper losses, that reduce the overall output power of the motor.

The phase currents, which are labelled A, B and C for this investigation are determined from the symmetrical components of the equivalent circuit. This involves the determination of the voltage sequence components, both positive
and negative, and the respective per phase impedance of the motor for each sequence. These quantities are used to determine the positive and negative sequence stator current components.

The actual phase currents, $I_A$, $I_B$ and $I_C$, are determined by the stator current components using an inverse transformation to that used in finding the sequence components. Appendix 11.3 displays the MATLAB code used in this calculation. The unbalanced phase currents calculated form the basis for the overall line current through the motor (Singh, 2013).

4.3 Standards

There are standards in place to reduce voltage unbalance. Various standards ranging from Standards Australia, NEMA and the International Electrical Commission, IEC, have set guidelines to avoid unbalances. These guidelines involve both the use of percentage voltage imbalance and derating. For this investigation standards from Standards Australia are used in regards to unbalanced voltages.

4.3.1 Percentage Voltage Imbalance

For voltage imbalances over 5 percent a study of the negative sequence component of the currents is necessary.

The percentage voltage imbalance, PVI, can easily be determined by a motor user from the voltage reading of the three phases. It is calculated by the following formula from (1359.31-1997, 1997);

$$PVI = \frac{\text{Maximum Voltage Deviation}}{\text{Average Voltage}} \times \frac{\text{Average Voltage}}{\text{Average Voltage}} \times 100\%$$
The true negative sequence voltage component may be up to 18 percent higher than the value obtained from the formula.

The percentage voltage imbalance is given for the convenience of the motor user, and is only an approximation of the percent negative sequence voltage component. A more accurate determination can be made with the aid of symmetrical components (1359.31-1997, 1997).

4.3.2 Derating and Voltage Unbalance Factor

As stated in (60034.12, 2009), when a motor for use on a power supply of rated frequency is connected to a three-phase voltage system having a negative sequence component exceeding 1 percent of the positive sequence component of the voltages over a long period, (at least the thermal time constant of the machine), the permissible power of the motor is less than the rated power to reduce the possibility of damage to the motor. A typical derating factor for motors of design N within the scope of (AS 60034.12-2009) is given in Figure 7. This is on the supposition that the positive sequence component of the supply voltage is close to the rated voltage. Operation of the motor above a 5 percent voltage unbalance condition is not recommended.
The unbalance factor $f_u$ in Figure 11 is defined as:

$$f_u = \frac{U_n}{U_p}$$

Equation 17: Unbalance Factor

Where, $U_n$ is the r.m.s. value of the negative sequence component of the supply voltage and $U_p$ is the r.m.s. value of the positive sequence component of the supply voltage (60034.26-2009, 2009).

4.4 Reducing the effects of Unbalanced Voltages

Three-phase induction motors never operate exactly at balanced conditions for their life span. There is always imbalance within the motor, albeit a small amount of it. Even a small amount of unbalance can cause problems due to the randomness of the connection and disconnection of the single-phase

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6 For the derating curve, $X$ is the unbalance factor $f_u$ and $Y$ represents the derating factor.
loads, uneven distribution of single-phase loads on the three phases and inherent asymmetry of the power system (Vic Gosbell, 2002).

There are various ways to improve the voltage imbalance in an induction motor. However, some are more desirable to use than others.

4.4.1 Derating
While derating can reduce the voltage unbalance it is also undesirable. If the voltage unbalance exceeds 1 percent then the motor must be derated for it to operate successfully. As the voltage unbalance increases, the motor is usually derated to a significantly reduced rated operating power, which will affect the motors use for large projects that require significant amounts of power.

4.4.2 Automatic Voltage Regulators
Voltage regulators can be used to appropriate the unbalanced voltage. The device tries to compensate for the fluctuations experienced by the voltage. This works only if the input voltage is within the regulators range of magnitude and adjustment speed. Rather than using one large regulator to try and remove all the unbalanced voltages, several smaller regulators are preferred to protect various parts of the circuit it may be connected on (Bishop, Unbalanced Voltages and Electric Motors, 2014).
5 Variable Speed Drives

A variable speed drive is a piece of equipment that regulates the speed and rotational force, or torque output, of an electric motor (ABB, 2008). For this investigation, the variable speed drive in use will regulate the speed of the motor at a speed of 1500 revolutions per minute and at a frequency of 50 Hertz. The variable speed drive used in this investigation is the Movitrac 07, which is used for three-phase machines.

5.1 Operation of VSD

Adjusting the speed of the motor can be done by changing the poles of the motor. However this can only be done through machines dedicated to electrically changing the poles or physically changing the motor. A cost effective method to do this is by using a variable speed drive, which also produces more precise results. Through the use of the variable speed drive (VSD) the three-phase induction motor’s speed can be adjusted by changing the frequency applied to the motor (ABB, What is a Variable Frequency Drive? How Does a VFD Work?, 2014). Though, the VSD used in this investigation is using a set frequency and then adjusting the speed of the motor.

5.2 Total Harmonic Distortion

The input current is non-sinusoidal and consists of two pulses per half period. This current waveform has a high level of harmonic distortion. As such, due to the current not being proportional to the supplied voltage, the loads are known as non-linear loads. The impedance which lies between the source and load, has a current which is drawn from nonlinear loads that flow through the
impedance. As this current flows through the impedance it produces voltage harmonics.

The summation of these voltage harmonics then produce a distorted voltage that combines with the fundamental voltage. This creates what is known as total harmonic distortion. The source impedance and voltage harmonics are relied upon by the harmonic distortion. Lower impedances will produce a smaller harmonic distortion for a given level of harmonic current (Hapeshis, 2005). However, if the harmonic current increased, then the harmonic distortion could increase drastically. The total harmonic distortion produced from a variable speed drive has significant effects on a three-phase induction motor, which will be explained later on in the report.
6 Voltage Harmonics

The effect of voltage harmonics on three-phase induction motors is one of the major concerns in industrial power systems. When induction motors are fed by non-sinusoidal currents they produce harmonic distortions. These distortions originate from the harmonic content of the magnetic flux density spatial distribution produced by its coils along the air gap of the motor (L. M. Neto).

Harmonic distortions affect the motors operation, such as torque oscillations with time and spatial harmonic components. However, this investigation only examines time harmonics. They are called time harmonics as they are generated by a source that varies non-sinusoidally in time. Due to these harmonic components it is common practice to manipulate of the coil pitch and distribution, when designing motors (L. M. Neto).

6.1 Operating Performance

Rotating machines such as induction motors are considered a source of harmonics. This is due to the windings being embedded in slots that can never be distributed sinusoidally, so the magnetomotive force is distorted. Though, low order harmonics are a greater risk to a three-phase motor than higher order harmonics (Yasar Birbir, 2007).

6.1.1 Torque-Speed Characteristics

When voltage that is supplied to a three-phase induction motor contains time harmonics, the air-gap flux will have components that will rotate at speeds other than that corresponding to the fundamental frequency (Equations 18 and 19).
\[ n_{\text{sync}} = k \times n_1 \]

*Equation 18: Mechanical Speed under Voltage Harmonics*

\[ w_{\text{sync}} = k \times w_1 \]

*Equation 19: Angular Speed under Voltage Harmonics*

The harmonic speed of the motor, \( n_{\text{sync}} \), is equivalent to the harmonic factor, \( k \), multiplied by the fundamental speed of the motor. Under the second time harmonic, the flux would rotate around the motor at twice the rate of the fundamental. However, the phase voltage at any time harmonic greater than the fundamental is greatly diminished. As seen in Equation 20, the amplitude of the kth harmonic, \( V_k \), is calculated by the fundamental voltage, \( V_1 \), over the harmonic, \( k \). Examples demonstrating this are the 5th harmonic, negative sequence, and the 7th harmonic, positive sequence. They produce phase voltages equivalent to 20% and 15% of the fundamental respectively. (Appendix 11.4).

\[ V_k = \frac{V_1}{k} \]

*Equation 20: Phase Voltage under Voltage Harmonics*

The reactance of the motor at these harmonics is quite high. This results in the current flowing through the windings being significantly reduced, shown in equation 21. This affects the torque, in that harmonics other than the fundamental decrease drastically due to the reduced line current, as shown by 22, (Bhattacharya, 2009).
\[ I_{1k} = \frac{V_k}{(R_1 + jkX_1) + \frac{(R_2^2 + jkX_2)(jkX_M)}{s_k^2 + jk(X_2 + X_M)}} \]

Equation 21: Line Current under Voltage Harmonics

\[ \tau_{emk} = \frac{3 \times R_2 I_{1k}^2}{\Omega_k (kS_k)} \]

Equation 22: Induced Torque under Voltage Harmonics

6.1.2 Sequence Effects on the Rotor

The damage that could potentially be caused by voltage harmonics in a three-phase induction motor mainly comes from the rotor of the motor. A combination of vibration and overheating may occur in the rotor. The vibrations in the rotor originate from the pulsation of torque due to the positive and negative sequence harmonics. As with unbalanced voltages, the positive sequence will produce torque in the clockwise direction, whereas the negative sequence component will act against the direction of the positive sequence. This results in torque pulsations, which in turn lead to torsional vibrations in the rotor. This vibration can increase the friction losses in the bearings, which in turn will reduce the operating life of the bearings. This leads to an increased risk of mechanical failure from the induction motor. The vibration will force the rotor to rub against the stator, if the rotor axis does not have enough strength. This will eventually cause overheating until the wedges are damaged (Ching-Yin Lee, 1998).
6.2 Equivalent Circuit

In circuit analysis under voltage harmonics, the equivalent circuit is identical to that under a balanced system, with slip \( s \), corresponding to the fundamental, as shown in Figure 8, replaced by, \( s_k \), corresponding to the \( k \)th harmonic, shown in Figure 9.

In calculating the \( s_k \) value for each harmonic, attention has to be paid as to whether that harmonic creates a forward rotating magnetic field, positive sequence, or a backward rotating magnetic field, negative sequence.

![Figure 8: Per-phase Equivalent Circuit Corresponding to Fundamental Frequency](image1)

![Figure 9: Per-phase Equivalent Circuit Corresponding to Harmonic Frequencies](image2)
If the harmonic creates a forward rotating magnetic field, the slip is given by Equation 23. Examples are the 5th, 11th and 17th harmonics.

\[ s_k = \frac{(k - 1) + s_1}{k} \]

Equation 23: Positive Sequence Slip Calculation

If the harmonic creates a backward rotating magnetic field, the slip is given by Equation 24. Examples are the 7th, 13th and 19th harmonics.

\[ s_k = \frac{(k + 1) - s_1}{k} \]

Equation 24: Negative Sequence Slip Calculation

6.3 Derating and Temperature

To maintain the rated design temperature rise of a three-phase induction motor operating under voltage harmonics, the motor rating has to be reduced through derating. This reduces the excessive heat build-up due to voltage harmonics that can be severe if the magnitudes of the harmonics are large enough. The heat increase is due to the increases of losses, mainly copper losses and iron losses in the motor from the harmonics. The surge in these losses results in the operating temperature of the motor rising in order to keep the winding temperature down (What do Harmonics Do?, 2010). However, derating standards do not exist for harmonic conditions. The derating factor, as shown in Equation 25, is so that the maximum temperature rise with or without harmonics in the supply for a specific insulation class is kept constant. Through

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7 There are no standards directly examining voltage harmonics and hence will not be illustrated in this report.
the use of thermal modelling and calculating the output power with restriction in the temperature rise limit, derating can be used (Choobari).

\[ D_h = 1 - \frac{P_{outh}}{P_{out}} \]

Equation 25: Derating Equation for Induction Motors under Voltage Harmonics (Choobari)  

6.4 Mitigating Voltage Harmonics

There are various ways to combat voltage harmonics and reduce their severity. Through the use of either filters or reactors on the input and output sides of the variable speed drive of a motor, voltage harmonics can be reduced to reduce their damaging effects on the three-phase induction motor.

6.4.1 Input Reduction Techniques

Line reactors, which are also commonly known as inductors, chokes and line filters as stated in (Fuseco, 2011), help to reduce harmonic distortion. By reducing the total harmonic distortion in a motor, the additional core losses and excessive heating of the motor core can be decreased. Although, it does not eliminate harmonic distortion, it can reduce it to a respectable amount within the five percent limit for voltage harmonics.

6.4.2 Output Reduction Techniques

The use of sine wave filters not only provides protection for a variable speed drive, but they also increase the operating life of the motor and reduces the motor noise, vibration and heat dissipation. The decline of vibration within

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\( P_{outh} \) represents the output power under a non-sinusoidal supply and \( P_{out} \) represents the output power with a sinusoidal supply.
the induction motor is pivotal, as the torque pulsations from the positive and negative sequences that produce this vibration increases the noise and heat of the motor. By reducing the vibration, in the motor will emit less noise and radiate less heat. The reduction of all three of these factors leads to a longer operating life for the motor (Fuseco, 2011).
7 Determination of Motor Parameters

Through motor parameter testing the stator and rotor resistance, stator and rotor reactance and magnetisation reactance can be determined. Obtaining these important parameters allows for a real life motor to be modelled through simulations. However, to obtain these parameters several tests must be done on the induction motor. These tests include:

- The DC Test;
- Blocked-Rotor Test;
- No Load Test.

The tests are done in the following order as each one allows the next variable to be determined from the parameter obtained in the previous.

There are other ways to determine induction motor parameters, which include the use of algorithms and the steady state slip curve. However, this investigation only examined live testing on the motor due to the complexities involved with the other methods for motor parameter determination.

In conjunction with the parameter tests for the induction motor a laboratory write up was designed for this investigation to assist with the setup. The laboratory write up could possibly be used for classes and to help with investigations involving three-phase induction motors (see Appendix 11.10).

7.1 DC Test

The rotor resistance, $R_2$, plays a significant role in the operation of an induction motor as it determines the shape of the torque-speed characteristic curve as well as determining the speed at which the pull-out torque occurs. To
determine the rotor resistance, the stator resistance, $R_1$, must be determined first. The DC test allows for the determination of the stator resistance.

When undertaking the DC test, a direct current voltage is applied to the induction motor. Due to the current being DC, the rotor does not have any current flow or voltage through it. Whilst under a DC current, the reactance of the motor is zero.

Setting up the DC test involves connecting a DC power supply to only two of the three phases of a wye connected three phase induction motor as seen in Figure 10. The current is adjusted to the rated current of the motor, or close to it, to perform the test. This is done to heat the stator windings to a temperature that would be achieved during normal operation of the motor (Chapman S. J., 2005).

Due to the current flowing through two of the windings, the total resistance in the current path is $2R_1$. The restructuring of the DC equations leads to the determination of $R_1$ (see Appendix 11.5).

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**Figure 10: DC Test set-up (Y. Yanawati, 2012)**
7.2 Blocked-Rotor Test

After obtaining the stator resistance, the stator and rotor reactances, $X_1$ and $X_2$, can be determined. Though, both the stator and rotor reactance are equal to each other, for a Design Class N motor, this means both will have the same value when obtained.

The reactance values are obtained from the blocked-rotor test, also known as the locked-rotor test. This test involves the rotor of the motor being blocked. For this investigation an attachment was added to the rotor to lock it in place. Once the rotor is blocked, a voltage is applied to the motor and the resulting voltage, current, power values (from wattmeters P) are measured on each phase, as shown in Figure 11. The measured values are averaged out across the three phases (see Appendix 11.5).

![Figure 11: Blocked-Rotor Test set-up (Y. Yanawati, 2012)](image)

Performing the blocked-rotor test requires the application of an AC voltage to the stator, whilst current flow is adjusted to be approximately full-load value. Once the current is at the rated value, then the required values are measured. Once these values are measured the stator and rotor reactance can be determined (see Appendix 11.5). The stator and rotor reactance are
inseparable, they are being broken down to solve for their respective values.

Due to this, a rule of thumb is used as a way of determining them with more simplicity (see Appendix 11.5). The use of the NEMA Standards allows for the design class of the motor to determine an approximation of the reactance values (see Table 1)

**NEMA Rule of Thumb Table:**

<table>
<thead>
<tr>
<th>Rotor Design</th>
<th>$X_1$</th>
<th>$X_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wound Rotor</td>
<td>$0.5X_1$</td>
<td>$0.5X_2$</td>
</tr>
<tr>
<td>Design A</td>
<td>$0.5X_1$</td>
<td>$0.5X_2$</td>
</tr>
<tr>
<td>Design B</td>
<td>$0.4X_1$</td>
<td>$0.6X_2$</td>
</tr>
<tr>
<td>Design C</td>
<td>$0.3X_1$</td>
<td>$0.7X_2$</td>
</tr>
<tr>
<td>Design D</td>
<td>$0.5X_1$</td>
<td>$0.5X_2$</td>
</tr>
</tbody>
</table>

Table 1: NEMA Rule of Thumb Table for Determining Stator and Rotor Reactance (Chapman S. J., Electric Machinery Fundamentals 4th Ed., 2005)

However, under normal operation when the rotor is not blocked, the stator frequency is the line frequency of the power system. At starting conditions the rotor frequency is at line frequency. Under normal operating conditions the slip of most motors is only two to four percent and the resulting rotor frequency is less than ten percent of the original line frequency. This causes a problem in that the line frequency does not represent the normal operating conditions of the rotor. By having the incorrect rotor frequency,
results obtained in the test can be misleading. To overcome this problem, a frequency of twenty five percent or less of the rated frequency is used. This investigation uses a frequency that is ten percent of the rated frequency. Due to the rated frequency being 50 Hz, the frequency used for the blocked-rotor test is 5 Hz (Chapman S. J., 2005).

7.3 No Load Test
The no load test is used to determine the magnetisation reactance, $X_M$, in the motor. Unlike the blocked-rotor test, the rotor is not locked in place, rather it is allowed to spin freely. The no load test also measures the rotational losses of the motor. However, only the core losses are considered, since they amount to the bulk of the losses for this investigation. Though the rotor copper losses are ignored due to the rotor current, $I_2$, being extremely small.

Conducting the no load test requires an AC voltage like the blocked-rotor test. However, instead of adjusting the current to the rated point, the voltage and frequency are set to their rated requirements. Once these variables are at their required rated values measurements are taken on the voltage, current, power (from wattmeters P1 and P2) and frequency of each phase, as shown in Figure 12. As with the blocked-rotor test, the average of the values is taken across the three phases (see Appendix 11.5).
Determining the magnetisation current not only requires the values measured during the no load test, but it also requires the use of both the parameters and values that were calculated in the DC test and blocked-rotor test. Due to the stator and rotor reactances, $X_1$ and $X_2$, being equal, either parameter can be used to determine the final magnetisation current (Chapman S. J., 2005).

Once all three tests and parameters have been completed and calculated, the final simulation of the three-phase induction motor can take place.

7.4 Parameter Determination Problem
During the investigation a problem arose when determining the motor parameters. This problem in turn affected the final simulation results which were a crucial part of the investigation.

After determining the motor parameters for the Toshiba three-phase induction motor, it was found that the power meter, WT2030 Digital Power Meter, measuring the variables such voltage, current and power were incorrect due to a connection error. The Toshiba premium efficiency motor, model...
S95056102 (Toshiba), was then stripped for parts before any further testing could be done on it. At this point, a Leroy Somer Australia LSMV motor, model 1205360345, was used as a replacement, as shown in Figure 1, as a way to continue the investigation. The parameter testing was undertaken for the new motor with surprising results. The parameters that were calculated were not of sufficient value that would allow the investigation to continue. This was due to the magnetisation reactance being almost half of the stator and rotor reactance. In three-phase induction motors, the magnetisation reactance is the largest variable as it blocks the brunt of the current coming through the stator. This occurs as the rotor reactance and resistance are small.

Due to this significant problem parameters from (Chapman S. J., Electric Machinery Fundamentals, 2005) were used. However, these parameter values displayed results that were unrealistic for the motor used in the parameter testing. This resulted in the original Toshiba motor’s parameters being used. The rated variables of the Toshiba motor were similar to the Leroy Somer motor that would allow for the simulated results to be as close to realism as possible.
8 Simulation Results

Due to problems with determining the motor parameters of the induction motor, the simulation results, whilst seemingly demonstrating the characteristics of a three-phase induction motor, did not correctly illustrate the starting torque, and pull-out torque and operating of the motor. With the torque being incorrect, this in turn produced plots for the line current and output power that did not exhibit the characteristics of an induction motor.

The problems associated with the torque-speed characteristics are based on the true operating torque of the motor being significantly higher than the starting torque for balanced voltages. This in turn will affect the torque-speed characteristics of the motor under unbalanced voltages and voltage harmonics, in that they will not display their true torque results. The line currents for each set of results seem correct, however due to the wrong parameters being used, they may not display the correct plots associated with each case.

The output power results have been greatly affected by this problem, predominantly due to the output power being directly proportional to the torque. The input power is not displayed on the plots, as the required way of determining the input power for an induction motor requires the stator resistance parameter, $R_1$, to be correct according to the motor. Due to this, only the output power was calculated (Equation 33). The output power, $P_{out}$, was determined by the torque, $\tau_{ind}$, and speed of the motor, $\omega_m$, instead of examining taking into account of the losses in the motor that affect the input power. This in turn means the output power cannot be compared to the input power, whilst also examining the characteristics of the motors power.
\[ P_{out} = \omega_m \times \tau_{ind} \]

Equation 26: Output Power of the Induction Motor

8.1 Torque Speed Characteristics

8.1.1 Balanced Voltages

Figure 13 displays the torque-speed characteristics of a three-phase induction motor operating under ideal conditions and with balanced voltages, obtained using MATLAB code in Appendix 11.6.

Due to the motor parameter problem encountered earlier, the starting torque is almost half of the operating torque. The starting torque should be around 150% of the operating torque due to the high starting current.

However, the torque-speed characteristics demonstrate the torque induced by the motor as the speed increases. The pull-out torque of the motor is the maximum possible torque attainable, which in this case is 4.331 N.m, which is also at its full-rated speed of 1385 rpm. This peak torque is only possible for a brief period of time, approximately one minute, since the motor exceeds its
rated current. The torque decreases as it cannot sustain such an amount of
torque based on its rated variables, if it did, then the motor would stall.

The three-phase induction motor’s operating torque, which is around
two to three times smaller than the pull-out torque, is where the motor can
sustain operation and produce its rated torque. Based on the torque-speed
characteristics of Figure 13, the operating torque is around 1.459 N.m. Whilst
incorrect due to the motor parameters, if the motor operation was within that
region, it motor would continue to operate without any problems of breakdown
or overheating. However, if the speed of the motor were to continue past its
synchronous speed, $s = 0$, then it would begin to operate as a generator.

8.1.2 Unbalanced Voltages

The torque-speed characteristics of a three-phase induction motor under
unbalanced voltages produces a similar plot to that under balanced voltages,
with the exception of the negative sequence torque and net torque.

Figure 14, demonstrates the torque-speed characteristics of an induction
motor under unbalanced voltages. To produce an imbalance, Phase B of the
three-phase voltages was displaced by eighty degrees in the MATLAB code (see
Appendix 11.7). The three phase voltages under unbalance were:

$$V_{sa} = 240 \times \cos(2\pi ft)$$

*Equation 27: Phase Voltage A for Zero Sequence*

$$V_{sb} = 240 \times \cos(2\pi ft - (120^\circ - 80^\circ))$$

*Equation 28: Phase Voltage B for Positive Sequence*
\[ V_{sc} = 240 \times \cos(2\pi ft + 120^\circ) \]

Equation 29: Phase Voltage C for Negative Sequence

Whilst the positive sequence produces a larger torque, the negative sequence produces a torque that reduces the net torque, but its effect is minimal due to its almost small, but constant torque as shown in Figure 14.

The net torque of the motor demonstrates what the motor would output under this unbalanced situation, shown in Figure 14. The starting torque, pull-out torque and operating torque are almost half of the torque produced under a balanced voltages in comparison to Figure 13.

![Figure 14: Torque-Speed Characteristics under Unbalanced Voltages](image)

8.1.3 Voltage Harmonics

Voltage harmonics in a three-phase induction motor reduce the torque, current and power that can be achieved by the motor. Since harmonics increase the torque and line current become negligible as the motor cannot operate

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9 The starting torque should be larger than the operating torque, however due to parameter errors this was not shown through the simulation.
under such small loads. The forward rotating magnetic field starts at the seventh harmonic, in comparison to the backward rotating magnetic field, which starts at the fifth harmonic.

The fundamental torque-speed characteristic is identical to that of a motor under balanced conditions. In comparison to the torque-speed characteristics under balanced voltages (Figure 13) the torques produced by forward rotating and backward rotating magnetic fields under voltage harmonics are dramatically reduced as seen in Figures 15 and 16. This is due to the larger reactance created by the harmonic frequency, see Appendix 11.8 and 11.9, which results in reduced line current and in turn a significantly decreased torque output. However, the forward rotating magnetic field created by the seventh harmonic, reaches its pull-out torque at a point that is well past the synchronous speed of the fundamental harmonic.

The backward rotating magnetic field created by the fifth harmonic, produces a slightly larger torque than the forward wave (Figure 15 and 16). Though, as with the positive sequence, its torque is greatly diminished as well in comparison to the torque under balanced voltages. The pull-out torque for the fifth harmonic is also reached at a rotation rate longer than the starting torque of the fundamental. This is in part due to the backward wave operating in the opposite direction to the forward wave.
Figure 15: Torque-Speed Characteristics for Forward Rotating Magnetic Field

Figure 16: Torque-Speed Characteristics for Backward Rotating Magnetic Field

8.2 Line Current

8.2.1 Balanced Voltages

Figure 17, shows the plot for the line current of the three-phase induction motor under balanced conditions. The current demonstrates its starting point, when the pull-out torque occurs and the start of its operating region.

\[10\] The pull-out torques for both the forward and backward rotating magnetic fields could not be displayed due to program difficulty.
The balanced induction motor line current has a high starting current as a way to excite the rotating magnetic fields to produce the torque required for the motor. In comparison to the rated current of the motor, at 1.02 A, the line current is four to five times the rated current when starting. However, this starting current does not last long nor can it. If the starting current maintained its high magnitude the motor will breakdown due to excessive heating.

Whilst the pull-out torque produces the maximum available torque for the motor, the line current does not seem to be at a value that would be sufficient to obtain that peak torque. Whilst the line current may not seem high, the slip of the motor at the speed, 1385 rpm, is quite small, $s = 0.0767$, compared to the starting current slip, which is at, $s = 1$. The low slip value creates a significantly reduced reactance in the motor in comparison to a high slip value, which creates a larger reactance. This low slip value combined with the still large line current produces the maximum possible torque the induction motor can attain.
As the motor passes its peak torque, the motor’s load increases since it begins to reach its operating current. Whilst the operating current displayed above is high due to the parameters, the start of the operating region would be closer to the motor’s rated current of 1.02 A. However, unlike the torque of the motor, once the line current reaches its operating region, it does not tend to zero. Rather the current levels off close to the rated current at which it can continuously operate.

8.2.2 Unbalanced Voltages

In comparison to balanced voltages, where each phase has the same current, unbalanced line currents take into account three different phases of currents in the A, B and C phases, as shown by Figures 18-20. These three phase currents are derived from the positive and negative sequence line currents, which were derived using the corresponding sequence voltages (see Appendix 11.7). The sequence currents can be used to find the unbalanced phase currents using symmetrical component analysis (see Appendix 11.7).

In comparison to balanced voltages, the combined line current from phases A, B and C results in a larger starting current, 7.3 A, see Figure 18, as opposed to 4.8 A and a hefty operating current, 3.9 A, as shown in Figure 20, as opposed to 1.45 A. The pull-out torque of the motor under unbalanced voltages also results in a higher current of 5.4 A, see Figure 19, in comparison to 3.45 A for balanced voltages. These sizable currents that occur under unbalanced voltages are undesirable due to the current instigating significantly larger losses.

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11 The unbalance used in this investigation was an eighty degree phase change for Phase B (see Appendix 11.7).
through the stator and rotor that, in turn, produce a larger input power whilst also reducing the output power of the motor due to the increased losses.

Figure 18: Line Current Plot Starting Current under Unbalanced Voltages

Figure 19: Line Current Plot Pull-out Torque under Unbalanced Voltages
8.2.3 Voltage Harmonics

The main effect voltage harmonics have on three-phase induction motors is due to the torque being directly proportional to the square of the line current of the motor squared, as seen by Equation 22. Any reduction in the line current (Equation 22) passes to the torque, \( \tau_{emk} \), in which it is multiplied by the speed of the motor, \( \omega_m \), to calculate the output power, see Equation 37, which in turn is reduced.\(^{12}\)

\[
P_{out} = \omega_m \times \tau_{emk}
\]

Equation 30: Output Power equation under Voltage Harmonics

The harmonic, which creates a backward turning magnetic field, produces significantly reduced current (Figure 22) in comparison to the balanced voltage line current. However, it still achieves a current that is double the forward rotating field produced by the 7\(^{th}\) harmonic (Figure 21). This is due to

\(^{12}\) The zero sequence is not examined as there is no current or rotating magnetic field under voltage harmonics.
the 5\textsuperscript{th} harmonic producing a slightly reduced reactance compared to the 7\textsuperscript{th} harmonic.

Both the forward and backward rotating fields display line currents that are constant from the motor’s start up until its synchronous speed point. This is due to the impedance in the motor being almost constant itself across the slip points from, $s = 0$ to $s = 1$, at harmonic frequencies.

Due to the motor parameter problem, the line currents do not display their entire current over the speed of the motor. This is in part due to the torque-speed characteristics themselves not demonstrating to a satisfactory level, its true nature over the speed of the motor under voltage harmonics as well.

Figure 21: Line Current Plot for Forward Rotating Magnetic Field under Voltage Harmonics
8.3 Output Power

8.3.1 Balanced Voltages

The output power characteristics, as illustrated in figure 23, of the three-phase induction motor display incorrect points. Primarily the operating region is lower than its rated output power of 0.37 kW. This is due to the motor parameters not being correct for the motor being investigated.
When the motor is stationary, \( s = 1 \), the output power is zero since the speed of the motor is zero.

The output power characteristics follow a similar pattern to the torque-speed characteristics, in that is follows the same motion, excluding the starting points of each plot, in which the power starts from zero. Whilst the output power, \( P_{out} \), is directly proportional to the torque of the motor, \( \tau_{ind} \), it is not necessary that both their maximum values be attained at the same speed. The output power, is based upon the rotor whose speed lags the magnetic field speed. Due to this lagging rotor speed, the peak output power will not be achieved until after the peak torque has been attained. Equation 38 demonstrates that the speed, \( \omega_m \), is multiplied by the torque, to produce the output power of the motor.

\[
P_{out} = \tau_{ind} \times \omega_m
\]

\textit{Equation 31: Output Power of an Induction Motor under Balanced Voltages}

The motor parameters being used for the investigation cause the full load power and operating power, to be less than the actual rated power of the motor, 0.37 kW. The full load output power of the motor should include its rated power, whereby the motor will be able to operate continuously closer to its rated power.

The issue of the motor parameters affected the input power results as the plot could not be displayed. The input power was to be used as a direct comparison against the output power to demonstrate the losses sustained
during a motor’s operation. The losses considered in this investigation are the stator copper losses and rotor copper losses, which exemplify how much power is lost from the input to the output of the motor.

8.3.2 Unbalanced Voltages

Due to the motor parameter problem, the input power could not be displayed to compare against the output power as well as hampering the start of the operating power region.

Unbalanced voltages cause a larger line current in comparison to balanced voltages, which in turn directly affects the output power of the motor. The larger line current caused by an imbalance will give rise to an increased input power. Whilst an increase in input power may seem to indicate an increased output power, it is not the case with unbalanced voltages. The large line current also increases losses in the motor, primarily the stator and rotor, which in turn reduce the output power of the induction motor.

The output power under unbalanced voltages (Figure 24) in comparison to balanced voltages (Figure 23) demonstrates the reduced output power caused by an unbalance in the voltage. Whilst the starting power remains identical, the peak power and the beginning of the operating power region are reduced by 243 W and 94 W respectively. Such a reduction in the output power of the motor is undesirable as the peak power is only 18.3 W above the rated power of the motor itself, 370 W, which means the motor will always operate below its rated output under unbalanced voltages.
8.3.3 Voltage Harmonics

For this investigation the converted power is equivalent to the output power, see equation 40. The torque of the motor is directly proportional to the converted power of the motor, see equation 39. With a decrease in the torque of the motor, the output power will directly be affected, which results in a reduction of power the motor can output. However, due to voltage harmonics, which causes the line current and in turn the torque to drastically decrease, the three-phase motor suffers a severe reduction in its output power, see Figures 25 and 26.

\[ P_{\text{conv}} = \frac{T_{\text{em}}}{\omega_{\text{sync}}} \]

Equation 32: Converted Power under Voltage Harmonics

\[ P_{\text{out}} = P_{\text{conv}} \]

Equation 33: Output Power of an Induction Motor under Voltage Harmonics
As with the line currents, the 5th harmonic produces a slightly greater output power than the 7th harmonic as the speed increases. The extreme reduction of the output power due to harmonics exasperates the problem that harmonics, if larger enough, can possibly render the motor inoperable as such an insignificant output of power will not operate a machine rated sufficiently higher. This is exemplified in a comparison of the output power of the balanced voltages against the output power of the voltage harmonics. The peak power output under a balanced system is 631.2 W, whilst the highest power attained under harmonics for the positive and negative sequence is 0.012 W and 0.045 W respectively.

It is to be noted that due to the motor parameter problem the input power could not be attained to compare against the output power, whilst the net power under voltage harmonics was not able to be achieved due to timing and programming issues.

![Figure 25: Output Power Plot for Forward Rotating Magnetic Field under Voltage Harmonics](image)
Figure 26: Output Power Plot for Backward Rotating Magnetic Field under Voltage Harmonics
9 Conclusion

9.1 Thesis Conclusion

This investigation sets out to examine and understand the effects of unbalanced voltages and voltage harmonics, in relation to performance of a three-phase induction motor. To understand the effects of these cases a thorough analysis of motor operation was carried out alongside motor parameter testing as a means to pursue graphical demonstrations through simulation.

Through the use of the programming software MATLAB, simulations of the torque-speed characteristics, line current, input power and output power were displayed to demonstrate the performance of a three-phase induction motor under these two cases. A third case, balanced voltages, was simulated as a way to exemplify the reduction in performance of a three-phase induction motor, whilst under unbalanced voltages and voltage harmonics.

Unfortunately, due to complications with the motor parameter testing, incorrect parameters, and the three-phase induction motor being stripped for parts, the results did not represent the true characteristics of a three-phase induction motor. This lead to the omission of the input power characteristic of the motor in the report, as well as the results under voltage harmonics being under done.

Overall the investigation of unbalanced voltages and voltage harmonics provides a base for future work examining the combination of unbalanced voltages and voltage harmonics together, which is ever present in induction motor operation today.
9.2 Improvements

Whilst this investigation may be deemed a partial success, it needs to be improved upon to avoid problems in the future. A suggestion for improvement would be in the undertaking of motor parameter testing. The parameter testing for a three-phase induction motor relies solely on the equipment being used, such as a three-phase power meter, variable-speed drive and even the induction motor, to be in working condition and operating properly. If any of the equipment were to suffer from a fault, it instantaneously jeopardises the entire investigation as the required parameters may produce results that could possibly not be satisfactory for examination. This requires that all equipment be maintained and inspected regularly before any work is undertaken with them.

9.3 Future Work Recommendations

The foundation of this investigation allows for further investigation of not only the performance of a three-phase induction motor under unbalanced voltages and harmonics, but also of larger industrial induction motors under these conditions can be suggested as future work if professionally required.

Three-phase induction motors suffer greatly from unbalanced harmonics which are unavoidable. By investigating the torque-speed characteristics, line currents, input power and output power of an induction motor under unbalanced harmonics, it will allow for a greater understanding on not only the effects on a motor’s performance, but also how to implement preventative measures to reduce unbalanced harmonics to a level that does not affect the motor’s performance greatly.
Investigating larger industrial motors could provide a greater insight into the effects of unbalanced voltages and voltage harmonics as they are under heavy amounts of stress in environments such as power systems and plants. This increases their chances of being affected by unbalanced voltages and voltage harmonics, which could be detrimental if not addressed adequately.
References


Y. Yanawati, I. D. (2012). *Efficiency Increment on 0.35 mm and 0.50 mm Thicknesses of Non-oriented Steel Sheets for 0.5 Hp Induction Motor*. Scientific & Academic Publishing.

11 Appendix

11.1 Turns Ratio

\[
V_P = V'_S = aV_S
\]
\[
I_P = I'_S = \frac{I_S}{A}
\]
\[
Z'_S = a^2 Z_S
\]
\[
E_1 = E'_R = a_{\text{eff}} E_{RO}
\]
\[
I_2 = \frac{I_R}{a_{\text{eff}}}
\]
\[
Z_2 = a_{\text{eff}}^2 \left( \frac{R_R}{S} + jX_{RO} \right)
\]
\[
R_2 = a_{\text{eff}}^2 R_R
\]
\[
X_2 = a_{\text{eff}}^2 X_{RO}
\]

11.2 Negative Sequence Slip

\[
s_{\text{positive}} = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}}
\]
\[
s_{\text{negative}} = -\frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} = (2 - s_{\text{positive}})
\]

11.3 Phase Currents: Unbalanced Voltages

% Line Current Calculation for Positive and Negative Sequence as well as Phase A, B and C currents

% Phase vectors
alp=-0.5+j*0.866; % Phase vector alpha
alp2=-0.5-j*0.866; % Phase vector for alpha^2

Zinpos=R1+j*X1+(j*Xm*(R2./s+j*X2))./(R2./s+j*(X2+Xm)); % Impedance going into the Induction Motor for the Positive Sequence

Zinpos=real(Zinpos); % Real part of the impedance in
Zin2pos=imag(Zinpos); % Imaginary part of the impedance in
Ils1=Vpos./Zinpos; % Positive Sequence Line Current
Ilpos=sqrt(3)*(abs(Ils1)); % Line current, where Ils1 is the absolute value of the Positive Sequence Line Current

Zinneg=R1+j*X1+(j*Xm*((R2./(2-s))+j*(X2+Xm)))/(s/(2-s)+j*(X2+Xm)); % Impedance going into the Induction Motor for the Negative Sequence

Zin2neg=real(Zinneg); % Real part of the impedance in
Zin2neg=imag(Zinneg); % Imaginary part of the impedance in
I1s2=Vneg./Zin2neg; % Negative Sequence Line Current
Ilneg=sqrt(3)*(abs(I1s2)); % Line current, where I1s2 is the
% Absolute value of the Negative Sequence Line Current

% Final Line Currents for Phase A, B and C
ILsa=I1s1+I1s2; % Phase A Line Current Component
ILsanew=abs(ILsa); % Phase A Line Current

ILsb=alp2*I1s1+alp*I1s2; % Phase B Line Current Component
ILsbnew=abs(ILsb); % Phase B Line Current

ILsc=alp*I1s1+alp2*I1s2; % Phase C Line Current Component
ILscnew=abs(ILsc); % Phase C Line Current

11.4 Phase Voltage: Voltage Harmonics
11.4.1 Fifth Harmonic: Negative Sequence

\[ V_5 = \frac{240}{5} = 48 \text{ V} \]

\% = \frac{V_5}{V_1} = \frac{48}{240} \times 100\% = 20\%

11.4.2 Seventh Harmonic: Positive Sequence

\[ V_7 = \frac{240}{7} = 34.3 \text{ V} \]

\% = \frac{V_7}{V_1} = \frac{34.3}{240} \times 100\% = 14.3\%

11.5 Parameter Measurement
% Calculating the Motor Parameters for a 3-Phase Induction Motor
%Based on Original Toshiba Motor
% Clear
clc

frated=50; % Rated frequency of the motor
ftest=5; % Test frequency of the motor for the blocked rotor test
Ir=1.1; % Rated current of the motor

% DC Test
Vdc1=28.78; % Measured DC voltage at rated current between phases
Vdc2=28.69; % Measured DC voltage at rated current between phases
Vdc3=28.64; % Measured DC voltage at rated current between phases
Vdcav=(Vdc1+Vdc2+Vdc3)/3; % Average of the DC voltages
R1=Vdcav/(2*Ir); % Calculation of R1 (Stator Resistance)
%No-load Test
Vnl1=418.73; %Phase one no load voltage
Vnl2=418.77; %Phase two no load voltage
Vnl3=419.91; %Phase three no load voltage
VNL=(Vnl1+Vnl2+Vnl3)/3; %No load voltage average across 3-phases

Inl1=0.7095; %Phase one no load current
Inl2=0.7474; %Phase two no load current
Inl3=0.7154; %Phase three no load current
INL=(Inl1+Inl2+Inl3)/3; %No load current average across 3-phases

PNL=-366; %no load active power average
QNL=836.7; %no load reactive power average
SNL=941.3; %no load apparent power average

%Block-Rotor Test
Vbr1=50.22; %Phase one blocked rotor voltage
Vbr2=50.16; %Phase two blocked rotor voltage
Vbr3=50.33; %Phase three blocked rotor voltage
VBR=(Vbr1+Vbr2+Vbr3)/3; %blocked rotor voltage average across 3-phases

Ibr1=1.0913; %Phase one blocked rotor current
Ibr2=1.0819; %Phase two blocked rotor current
Ibr3=1.0833; %Phase three blocked rotor current
IBR=(Ibr1+Ibr2+Ibr3)/3; %blocked rotor current average across 3-phases

PBR=106.8; %blocked rotor active power average
QBR=132.6; %blocked rotor reactive power average
SBR=170.5; %blocked rotor apparent power average

%No-load Calculations first
VNLnew=VNL/sqrt(3);
ZNL=VNLnew/INL; %This is equal to X1 + XM

PSCL=3*(INL)^2*(R1); %Stator copper losses
PROT=PNL-PSCL; %Rotor power

%Blocked Rotor calculations
ZBR=VBR/(sqrt(3)*IBR);

theta=acosd(PBR/(sqrt(3)*VBR*IBR)); %Angle calculation to determine other parameters

RBR=ZBR*cosd(theta); %The value for the blocked rotor resistance, RBR, is equal to R1+R2
R2=RBR-R1; %This gives the rotor resistance of the motor

X1BR=ZBR*sind(theta); %This is used to find the blocked rotor reactance

XBR=(frated/ftest)*X1BR; %The value of the blocked rotor reactance is used to calculate the magnetisation reactance as well as the rotor and stator reactance
%Basing the motor off of a Design Class A induction motor
%Where X1=X2
X1=abs(0.5*XBR);  %The stator and rotor reactance are determined
X2=abs(0.5*XBR);  %The stator and rotor reactance are determined
XM=ZNL-X1;  %The value of the magnetisation is calculated

disp(['R1=',num2str(R1)])
disp(['R2=',num2str(R2)])
disp(['X1=',num2str(X1)])
disp(['X2=',num2str(X2)])
disp(['XM=',num2str(XM)])

11.6 Balanced Voltages: Simulation
%Induction Motor Under Balanced Conditions

clear  
clc  

%Motor Parameters
VT=415;  %Rated Motor Voltage
Pow=370;  %Rated Power
phV=VT/sqrt(3);  %Phase voltage
Pha=3;  %Phase Number
Pol=4;  %Number of Poles
f=50;  %Rated Frequency
Ir=1.02;  %Rated Motor Current

%DC Test
Vdc1=24.5;  %Measured DC voltage at rated current between phases 3 and 2
Vdc2=24.3;  %Measured DC voltage at rated current between phases 3 and 1
Vdc3=24.1;  %Measured DC voltage at rated current between phases 2 and 1
Vdcav=(24.5+24.3+24.1)/3;  %Average of the DC voltages
%R1=Vdcav/(2*Ir);  %Calculation of R1 (STATOR RESISTANCE)
R1=13.641;  %Stator Resistance
X1=44.50;  %Stator Reactance
R2=6.3;  %Rotor Resistance
X2=44.5;  %Rotor Reactance
Xm=280.3;  %Magnetization Reactance

%Synchronous Speeds
wsync=4*pi*f/Pol;  %Angular Speed
nsync=120*f/Pol;  %Rpm Speed

%Thevenin Equivalent
Zth=(j*Xm*(R1+j*X1))/(R1+j*(X1+Xm));  %Thevenin Impedance
Rth=real(Zth);  %Real part of the Thevenin Impedance
Xth=imag(Zth);  %Imaginary part of the Thevenin Impedance
Vth=abs((phV*j*Xm)/(R1+j*(X1+Xm))));  %Thevenin Voltage

%Any Line Voltage can be used as they are all the same under Balanced
%Conditions
% Line voltage First Phase
VL1=phV;
% Line voltage Second Phase
VL2=phV;
% Line voltage Third Phase
VL3=phV;

% Slip Calculation
s=1/1000:1/1000:1; % Slip
nm=(1-s)*nsync; % Mechanical Speed
wm=(1-s)*wsync; % Angular Speed

% Calculation of the Electromagnetic Torque
indTor=(3*Vth^2.*(R2./s))./(wsync*((Rth+R2./s).^2+(Xth+X2).^2));
% Electromagnetic Torque

% Line Current for each Phase (Under balanced conditions the line currents are the same for each phase)
Zin=R1+j*X1+(j*Xm*(R2./s+j*X2))./(R2./s+j*(X2+Xm)); % Impedance going into the Induction Motor
Zin1=real(Zin); % Real part of the impedance in
Zin2=imag(Zin); % Imaginary part of the impedance in
I1=VL1./Zin; % Line Current
IL=sqrt(3)*abs(I1); % Absolute Line current, where I1 is the absolute value of the original Line Current

% Output Power of the Motor
Pout=wm.*indTor; % Output Power

% Plots of the Torque, Line Current and Output Power
subplot(2,2,1)
plot(nm,indTor) % Electromagnetic Torque Plot
xlabel('Mechanical Speed, r/min')
ylabel('Induced Torque, N.m')
title('Balanced Induction Motor Torque-Speed Characteristics')
legend('Torque')
grid on
subplot(2,2,2)
plot(nm,IL) % Line Current Plot
xlabel('Mechanical Speed, r/min')
ylabel('Line Current, A')
title('Balanced Induction Motor Line Current')
legend('IL')
grid on
subplot(2,2,3)
plot(nm,Pout) % Output Power Plot
xlabel('Mechanical Speed, r/min')
ylabel('Output Power, W')
title('Balanced Induction Motor Output Power')
legend('Pout')
grid on
hold

11.7 Unbalanced Voltages: Simulation
% Induction Motor Under Unbalanced Conditions

clear
clc
%Motor Parameters
VT=415; %Rated Motor Voltage
phV=VT/sqrt(3); %Phase Voltage
Pow=370; %Rated Power
Pha=3; %Phase Number
Pol=4; %Number of Poles
f=50; %Rated Frequency
Ir=1.02; %Rated Motor Current
Vsa=phV*(cosd(0)+j*sind(0)); %Phase Voltage 1 for Zero Sequence
Vsb=phV*(cosd(-40)+j*sind(-40)); %Phase Voltage 2 for Positive Sequence
Vsc=phV*(cosd(120)+j*sind(120)); %Phase Voltage 3 for Negative Sequence

%Phase vectors
alp=-0.5+j*0.866; %Phase vector alpha
alp2=-0.5-j*0.866; %Phase vector for alpha^2

%Positive Sequence Voltage Component
Vpos1=(Vsa); %Phase voltage 1
Vpos2=(Vsb); %Phase voltage 2
Vpos3=(Vsc); %Phase voltage 3
Vpos=(Vpos1+alp*Vpos2+alp2*Vpos3)/3; %Positive Sequence Voltage Component

%Negative Sequence Voltage Component
Vneg1=(Vsa); %Phase voltage 1
Vneg2=(Vsb); %Phase voltage 2
Vneg3=(Vsc); %Phase voltage 3
Vneg=(Vneg1+alp2*Vneg2+alp*Vneg3)/3; %Negative Sequence Voltage Component

%DC Test
Vdc1=24.5; %Measured DC voltage at rated current between phases 3 and 2
Vdc2=24.3; %Measured DC voltage at rated current between phases 3 and 1
Vdc3=24.1; %Measured DC voltage at rated current between phases 2 and 1
Vdcav=(24.5+24.3+24.1)/3; %Average of the DC voltages
R1=Vdcav/(2*Ir); %Calculation of R1 (STATOR RESISTANCE)
R1=13.641; %Stator Resistance
X1=44.750; %Stator Reactance
R2=6.3; %Rotor Resistance
X2=44.5; %Rotor Reactance
Xm=280.3; %Magnetization Reactance

%Synchronous Speeds
wsync=4*pi*f/Pol; %Angular Speed
nsync=120*f/Pol; %Rpm Speed

%Thevenin Equivalents For Positive and Negative Sequence
%Positive Sequence Thevenin Voltage
Zth=j*Xm*(R1+j*X1)/(R1+j*(X1+Xm)); %Thevenin Impedance
Rthpos=real(Zth); %Real part of Thevenin Impedance
Xthpos=imag(Zth); %Imaginary Part of Thevenin Impedance
Vthpos=abs((Vpos*j*Xm)/(R1+j*(X1+Xm)))); %Positive Sequence Thevenin Voltage
%Negative Sequence Thevenin Voltage
Zth=(j*Xm* (R1+j*X1))/(R1+j*(X1+Xm)); %Thevenin Impedance
Rthneg=real(Zth); %Real part of Thevenin impedance
Xthneg=imag(Zth); %Imaginary part of Thevenin impedance
Vthneg=(abs((Vneg*j*Xm)/(R1+j*(X1+Xm)))); %Negative Sequence Thevenin Voltage

%Voltage Unbalance Factor: %Calculating the Positive and Negative Sequence Unbalance
Z1=R1+j*X1;
Z2=R2+j*X2;
V1unpos=abs(Vpos/((1-alp2)*(1+Z2/Z1))); %Positive sequence unbalance factor
V2unneg=abs(Vneg/((1-alp)*(1+Z1/Z2))); %Negative sequence unbalance factor
%Calculating the positive and negative sequence voltages
disp(['V1un=',num2str(V1unpos)])
disp(['V2un=',num2str(V2unneg)])
VUF=abs(V2unneg/V1unpos)*100; %Voltage unbalance factor where V2un and V1un are the Negative and Positive Sequence Unbalance Factors Respectively
disp(['VUF=',num2str(VUF)]) %Final overall UNbalance Factor affecting the motor

%Calculation of the Slip and Speeds
s=1/1000:1/1000:1; %Slip
nm=(1-s)*nsync; %Mechanical Speed
wm=(1-s)*wsync; %Angular speed

%Positive Sequence Torque
indTorpos=(3*Vthpos^2*(R2./s))./(wsync*((Rthpos+R2./s).^2+(Xthpos+X2)^2)); %Calculate Induced Torque for Positive Sequence

%Negative Sequence Torque
indTorneg=(3*Vthneg^2*(R2./(2-s)))./(wsync*((Rthneg+(R2./(2-s))).^2+(Xthneg+X2)^2)); %Calculate Induced Torque for Negative Sequence

%Net Output Torque Produced
indTorToT=indTorpos-indTorneg; %Net Torque

%Net Output Power
Pout=wm.*indTorToT; %Net Output Power

%Line Current Calculation for Positive and Negative Sequence as well as Phase A, B and C currents
Zinpos=R1+j*X1+(j*Xm*(R2./s+j*X2))./(R2./s+j*(X2+Xm)); %Impedance going into the Induction Motor for the Positive Sequence
Zinpos=real(Zinpos); %Real part of the impedance in Zin
Zin2pos=imag(Zinpos); %Imaginary part of the impedance in Zin
I1s1=Vpos./Zinpos; %Positive Sequence Line Current
Ilpos=sqrt(3)*abs(I1s1); %Line current, where I1s1 is the absolute value of the Positive Sequence Line Current
\[ Z_{\text{inneg}} = R_1 + jX_1 + (jX_m \cdot \frac{(R_2 / (2 - s) + jX_2)}{((R_2 / (2 - s)) + j(X_2 + X_m))}) \] % Impedance going into the Induction Motor for the Negative Sequence

\[ Z_{\text{in1neg}} = \text{real}(Z_{\text{inneg}}); \] % Real part of the impedance in
\[ Z_{\text{in2neg}} = \text{imag}(Z_{\text{inneg}}); \] % Imaginary part of the impedance in
\[ I_{\text{ls2}} = \text{V}_{\text{neg}} / Z_{\text{inneg}}; \] % Negative Sequence Line Current
\[ I_{\text{lng}} = \sqrt{3} \cdot \text{abs}(I_{\text{ls2}}); \] % Line current, where \( I_{\text{ls2}} \) is the absolute value of the Negative Sequence Line Current

% Final Line Currents for Phase A, B and C
\[ I_{\text{lsa}} = I_{\text{ls1}} + I_{\text{ls2}}; \] % Phase A Line Current Component
\[ I_{\text{lsanew}} = \text{abs}(I_{\text{lsa}}); \] % Phase A Line Current

\[ I_{\text{lsb}} = \alpha_2 \cdot I_{\text{ls1}} + \alpha_1 \cdot I_{\text{ls2}}; \] % Phase B Line Current Component
\[ I_{\text{lsbnew}} = \text{abs}(I_{\text{lsb}}); \] % Phase B Line Current

\[ I_{\text{lsb}} = \alpha_1 \cdot I_{\text{ls1}} + \alpha_2 \cdot I_{\text{ls2}}; \] % Phase C Line Current Component
\[ I_{\text{lsnew}} = \text{abs}(I_{\text{lsb}}); \] % Phase C Line Current

% Plot
\[ \text{subplot}(2,2,1) \]
\[ \text{plot}(\text{nm}, \text{indTorneg}, '-', \text{nm}, \text{indTorpos}, '-.', \text{nm}, \text{indTorToT}, '-') \] % Plot of the Net Torque of the Motor
\[ \text{xlabel('Mechanical Speed, \text{r/min}')}
\[ \text{ylabel('Induced Torque, N.m')}
\[ \text{title('Unbalanced Induction Motor Torque-Speed Characteristics for Positive and Negative Sequence')}
\[ \text{legend('Negative Sequence','Positive Sequence','Net Torque')}
\[ \text{grid on} \]
\[ \text{subplot}(2,2,2) \]
\[ \text{plot}(\text{nm}, \text{Pout}) \] % Plot of the Net Output Power
\[ \text{xlabel('Mechanical Speed, \text{r/min}')}
\[ \text{ylabel('Output Power, W')}
\[ \text{title('Unbalanced Induction Motor Total Output Power')}
\[ \text{legend('Pout')}
\[ \text{grid on} \]
\[ \text{subplot}(2,2,3) \]
\[ \text{plot}(\text{nm}, I_{\text{lsanew}}, \text{nm}, I_{\text{lsbnew}}, \text{nm}, I_{\text{lsnew}}) \] % Plot of the Phase A, B and C Line Current Components
\[ \text{xlabel('Mechanical Speed, \text{r/min}')}
\[ \text{ylabel('Line Current, Amps')}
\[ \text{title('Unbalanced Line Current Plot')}
\[ \text{legend('A phase','B Phase','C Phase')}
\[ \text{grid on} \]

11.8 Voltage Harmonics: Positive Sequence Simulation

% Induction Motor Under Voltage Harmonic Conditions
% Created by: AV (June 6th 2014)

clear
clc

% Motor Parameters
VT=415; % Rated Motor Voltage
Pow=370; % Rated Power
Pha=3; % Phase Number
Pol=4; % Number of Poles
f=50; %Frequency
Ir=1.02; %Rated Motor Current
phV=VT/sqrt(3); %Phase Voltage (Balanced voltage coming into the motor)
k=7; %Positive Sequence Hamronic Factor

%DC Test
Vdc1=24.5; %Measured DC voltage at rated current between phases 3 and 2
Vdc2=24.3; %Measured DC voltage at rated current between phases 3 and 1
Vdc3=24.1; %Measured DC voltage at rated current between phases 2 and 1
Vdcav=(Vdc1+Vdc2+Vdc3)/3; % Average of the DC voltages
R1=Vdcav/(2*Ir); %Calculation of R1 (STATOR RESISTANCE)
R1=13.641; %Stator Resistance
X1=44.750; %Stator Reactance
R2=6.3; %Rotor Resistance
X2=44.5; %Rotor Reactance
Xm=28.0; %Magnetization Reactance

%Synchronous Speed
wsync=4*pi*f/Pol; %Angular Speed
nsync=120*f/Pol; %Rpm Speed

%Positive sequence speeds
wsynck=k*wsync; %Angular Speed under harmonics
nsynck=k*nsync; %Rpm under harmonics

%Slip for Harmonics
spos = 1:-1/1000:1/1000; %Slip
skpos =((k-1)+spos)/k; %Negative Sequence Slip Calculation due to Harmonics

%Synchronous Speed for Positive Sequence Harmonics
nmpos=(1-spos)*nsync; %Mechanical Speed
wmpos=(1-spos)*wsync; %Angular Speed

%R2new1=R2./skpos; %ROTOR RESISTANCE OVER THE SLIP: CALCULATES EACH SLIP POINT

%Positive Sequence Line Currents
Ilkpos1=(((R2./skpos+1i*k*X2)*1i*k*Xm)./(R2./skpos+1i*k*(X2+Xm)));
Ilkpos2=(R1+1i*k*X1); %Single value calculation for the Line Current
Ilkpos3=Ilkpos1+Ilkpos2; %The combination of the two create the Impedance in the Motor
Ilkposfin=Vk1./Ilkpos3; %Final Line Current Calculation under the specific Harmonic
Ilkposnew=abs(Ilkposfin); %Absoluting the Line Current for plotting (Plots don't display imaginary values)

%Positive Sequence Torques
emTorpos=(3*R2*(Ilkposnew).^2)./(wsync*k*skpos); %Calculate Electromagnetic Torque for Negative Sequence Harmonics
Poutpos=wmpos.*emTorpos; %Output Power of the Motor under Positive Sequence Harmonics
%Plots of the Electromagnetic Torque, Line Current and Output Power for the Positive Sequence Harmonics
subplot(2,2,1)
plot(nmpos,emTorpos) %Electromagnetic Torque Plot
xlabel('Mechanical Speed, r/min')
ylabel('Harmonic Induced Torque, N.m')
title('Voltage Harmonic Induction Motor Positive Sequence Torque-Speed Characteristics')
legend('Tor Pos')
grid on
subplot(2,2,2)
plot(nmpos,I1kposnew) %Line Current Plot
xlabel('Mechanical Speed, r/min')
ylabel('Harmonic Line Current, A')
title('Voltage Harmonic Positive Sequence Line Current')
legend('IL')
grid on
subplot(2,2,3)
plot(nmpos,Poutpos) %Output Power Plot
xlabel('Mechanical Speed, r/min')
ylabel('Harmonic Output Power, N.m')
title('Voltage Harmonic Positive Sequence Output Power')
legend('Pout')
grid on

11.9 Voltage Harmonics: Negative Sequence Simulation

%Induction Motor Under Voltage Harmonic Conditions
%Created by: AV (June 6th 2014)

clear
clc

%Motor Parameters
VT=415; %Rated Motor Voltage
Pow=370; %Rated Power
Pha=3; %Phase Number
Pol=4; %Number of Poles
f=50; %Frequency
Ir=1.02; %Rated Motor Current
phV=VT/sqrt(3); %Phase Voltage (Balanced voltage coming into the motor)
k=5; %Negative Sequence Hamronic Factor
Vk=phV/k; %Phase Voltage at each Harmonic

%DC Test
Vdc1=24.5; %Measured DC voltage at rated current between phases 3 and 2
Vdc2=24.3; %Measured DC voltage at rated current between phases 3 and 1
Vdc3=24.1; %Measured DC voltage at rated current between phases 2 and 1
Vdcav=(Vdc1+Vdc2+Vdc3)/3; % Average of the DC voltages
%R1=Vdcav/(2*Ir); %Calculation of R1 (STATOR RESISTANCE)
R1=13.641; %Stator Resistance
Xl=44.750; %Stator Reactance
R2=6.3; %Rotor Resistance
X2=44.5; %Rotor Reactance
Xm=280.3; %Magnetization Reactance

%Synchronous Speed
wsync=4*pi*f/Pol; %Angular Speed
nsync=120*f/Pol; %Rpm Speed

%Negative sequence speeds
wsynck=k*wsync; %Angular Speed under harmonics
nsynck=k*nsync; %Rpm under harmonics

%Slip for Harmonics
sneg=1:-1/1000:1/1000; %Slip
skneg=((k+1)-sneg)/k; %Negative Sequence Slip Calculation due to Harmonics

%Synchronous Speeds for Negative Sequence Harmonics
nmneg=(1-sneg)*nsync; %Mechanical Speed
wmneg=(1-sneg)*wsync; %Angular speed

%Fundamental Line Current
I1kneg1=((R2./skneg+1i*k*X2)*1i*k*Xm)./(R2./skneg+1i*k*(X2+Xm))
I1kneg2 = (R1+1i*k*X1); %Single value calculation for the Line Current
I1kneg3=I1kneg1+I1kneg2; %The combination of the two create the Impedance in the Motor
I1knegfin=Vk1./I1kneg3; %Final Line Current Calculation under the specific Harmonic
I1knegnew=abs(I1knegfin); %Absoluting the Line Current for plotting (Plots don't display imaginary values)

%Fundamental Torque and Output Power
emTorneg=(3*R2*(I1knegnew).^2)./(wsync*k*skneg); %Calculate Electromagnetic Torque for Negative Sequence Harmonics
Poutneg=wmneg.*emTorneg; %Output Power of the Motor under Negative Sequence Harmonics

%Plots of the Electromagnetic Torque, Line Current and Output Power for the %Negative Sequence Harmonics
subplot(2,2,1)
plot(nmneg,emTorneg) %Electromagnetic Torque Plot
xlabel('Mechanical Speed, r/min')
ylabel('Harmonic Induced Torque, N.m')
title('Voltage Harmonic Induction Motor Negative Sequence Torque-Speed Characteristics')
legend('Tor Neg')
grid on

subplot(2,2,2)
plot(nmneg,I1knegnew) %Line Current Plot
xlabel('Mechanical Speed, r/min')
ylabel('Harmonic Line Current, A')
title('Voltage Harmonic Negative Sequence Line Current')
legend('IL')
grid on

subplot(2,2,3)
plot(nmneg,Poutneg) %Output Power Plot
xlabel('Mechanical Speed, r/min')
ylabel('Harmonic Output Power, N.m')
11.10 Laboratory Design

The following laboratory is designed to demonstrate how to conduct the measurement of parameters for a three-phase induction motor.

**DC Test Procedure:**

Before the test is conducted, the circuit must be organised according to Figure 38, with the following pieces of equipment:

- Adjustable DC Power Supply;
- Ammeter and Voltmeter.

The following steps allow for the correct setup of the test:

1. Connect the DC power supply to two of the stator windings of the induction motor. Only two terminals should be connected i.e. one terminal of the motor is not connected.
2. Place a voltmeter across the two connected stator resistors.
3. Place an ammeter between DC power supply and voltmeter.

The circuit should be organised as shown in Figure 38.

![Figure 27: DC Test set-up (Y. Yanawati, 2012)](image-url)
4. Once the circuit is organised, turn the power supply on and allow the motor to run at its rated current. Do not go above the rated current of the induction motor.

5. Once the rated current has been obtained, take down the results for the voltage and current.

6. Turn off the power supply and determine the stator resistance using Equation 26: (For a star connected motor)

\[
R_1 = \frac{V_{dc}}{2I_{dc}}
\]

**Equation 34: Calculation to determine the Stator Resistance**

**Blocked-Rotor Test Procedure:**

To conduct this test the materials required include:

- Three ammeters;
- Two power meters;
- One voltmeter;
- AC power supply.

The following steps allow for the correct setup of the test:

1. Before turning the power supply on use a device to lock the rotor in place.
2. Connect the AC power supply through each phase of the induction motor.
3. Each phase of the induction motor will require an ammeter measuring the current.
4. Two of the phases, A and C require power meters to be connected.

5. The voltmeter will be connected between phases A and B.

The circuit should be organised as illustrated in Figure 21:

![Figure 28: Blocked Rotor Test set-up (Y. Yanawati, 2012)]

6. Set the frequency to 25% of the rated frequency, this will become $f_{test}$

7. Turn on the AC Power supply and adjust it until close to the rated current.
   (Do not let the current go above its rated value)

8. Quickly take down the values for the power, voltage and current.

9. Turn off the AC power supply and determine the stator and rotor parameters.

**Calculating the resistance and reactance:**

Equations 27, 28 and 29 are used to determine the rotor resistance and reactance as well as the stator reactance. The reactance are determined by a rule of thumb depending on the design class of the motor. (See Table 2)
NEMA Rule of Thumb Table:

<table>
<thead>
<tr>
<th>Rotor Design</th>
<th>$X_1$</th>
<th>$X_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wound Rotor</td>
<td>$0.5X_1$</td>
<td>$0.5X_2$</td>
</tr>
<tr>
<td>Design A</td>
<td>$0.5X_1$</td>
<td>$0.5X_2$</td>
</tr>
<tr>
<td>Design B</td>
<td>$0.4X_1$</td>
<td>$0.6X_2$</td>
</tr>
<tr>
<td>Design C</td>
<td>$0.3X_1$</td>
<td>$0.7X_2$</td>
</tr>
<tr>
<td>Design D</td>
<td>$0.5X_1$</td>
<td>$0.5X_2$</td>
</tr>
</tbody>
</table>

Table 2: NEMA Rule of Thumb Table for Determining Stator and Rotor Reactance (Chapman S. J., Electric Machinery Fundamentals 4th Ed., 2005)

\[ R_{BR} = R_1 + R_2 \]

Equation 35: Blocked-Rotor Resistance Equation

\[ X_{BR} = X_1 + X_2, \text{ where } X_{BR} = \frac{f_{\text{rated}}}{f_{\text{test}}} \times X_{BR}' \]

Equation 36: Blocked-Rotor Reactance Equation

\[ \frac{Z_{BR}}{\text{Rule of thumb for Design Class}} = Z_1 = Z_2 \]

Equation 37: Rule of Thumb to determine the Stator and Rotor Reactance
**No-Load Test Procedure:**

The no load test is exactly setup like the blocked rotor test, however instead of the ammeters being connected straight from the variable AC power supply, they will be placed just after the power meters.

The following steps allow for the correct setup of the test:

1. Connect the circuit as in the blocked rotor test, but with the ammeters being placed after the power meters as shown in Figure 41;

![Figure 29: No Load Test set-up (Y. Yanawati, 2012)](image)

2. Turn the variable power supply on without going over the rated voltage and frequency of the motor.

3. Once the motor is operating, obtain the voltage, input power and stator current values.

4. Turn the variable supply off.

**Calculating the magnetizing reactance and rotational losses:**

To determine the rotational losses, the stator resistance must be known.

This resistance value is obtained from the DC test. With this the stator copper
losses can be obtained. Therefore, input power known from the test, the rotational losses can be determined:

\[ P_{in} = P_{SCL} + P_{rot} \]

*Equation 38: Determining Rotational Losses*

To determine the magnetizing reactance, the stator reactance that was obtained from the blocked rotor test is needed along with the no load reactance. The no load reactance can be obtained by the no load voltage and no load current. Once the no load reactance has been calculated, the magnetizing reactance can be obtained;

\[ X_{NL} = \frac{V_{NL}}{\sqrt{3} \times I_{NL}} \]

*Equation 39: Determining the No Load Reactance*

\[ X_M = X_{NL} - X_1 \]

*Equation 40: Determining the Magnetisation Reactance from the No Load Reactance*