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AN IMPROVED ALGORITHM FOR BORDER FOLLOWING OF BINARY IMAGES

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INTRODUCTION

Border following techniques have been extensively studied, and have a wide variety of applications. These applications include picture recognition, topological analysis, object counting, and image compression. For example, this work was motivated by research into the reconstruction of biomedical objects from serial sections. Biomedical images pose a severe test for border following algorithms due to the complex nature of such images, and due to the residual noise that remains after image processing. Border following has been applied to a variety of image types. Algorithms for binary, 2D raster images have been studied in most detail, and works by Rosenfeld and Kak (1), and Pavlidis (2), offer several different techniques. More recent algorithms have been published, such as Suzuki and Abe (3), and by and Attikiouzel (4). Methods for tracing multi-level images have also been studied, by Kruse (5) and Danielsson (6), and methods for tracing 3D voxel image sets have also been presented, by Toriwaki and Yokoi (7). Chen and Siy (8) have proposed a technique which uses a priori knowledge and feedback to gradually improve the border extraction.

These existing methods can satisfactorily find and trace outermost borders. However, it will be shown that inner borders are often incorrectly traced or follow sub-optimal paths. This paper will examine the reasons for this, and put forward a new method that overcomes these deficiencies. The paper will examine only the case of binary 2D raster images. However, since such images form the basis for the more complex image types, the techniques described in this paper can be readily extended to the more complex image forms.

DEFINITIONS AND NOTATION

The image \( \Sigma \) consists of binary pixels in a regular raster. The shape of \( \Sigma \) may be any simple closed region with a known boundary \( \alpha \), although normally it is a square. Without loss of generality, we assume the objects to be found and traced are regions of 1-pixels. Let \( \Delta \) be the set of 1-pixels, and let \( \Psi \) be the set of 0-pixels. That component of \( \Psi \) that includes \( \alpha \) is called the background. Other components of \( \Psi \), if any, are called holes. Let \( \Gamma \) and \( \Phi \) be any subsets of \( \Sigma \). If any path from any point of \( \Gamma \) to the border must pass through \( \Phi \), then \( \Phi \) surrounds \( \Gamma \).

There are two types of adjacency for pixels. Pixels are 4-adjacent to the 4 horizontal and vertical neighbours. Pixels are 8-adjacent to the 4-adjacent pixels and also the 4 diagonal neighbours. Pixels are said to be 4-(8-) connected to 4-(8-) neighbours with the same value.

PREVIOUS BORDER DEFINITIONS

Past works (1-4) have defined 1-regions (solids) to be 8-connected, and 0-regions (holes) to be 4-connected. The asymmetric definition resolves the connectivity paradox: Borders are required not to cross, yet if both 1-regions and 0-regions were 8-connected then the borders could cross at diagonal intersections, as shown in Fig. 1. To resolve this, 0-pixels are defined to only be 4-connected, and thus not joined across such intersections.

Given this, it can be shown (1), that the borders to be traced are those 1-pixels which are 4-adjacent to a 0-pixel. Previous authors (1-4) have then proposed algorithms for searching and tracing such borders.

Problems With Previous Methods

These definitions satisfactorily define borders surrounding solids (solid borders), but borders surrounding holes (hole borders) are poorly formed and may follow sub-optimal paths. Consider, for example, the pixels shown in Fig. 2. (N.B. os are 0-pixels, ws are 1-pixels, and borders may be shown offset for clarity). Notice that the pixels on the left are simply complements of the pixels on the right. However, the shapes of the hole borders are often very different. The hole borders are longer and more numerous than the analogous solid borders. Furthermore, the tracing algorithms found on cross over existing borders. For example, the simple patterns in Fig. 3 failed to be correctly traced by the algorithms of (1) and (4) as well as (6) for other examples of incorrectly traced patterns.

PROPOSED NEW METHOD

This paper presents a new border searching and tracing method which overcomes these problems. Firstly, the border definition is changed, to require that borders that surround 1-regions (solid borders) must trace over 1-pixels, and borders that surround 0-pixels (hole borders) must trace over 0-pixels. The connectivity relation is also altered so that all pixels are 8-connected, except across an existing border.

This definition causes borders to be defined iteratively. Let \( \Omega_0 \) be a set of regions - initially containing only \( \{ \Sigma \} \). \( \Omega_0 \) is the set of borders surrounding these regions - initially \( \{ \alpha \} \). \( \gamma \) is the set of all borders found - initially an empty set(). Then:
1. Let \( \theta_1 \) be a set of borders comprising those 1-pixels in \( \theta_0 \) that are 4-adjacent to a 0-pixel connected to a pixel in \( \theta_0 \). These pixels surround a new set of disjoint regions \( \Omega_1 \). The borders in \( \theta_1 \) are added to \( \theta \).

2. Let \( \theta_0 \) be a set of borders comprising those 0-pixels in \( \Omega_1 \) that are 4-adjacent to a 1-pixel connected to a pixel in \( \theta_0 \). These pixels surround a new set of disjoint regions \( \Omega_0 \). The borders in \( \theta_0 \) are added to \( \theta \).

3. Continue from 1, stop immediately when \( \theta_0 \) or \( \theta_1 \) becomes empty.

This results in all border pixels being specified in set \( \theta \).

**Effects of New Definition**

This novel definition generates hole borders that differ from the previous methods, but still uniquely defines the image in terms of borders. Thus, these new borders may still be applied to various existing post-processing algorithms, with the advantage that the hole borders will be better shaped. Reconsider the images in Fig. 2 under the new definition, as shown in Fig. 4. The shape improvements include:

- The hole borders will be shorter. This results in better data compression and faster post-processing for later stages.
- There may be fewer hole borders, again resulting in better compression and faster post-processing.
- Holes and solids with identical size and shape will have the same border code.
- Both 0-pixels and 1-pixels may be connected to any of the 8 neighbors without causing any paradox.

These improvements represent a significant gain compared to the results of previous work (1-4). Furthermore, these improvements can be achieved for other image types (multi-level, 3D rasters, etc.) when this novel definition is extended to these methods (5-8).

**IMPLEMENTATION**

The definition above defines which pixels are borders. However, to be useful the borders must be located, and traced to yield a chain code. Additionally the topology of the image should be determined. The topology is in the form of the surroundness relation (3) defining which holes lie inside which solids. For example Fig. 5 illustrates an image and its topology tree.

**Border Tracing Algorithm**

The border tracing algorithm can be described as a pair of co-recursive procedures. Initial input to the first procedure is the raster image and the image border. The image border need not be the rectangular boundary of the raster image. The border could well be an arbitrary chain code, generated manually using a mouse, that defines a region of interest within a large image. If the region of interest is the entire image then a dummy rectangular border can easily be generated. This border should be specified counter-clockwise, to follow the convention that solids are traced clockwise, and holes counter-clockwise.

**Solid Search Procedure.** Each pixel on the border is then checked, in turn, to determine whether the pixel is a suitable starting location for a line scan (described below). A suitable pixel will have its right neighbor inside the border. This can be found from the local curvature of the border using the chain code shown in Fig. 6. If \( d_\theta \) is the direction from one pixel to the next, then

\[
(d_\theta - 4 \mod 8) = d_n' \quad \ldots (1)
\]

is the direction of the previous pixel to the current pixel. Given that the border is traced counter-clockwise, if

\[
d_n' = 0 \text{ or } d_n' = d_n \quad \ldots (2)
\]

then the angle subtended by the previous pixel and the next pixel does not include the pixel on the left, as shown in Fig. 7. Thus for each pixel, if Eqn. 2 is true, then the pixel is marked with a 0'-pixel (right pixel outside), otherwise it is marked with a 0''-border will re-enter a pixel marked 0' and separate that pixel from its right neighbour. If this occurs, then the second time the pixel is examined the local curvature will be 0", and the pixel is changed to a 0"-pixel. However, if a 0"-pixel is re-entered it should never be relabeled 0' since it has already been determined that the right pixel is outside the border.

After all pixels have been marked the region is searched for solids. Each 0'-pixel on the border is a suitable starting point for a left to right raster scan since the right neighbour is known to lie inside that region. The left to right scan continues until a 0'-pixel or 0"-pixel is encountered. Each time a 1-pixel is encountered the line scan is paused and the solid is traced (described below). As each pixel is scanned it is marked with a 0'-pixel to indicate that it has been tested. When all suitable border points have been line scanned the procedure and all internal borders will have been traced.

**Solid Trace.** Each time a 1-pixel is located during the line scan, a new border has been found. This 1-border can then be traced in a clockwise manner using the following procedure. The 1-pixel's neighbours are tested, starting with the one in direction 5, and continuing clockwise. The first 1-pixel found is the next pixel in the border, and its direction is added to the chain code for the border. The next border pixel will be found (even if the border must re-enter itself and pass over pixels previously added to the border) unless the region consists of a single 1-pixel, in which case the trace has finished. In the normal case, the trace then moves on to the next pixel and tests its neighbours, and so on, until the trace returns to initial pixel. Even then extra neighbours must be tested. Specifically, if the final direction is \( d_f \), then the directions

\[
(d_f + 2 \mod 8) \text{ to } (11) \mod 8 \quad \ldots (3)
\]

must be checked, as addition parts of the solid might lie in those directions, as shown in Fig. 8. The connectivity paradox is avoided by ensuring the border does not create a surrounding border as it is being traced. Since at any given border, the surrounding border has been marked with 0' or 0"-pixels, this can easily be tested. Whenever the
border tracing routine attempts to move diagonally the two orthogonal pixels are tested. If both are marked then the trace is blocked from moving in that direction and must try another direction, as shown in Fig. 9.

Having found the entire solid border this can be saved. Also the surroundings tree can be updated, by adding the solid border as a child of the hole border surrounding it. The solid border is then passed, co-recursively to a procedure that searches solid regions for holes.

Hole Search Procedure. The hole searching, and hole tracing procedures are essentially identical to solid searching and tracing. The differences being that the border is marked with 1'- and 1"-pixels, and the local curvature is tested by
\[ d_n = 0 \text{ or } d_n \neq d'_n \quad \ldots \quad (5) \]
instead of Eqs. 2. The search begins in direction 3 and continues counter-clockwise; the tracing may be blocked by two orthogonal 0'- or 0"-pixels. This results in hole borders being discovered and traced. These can be saved, and the surroundings tree updated to indicate that the border lies inside its solid border. The hole border is the passed co-recursively to the original procedure. Thus the algorithm can alternate between the two procedures until eventually all border are located and the surroundings tree is complete.

It should be noted that in practice the border marking can be performed while the borders are being traced. However, the line scan must be performed after the entire border has been checked. Also note that if the initial image border is known to be rectangular it can be quickly marked with 0'- and 0"-pixels.

This algorithm requires a frame buffer of 3 bit planes deep (to store the six possible values: 0,0',0",0',1',1") to extract full topological information of arbitrary depth. This compares favourably with those methods where the depth of the buffer is related to the number of contours traced.

This algorithm has been coded in "C" and occupies about 150 lines of source code.

RESULTS

This method has been extensively tested on artificial and biomedical images (see Figs. 10-13) and performs very well. It correctly locates and traces all the borders shown in Fig. 3 that were incorrectly traced by the previous methods. It also correctly traced real images that were found to cause the other methods to fail. Furthermore the hole borders are better shaped, and more compact than those generated by the other methods when they did work. Thus this algorithm has proven superior to the previous methods of binary border following.

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REFERENCES


Figure 1. Connectivity paradox.

Figure 2. Asymmetry in inverse patterns.
Figure 3. Incorrectly traced patterns.

Figure 4. Hole shape is improved (cf. Fig 2).

Figure 5. Topology of image.

Figure 6. Chain code directions.

Figure 7. Local curvature.

Figure 8. Initial point may be re-entered.

Figure 9. Examples of diagonal blocking.

Figure 10. MR image of a skull.
Figure 11. Same image after thresholding

Figure 12. Borders are traced.

Figure 13. Degenerate borders removed.