SIMPLE ARITHMETIC PROCESSING

Fact Retrieval Mechanisms and the Influence of Individual Differences, Surface form, Problem Type and Split on Processing.

by

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I declare that this thesis is my account of my research and contains as its main content work which has not previously been submitted at any tertiary education institution.
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ABSTRACT

Current theorising in the area of cognitive arithmetic suggests that simple arithmetic knowledge is stored in memory and accessed in the same way as word knowledge i.e., it is stored in a network of associations, with simple facts retrieved automatically from memory. However, to date, the main methodologies that have been employed to investigate automaticity in simple arithmetic processing (e.g., production and verification) have produced a wide variety of difficulties in interpretation. In an attempt to address this, the present series of investigations utilised a numerical variant of the well established single word semantic priming paradigm that involved the presentation of problems as primes (e.g., 2 + 3) and solutions as targets (e.g., 5), as they would occur in a natural setting. Adult university students were exposed to both addition and multiplication problems in each of three main prime target relationship conditions, including congruent (e.g., 2 + 3 and 5), incongruent (e.g., 2 + 3 and 13), and neutral conditions (X + Y and 5). When combined with a naming task and the use of short stimulus onset asynchronies (SOAs), this procedure enabled a more valid and reliable investigation into automaticity and the cognitive mechanisms underlying simple arithmetic processing.
The first investigation in the present series addressed the question of automaticity in arithmetic fact retrieval, whilst the remaining investigations examined the main factors thought to influence simple arithmetic processing i.e., skill level, surface form, problem type and split. All factors, except for problem type, were found to influence processing in the arithmetic priming paradigm. For example, the results of all five investigations were consistent in revealing significant facilitation in naming congruent targets for skilled participants, following exposure to Arabic digit primes at the short SOA. Accordingly, the facilitation was explained in terms of the operation of an automatic spreading activation mechanism. Additionally, significant inhibitory effects in incongruent target naming were identified in skilled performance in all of the studies in the present series of investigations. Throughout the course of these investigations, these effects were found to vary with operation, surface form and SOA, and in the final investigation, the level of inhibition was found to vary with the split between the correct solution and the incongruent target. Consequently, a number of explanations were put forward to account for these effects. In the first two investigations, it was suggested that the inhibitory effects resulted from the use of a response validity checking mechanism, whilst in the final investigation, the results were more consistent with the activation of magnitude representations in memory (this can be likened to Dehaene’s, 1997, ‘number sense’). In contrast, the results of the third investigation led to the proposal that for number word primes, inhibition in processing results from the activation of phonological representations in memory, via a reading based mechanism.

The present series of investigations demonstrated the utility of the numerical variant of the single word semantic priming paradigm for the investigation of simple arithmetic processing. Given its capacity to uncover the fundamental cognitive
mechanisms at work in simple arithmetic operations, this methodology has many applications in future research.
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1. INTRODUCTION

1.1 Overview

Cognitive arithmetic is the area of research that attempts to determine how number and arithmetic knowledge is organised in memory and how it is accessed and applied (Ashcraft, 1992). It has a relatively brief history (30-years), during which time, much of the research has been driven by the principle question of just how simple arithmetic facts are retrieved in adult simple addition and multiplication performance (Ashcraft, 1992; Dehaene, 1992). At present, most models of simple arithmetic processing are converging on the notion that simple arithmetic facts are stored in memory and retrieved through a process similar to that employed for word knowledge (Ashcraft, 1992; Dehaene, 1992; LeFevre, Bisanz & Mrkonjic, 1988). That is, they are stored in an interrelated network of associations that is based on the operands and their related nodes, and their solutions are retrieved via a process of automatic spreading activation (Ashcraft, 1992). This process is considered to be fast, accurate, and obligatory, and to require minimal cognitive load (LeFevre & Kulak, 1994). It contrasts with fact retrieval that occurs at a conscious level, that relies on rules \((N \times 0 = 0)\) or procedures (e.g., counting or transformations: \(6 + 7 = 6 + 6 + 1 = 12 + 1 = 13\)), and that is thought to be used primarily when automatic fact retrieval fails (Groen & Parkman, 1972).

However, to date, the main methodologies that have been employed to investigate automaticity in fact retrieval (i.e., verification, number matching, production, self-report and priming tasks) have produced a wide variety of difficulties in interpretation. For example, there are a number of theories to account for the processes utilised in verification tasks, which require participants to make
judgments as to whether given equations (e.g., $6 + 7 = 15$) are true or false (Campbell, 1987; Stazyk, Ashcraft & Hamann, 1982; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986; Zbrodoff & Logan; 1990). Rather than accessing simple arithmetic processing, it has been suggested that this procedure could be accomplished via familiarity judgements, plausibility judgements (based on approximate magnitude), or on the basis of the odd/even status of the given solution (e.g., see Campbell, 1987; LeFevre, Sadesky & Bisanz, 1996a). Furthermore, the need to make a binary decision about the relationship between the problem and the given solution could lead to the measurement of conscious, decisional processes, rather than automatic processes that occur without intention (Balota & Lorch, 1986; Smith, Besner, & Myoshi, 1994).

This latter criticism can also be levelled at the number matching task. In this task participants are first presented with a pair of numbers and then, following a short inter-stimulus interval, they are presented with a third number. The participant is then required to determine whether the third number is one of the first two numbers that were originally presented (e.g., LeFevre et al., 1988; LeFevre & Kulak, 1994; Thibodeau, LeFevre & Bisanz, 1996; Galfano, Rusconi & Umilta, 2003). Thus, the participant is first required to commit the original numbers to short-term memory. Then, following exposure to the third number, the participant must make a binary decision as to whether the numerical symbols presented on each occasion match. Consequently, as in the verification task, the line between conscious and automatic processing is again blurred.

The use of production tasks, in which participants are simply asked to produce correct solutions to problems (e.g., Campbell, 1987; LeFevre et al., 1996a, 1996b), can also be criticised. For example, there is little basis for determining an
appropriate cut-off point in reaction time or error rate measures that represents the boundary between automatic and strategic fact retrieval. This endeavour is made even more complex when the possible influences of problem size and skill level are considered. In view of this, a number of investigators have attempted to circumvent this problem by requesting that participants also provide self-reports of how they obtained their solutions (e.g., Campbell & Xue, 2001; LeFevre et al., 1996a, 1996b). Nevertheless, the research shows that the instructions employed within the self-report method can lead to reactivity and may influence people of varying skill levels differently (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003).

A final method employed in the investigation of simple arithmetic processing is a priming task (Ashcraft, 1992; Campbell; 1987, 1991). In this task, participants are first presented with either a prime that is the correct solution, a neutral prime (## or --), a related false prime (i.e., a frequent error) or an unrelated false prime (an infrequent error). The participants are then presented with a problem and required to produce the solution or they are presented with an equation that they must verify. Consequently, this method brings with it all of the difficulties associated with the interpretation of data obtained from production and verification tasks. Moreover, as noted by Campbell (1987, 1991), the inhibitory effects identified in this method suggest that participants consciously attempt to ignore interference caused by the prime.

Given the criticisms of the aforementioned tasks, the conclusion that simple arithmetic facts are retrieved via a process of automatic spreading activation appears to be somewhat premature. Furthermore, none of the methods is reliably able to distinguish between the effects associated with encoding a problem (i.e., the translation of the presented problem into an internal semantic representation) and the
effects associated with retrieving its solution (Noel & Seron, 1997). The ability to be able to reliably distinguish between these two processes is fundamental to being able to address the central questions within the literature of whether factors such as surface form (e.g., \(2 + 3 = 5\) cf. \(\text{two + three} = 5\)) and problem type (e.g., \(3 + 3 = 6\) cf. \(4 + 2 = 6\)) influence fact retrieval mechanisms (e.g., Blankenberger; 2001; Campbell & Gunter, 2002; Noel, Fias & Brysbaert, 1997; Sciama, Semenza & Butterworth, 1999).

The collective purpose of the present studies was thus to employ a new, more valid and reliable approach to the investigation of automaticity in adult simple arithmetic fact retrieval and the variables that are thought to influence it. Consistent with current theorising, this approach was based on a working assumption that simple arithmetic knowledge might be stored in memory and accessed in the same way as word knowledge (Ashcraft, 1992). Accordingly, the present studies employed a numerical variant of the well-established single-word semantic priming paradigm in five main investigations. In the first of these, the question of automaticity in fact retrieval was addressed, whilst in the remaining studies, the main factors thought to influence simple arithmetic processing, i.e., individual differences, surface form, problem type and split, were considered. The following subsections review the simple arithmetic processing literature and provide a rationale for these investigations.

1.2 Review of Research and Current Understanding

1.2.1 The Organization of Simple Arithmetic Knowledge and Access to this Information

Although the earlier methodologies employed in the investigation of simple arithmetic knowledge have difficulties in interpretation, the findings of production
and verification tasks reveal the highly interrelated nature of this knowledge. For example, production tasks reveal cross-operation confusion effects in which the correct solution to an alternative operation is often produced (e.g., $2 + 3 = 6$). Similarly, in verification tasks, it takes longer to determine that a cross-operation equation is false than it does to determine that other equations are false (Ashcraft, 1992; Campbell, 1987; Cipolotti & Butterworth, 1995; LeFevre & Kulak, 1994; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). What is more, verification tasks produce split effects in which the reaction time taken to respond to false equations reduces as the correct solution becomes more implausible (Dehaene, 1992). This finding fits well with network retrieval models and priming assumptions (Ashcraft & Stazyk, 1981). Specifically, it suggests that near neighbour nodes of correct solutions are activated in memory via a process of spreading activation from correct solutions, thus slowing reaction times (Stazyk et al., 1982). In contrast, more distant incorrect solutions receive little activation and responding occurs rapidly.

A similar effect is evidenced in number matching tasks. For example, Galfano et al. (2003) showed that the time taken to decide that a number (e.g., 8) adjacent to the correct product of two numbers (i.e., 10) is not one of the original numbers presented (i.e., 5 and 2) is longer than that observed for unrelated numbers (Galfano, et al., 2003). Nevertheless, the true utility of the number matching task is that it demonstrates the obligatory nature of simple arithmetic fact activation. That is, the presentation of the correct sum or product (e.g., 8) to be matched with the two originally presented numbers (e.g., $4 \times 2$) leads to lengthier decision times, irrespective of the intentions of the participants to simply match numerical symbols (LeFevre et al., 1988; LeFevre & Kulak, 1994; Thibodeau et al., 1996). Moreover,
this happens at very short SOAs, and occurs whether the two original numbers are presented with or without an arithmetic operator (e.g., $4 + 2$).

However, the notion that simple arithmetic facts are organised in an associative network and retrieved through a process of automatic activation is at odds with the findings of problem size (or difficulty) effects. Problem size effects are ubiquitous throughout the cognitive arithmetic literature and occur where the reaction times and error rate measures for larger facts (e.g., $7 + 8$) are greater than those for smaller facts (e.g., $2 + 3$) (Ashcraft, 1992; Brysbaert, 1995; Dehaene, 1992). This is different to the uniform reaction time and error rate patterns that would be expected, given that facts are automatically retrieved from memory. Furthermore, explanations of this effect in terms of a greater frequency of exposure to small problems in learning suggest that access to small problems may be more automated than access to large problems.

Finally, investigations undertaken by LeFevre and colleagues and involving relatively skilled university samples reported the use of fact retrieval strategies similar to those employed by children (e.g., counting and transformations) in up to 25% of trials (LeFevre et al., 1996a, 1996b). These findings, coupled with the past use of methodologies that have blurred the line between automatic and conscious processing, strongly indicate the need for a more valid and reliable investigation into the role of automatic processing in simple arithmetic performance.

1.2.2 Individual Differences in Access to Simple Facts

Intuitively, an influence of individual differences on simple arithmetic processing results because performance on simple arithmetic tasks is measured in terms of speed and accuracy. Correspondingly, those people who do well on tests of simple arithmetic processing should show greater levels of automaticity in
processing than those people who do not do well (LeFevre & Kulak, 1994). Nevertheless, until recently, it was widely assumed that most adults directly retrieve facts from memory, on most occasions (Ashcraft, 1992; Geary & Wiley, 1991; LeFevre et al., 1996a, 1996b; LeFevre & Kulak, 1994). Individual differences in processing were generally only considered in comparisons of children’s and adult performance i.e., as the end product of formal schooling (LeFevre & Kulak, 1994). The results from these studies showed that adults and older children have faster and more accurate access to simple facts, that adults show evidence of obligatory activation, and that adults require fewer cognitive resources in processing than do children (LeFevre & Kulak, 1994).

However, the comparisons of adult and children’s performance were undertaken at the expense of considering differences between adults in performance (LeFevre & Kulak, 1994). Only recently, did LeFevre and her colleagues attempt to address this oversight in the literature. The results of two number matching experiments showed that unintentional sum activation led to lengthier response times in high skilled performance than it did in low skilled performance (LeFevre, Kulak & Bisanz, 1991; LeFevre & Kulak, 1994). Furthermore, the results of a series of self-report investigations revealed a significant positive correlation between a high level of fluency and the reported use of direct fact retrieval (LeFevre et al., 1996a, 1996b; also see Geary & Wiley, 1991; Hecht, 1999).

Nevertheless, given the shortcomings of the number matching and self-report methodologies and the paucity of investigations in this area, the need for further investigation into the role of individual differences in simple addition and multiplication performance is indicated.

1.2.3 Surface Form Effects: Encoding or Fact Retrieval?
The question of whether surface form (i.e., Arabic digits cf. written word numbers) influences fact retrieval mechanisms is central to much of the research undertaken in the area of cognitive arithmetic over the past 30 years. It has implications for models describing the componential architecture of numerical knowledge and how it is accessed (Campbell, 1999). Four main models of numerical cognition are prominent in the literature and these differ on whether they assume that problems represented in different surface forms are first converted to a single representation before processing along a common pathway or remain unique and are processed along differing pathways (Ashcraft, 1992; Campbell, 1994; Campbell, 1999; Dehaene, 1992; Dehaene, Bossini & Giroux, 1993; Noel et al., 1997; Noel & Seron, 1997; Sciama et al., 1999). That is, these models differ on whether they assume that after encoding, fact retrieval for simple arithmetic problems represented in different forms occurs in the same or different ways. The following subsections describe and review these models.

1.2.3.1 Common Pathway Models

Arguably the most widely discussed of the common processing pathway models is McCloskey, Caramazza and Basili’s (1985; McCloskey, Macaruso & Whetstone, 1992; McCloskey & Macaruso, 1995) abstract-modular model. This model posits the existence of three, functionally independent, comprehension, calculation and production mechanisms that are organised in a modular architecture. The comprehension mechanism is thought to first translate problems represented in different surface forms into the same abstract representation that specifies their basic quantity or magnitude and the powers of 10 associated with them (e.g., 6 is represented by \( \{6\}10^{\text{EXP}0} \)). This abstract representation is then acted upon by a
calculation mechanism that performs cognitive processes that are specific to arithmetic, such as the comprehension of operation symbols and the retrieval of a solution. A production mechanism then converts the abstract representation of this solution into the appropriate digit or word output form. Thus, following initial encoding, digit and word problems assume an identical abstract representation and consequently follow a common processing pathway involving the same arithmetic fact retrieval and response production mechanisms.

Support for the abstract-modular model largely derives from the observation of selective functional deficits in the performance of acalculic patients that appear to correspond well to the workings of the three proposed mechanisms (Campbell, 1994; Dehaene, 1992; McCloskey et al., 1992). An important case in point is that of patient PS, who seemed able to comprehend and produce numerals in all formats, whilst at the same time demonstrating a clear deficit in simple multiplication fact retrieval (McCloskey et al., 1992; Sokol, McCloskey, Cohen & Aliminosa, 1991). In the latter case, PS was shown to err on exactly the same single digit multiplication problems regardless of the stimulus format to be comprehended or the response format required in production (McCloskey et al., 1992; Sokol et al., 1991). This finding provided strong support for the notion that surface form does not influence arithmetic fact retrieval and as such, that it occurs via only one processing pathway.

However, a major criticism of this model is that it cannot account for the finding that processes attributable to independent mechanisms may actually interact (Campbell, 1994). An example of this is demonstrated in the results of a study by LeFevre et al. (1988). In their study, participants were presented with two numbers and then required to decide if a target number was one of the original numbers presented. When the sum of the two numbers was presented as the target,
interference occurred and it took longer to decide that it was not one of the original numbers presented than if it was an unrelated number. Therefore, interference that the abstract-modular model assumes attributable to the calculation mechanism was identified in a number-matching task that should have only involved the comprehension mechanism (Campbell, 1994).

Further criticism of this model has centred on the assumption that the mental representation processed along its pathway is abstract in nature (Campbell, 1994). For example, Campbell and Clark (1992) argued, that it was unlikely that the brain has a uniquely numerical ‘power of ten’ mechanism and that it appears that a variety of verbal and non-verbal codes are necessary to explain how people represent numbers internally (pp. 487). Subsequently, two further common pathway models of numerical processing have been introduced that instead posit the existence of unique and specific mental codes. In the first of these models, by Noel and Seron (1993), it is suggested that individuals each have their own preferred entry code, whether Arabic digit or verbal, that all numerals are converted to before processing takes place. An example of this is shown in the case of patient NR, who reportedly transcoded all numerical information, regardless of surface form, into a verbal representation before accessing quantity and arithmetic knowledge.

The second model is Dehaene’s triple code model (1992; Chochon, Cohen, van de Moortele & Dehaene, 1999; Dehaene, et al., 1993; Dehaene et al., 1996; Dehaene, Piazza, Pinel & Cohen, 2003) in which it is posited that numbers may be represented in three different codes: a visual Arabic code, an analogue magnitude code and an auditory verbal code. In this model, as in Noel and Seron’s (1993), regardless of surface form, numerals must first be translated into the appropriate code necessary to perform a particular procedure. For example, for procedures such as the
identification of Arabic numerals, multidigit operations and parity judgements, numbers must first be converted to a visual Arabic code i.e., to strings of digits on an internal visuospatial scratchpad. In contrast, for magnitude comparison procedures, numbers first have to be represented implicitly as semantic analogical quantity codes specifying proximity relations along an oriented number line. Finally, the comprehension and production of spoken numerals is thought to occur via a verbal code in which numbers are initially encoded as sequences of words that are organised on the basis of syntax. An important proposition associated with this particular code is that it is thought to be “the obligatory entry code for accessing stored tables of rote arithmetic facts, encoded in the form of short sentences in verbal memory (e.g., two times three, six)” (Chochon et al., 1999, pp. 620). Furthermore, the Arabic and verbal processing modules associated with this model are thought to communicate directly via an asemantic transcoding route. That is, written numerals are translated (or connected) to a phonological representation of the words that may also be directly activated via Arabic numerals, without semantic mediation (Dehaene & Akhavein, 1995).

Support for the functional analysis offered by Dehaene and colleague’s model stems from studies that identify dissociable processing systems in patient samples. For instance, a study by Lemer, Dehaene, Spelke and Cohen (2003; see also Cohen, Dehaene, Chochon, Lehéricy & Naccache, 2000; Dehaene & Cohen, 1997) identified a dissociation between a verbal system of number words and a non-symbolic representation of approximate quantities. In this study, one patient was shown to be more impaired in subtraction (thought to be solved by mental manipulation of quantities) than in multiplication and exhibited impairments in approximation, subitising and numerical comparison tasks, with stimuli presented as Arabic digits or
arrays of dots. In contrast, a second patient was more impaired in multiplication (thought to be solved by access to a table of memorised facts represented as verbal associations, without access to quantity) than subtraction and exhibited preserved approximation and processing of non-symbolic numerosity abilities. Further support for this model stems from neuroimaging studies that demonstrate the activation of different cerebral pathways with different number processing tasks that appear to correspond to the models proposed mechanisms. For example, a study by Stanescu-Cosson et al. (2000; additionally, see Chochon et al., 1999; Dehaene et al., 1996; Dehaene et al., 2003) that measured cerebral activity using fMRI and event related potentials, revealed 2 cerebral networks for number processing in a non-patient sample. Greater activity in left lateralised regions, presumably relying on encoding of numbers in a verbal format, were evidenced for rote arithmetic operations with small numbers. Contrary to this, approximation and calculation with larger numbers appeared to put greater emphasis on left and right parietal cortices.

Dehaene and colleagues triple code model thus appears to be the most comprehensive and well researched accounts of numerical processing of all three of the common pathway models. However, the assumption that rote facts are accessed only through verbal codes is questionable and, in the literature, the possibility that simple arithmetic facts are elicited directly via visual codes has not yet been ruled out.

Notably, in both of the preceding models, as in the original abstract-modular model, following the encoding of the different surface forms and conversion to an appropriate mental representation, processing via only one pathway is maintained. Collectively, support for the three common pathway models has arisen from research involving the use of simple number manipulation procedures such as numerical
comparison, priming and number matching tasks. Outlined in Noel and Seron (1997), numerical comparison tasks have variously involved the independent comparison of Arabic digits, written word numerals or arrays of dots. A finding common to these studies is that, irrespective of their initial surface form, symbolic distance effects (where latency is an inverse function of the distance between two numbers) and serial position effects (given a constant distance, small numbers are compared more quickly than large numbers) are often observed (Dehaene & Akhavein; 1995; Noel & Seron, 1997).

Evidence in support of a common processing pathway stemming from priming research is provided in a study by Dehaene et al. (1998), who demonstrated notation independent masked priming effects. Participants in this experiment were sequentially presented with four stimuli, consisting of a random letter string (i.e., a mask), a prime number, a second mask and then a target number. The prime and the target numbers could be represented as either Arabic digits or written word numerals. The participants were simply required to make a number comparison judgement as to whether the target number was greater than or less than 5. The results showed that the amount of priming was the same regardless of the notation of either the prime or the target.

Similar results were revealed in LeFevre et al.’s (1988) number matching task, which showed that the strength of the interference effect did not differ whether the numbers were presented in digit or word format. This suggests again, that after encoding, digits and words are processed via a common pathway and in doing so, provides support to the three models described above. However, it is noteworthy that the results of this study were initially used to argue against the notion of the sharply separated modular architecture described by McCloskey’s model (see above for
interaction found between component comprehension and calculation processes). Such an example thus demonstrates the complexity and diversity required of a comprehensive model of number processing and, as Campbell (1994) noted, the inadequacy of the ‘simple architecture’ of McCloskey’s modular model in explaining the numerical processing data (pp. 5).

1.2.3.2 Separate Pathways: The Encoding Complex Hypothesis

In an attempt to account for the apparent interactivity of component processes and to reduce the emphasis placed on the role of abstract representations in McCloskey’s model, Campbell and Clark (1988, Campbell, 1992; Campbell, 1994; Clark & Campbell 1991) developed a new set of assumptions (cf. a specific architectural model) that they felt could be incorporated into alternative models of number processing. Referred to as the encoding complex hypothesis, it assumes the existence of multiple format and modality specific codes. Examples of these include verbal or word codes that encompass articulatory or auditory codes, visual and number word codes and unique codes such as sign language codes for numbers. Additionally, numerical processing is thought to implicate nonverbal codes such as visual and written codes for digits, imaginal analogue codes for magnitude, and visual-motor representations developed from counting on fingers or using an abacus. These codes are interconnected in a complex network and in tasks such as magnitude comparison, number fact retrieval and number naming, are thought to be accessed through learned associative relations and to directly activate one another to produce a multi-component representational structure called an encoding complex. Like Dehaene’s (1992) model, the notion that transcoding from one form to another can occur through a relatively autonomous asemantic route is suggested.
In addressing the question of whether separate or common processing pathways are utilised in numerical processing, the encoding complex hypothesis thus allows for both options (McCloskey et al., 1992). For example, in dealing with Arabic digit and written word stimuli, the assumptions of interactivity and autonomous transcoding routes make it possible that visual representations of the different forms may initially be converted to the same stronger or more automatic, representation before processing (e.g., to a phonological or verbal code). In such a situation, processing could be expected to occur in a fashion similar to that espoused by common pathway models. Alternatively, the encoding complex hypothesis also suggests that each of these codes can be employed in any type of numerical processing, with tasks performed qualitatively differently as a direct consequence of the different surface forms that problems may be encountered in. Thus, according to this view, problems represented as either digits or words will potentially be processed along differing pathways. Predictions as to just how these pathways will differ and hence, influence mechanisms that subserve calculation centre on frequency of exposure explanations. For instance, Campbell (1994) notes that arithmetic problems are more commonly represented in Arabic digit form than written word form, thereby making them more efficient at activating number fact representations. In contrast, written numerals are commonly encountered in reading contexts, which could possibly make them more likely to activate reading based associations. Such a position gains support not only from analyses of the various errors produced in performance (e.g., see Campbell, 1994) but has intuitive appeal when the strength of reading based mechanisms demonstrated in Stroop tasks is considered (MacLeod, 1991).
Empirical support for an influence of differential exposure to varying surface forms on processing is provided in a study by Sciama et al. (1999) that revealed surface form effects in a repetition priming task involving the addition operation. In two experiments, pairs of numbers were pre-exposed in a study phase and then presented again in a test phase, with participants required to produce the correct sum for each pair. Number pairs were presented in a vertical format and were represented as Arabic digits, number words (Experiment 1) or dot configurations (Experiment 2). The results showed that pre-exposure to the same number pair represented in the same form produced greater benefits in reaction time for word and dot stimuli than did pre-exposure of the same number pair in digit form. With addition problems seldom ever represented using number words or dots, the authors concluded that the influence of surface form on repetition priming was dependent on the typicality of the surface form for that task.

Interestingly, whilst the results of the Sciama et al. (1999) study identified form-specific effects, they also revealed priming effects across surface forms, with pre-exposure to the same number pair represented as digits, words or dots leading to the same amount of priming in digit stimuli. Such a finding is consistent with models that assume that after encoding, processing involves a common representation. In view of this, Sciama et al. (1999) proposed a general approach to mental numerical representation not too unlike that of the encoding complex hypothesis i.e., involving both common and form specific codes co-existing together. Unlike the earlier hypotheses however, Sciama et al. (1999) pertinently noted that it does not make sense to assume that specific codes for word problems exist in a stable semantic memory network due to the fact that problems represented as words are rarely, if ever, encountered in calculation tasks.
Further support for an influence of differential exposure to varying surface forms on processing was offered in an investigation of bilingual processing by Campbell, Kanz and Xue (1999). In this study, Chinese-English participants born and educated in China were asked to complete naming, magnitude selection and simple arithmetic tasks. They were first presented with either Arabic digit or mandarin symbols and then required to respond in either English or Chinese. The results revealed more efficient retrieval processing (measured in terms of reaction times, error rates, problem size and split effects), with presentation of the Arabic digit stimuli. Furthermore, naming and arithmetic reaction times were slower with English than Chinese responses and this was greatest with the mandarin stimuli. This suggested to the authors that the different ‘Notation Language’ combinations were mediated by separate associative pathways that varied in strength and efficiency as a result of prior experience.

Unfortunately, again, whilst the Campbell et al. (1999) findings initially appear to provide overall support to the notion of form-specific processing, as Noel et al. (1997) observed in a review of bilingual studies, they could just as easily be interpreted in support of common pathway models. For example, participants may first transcode stimuli presented in the non-preferred language to the preferred language and then perform the tasks so that processing effectively occurs along only one pathway. Such a possibility is supported in the previous study by the finding of slower English responses with the mandarin stimuli. It is not surprising then that perhaps, one of the greatest criticisms of the encoding complex hypothesis is that, in attempting to comprehensively account for all numerical processing, it does not make predictions that are easily tested (McCloskey et al., 1992). Moreover, as the preceding discussion shows, one of the biggest challenges faced by researchers in
this field is that of identifying a method that will enable them to reliably distinguish between the effects associated with encoding and the effects associated with fact retrieval mechanisms.

1.2.3.3 Simple Arithmetic and the Encoding Issue

In the simple arithmetic literature, an important finding is that of an interaction between problem size and surface form (Campbell & Clark, 1992; Campbell, 1994; Campbell, 1999; Campbell & Fugelsang, 2001; Noel et al., 1997). At the basis of this interaction is a greater increase in reaction times and error rates for larger problems represented as written number words than for larger problems represented as Arabic digits. This finding is not easily reconciled with the notion that number fact retrieval is mediated by a single, format independent code and thus, supports the notion of separate pathways in processing (Campbell & Clark, 1992). Nevertheless, as noted by McCloskey and colleagues (1992), the encoding of number words requires the processing of several characters represented over a greater physical length than Arabic digits. Thus, fact retrieval for word problems is carried out under greater speed pressure than for digits and necessarily results in longer encoding times for these problems. Moreover, there are substantial frequency differences between written number words (e.g., two and nine), and also between words and numbers that may account for the problem size by surface form interaction. Again, the difficulty faced by researchers in this field is that of being able to unambiguously identify the effects that are attributable to each of the encoding and fact retrieval processes.

Possibly as a consequence of this difficulty, researchers in this area have either overlooked the issue of the problem size and surface form interaction (e.g., Sciama et al., 1999) or they have undertaken investigations on the basis of some
rather tenuous assumptions. For example, in one study, Noel et al. (1997) reasoned that if the interaction found in multiplication performance was due to encoding, then this effect should be found in a non-arithmetic task involving similar encoding processes i.e., a number matching task. However, given that the number matching task leads to the obligatory activation of simple arithmetic facts, the assumption that it only involves encoding mechanisms is highly questionable.

In a second study, Campbell (1999) employed simultaneous (i.e., the standard presentation method) and sequential presentation conditions in a production task. In the sequential condition, the right operand in the arithmetic problem was presented 800 ms after the left operand to enable the left operand to be encoded before the right one was presented. Campbell then reasoned that encoding effects on the response in this condition should only arise in connection with the right operand and consequently, that if the interaction between problem size and surface form occurred at the encoding stage of processing, it should be halved. Nonetheless, such an assumption does not take into account the possibility that access to simple arithmetic facts, even in the sequential condition, requires whole problem encoding. If this is the case, the finding of the standard problem size and surface form interaction in this condition could easily be explained in terms of the integration of the sequentially presented stimuli into a single mental representation of the given problem in short term memory. However, it is important to note that even this interpretation of Campbell’s task may well be wrong. The point is that, in the literature, there is little evidence in support of either view and any assumptions made regarding the processes involved in this task, at this stage, are highly speculative.

1.2.3.4 Summary
In the cognitive arithmetic literature, four main models of numerical processing have been used to describe the organisation of numerical knowledge in the brain and access to this information. Although all of these models can be criticised, each model individually and successfully accounts for the differing research findings and theoretical perspectives that inform its approach. Nevertheless, any attempt to integrate these models into a single, more holistic and comprehensive model of numerical processing, has been hampered by disagreement on the fundamental issue of whether the surface characteristics of arithmetic problems influence later cognitive processing. That is, there is disagreement as to whether problems represented in different surface forms are first converted to a single representation before processing along a common pathway or remain unique, and are processed individually as specific codes. In research on simple arithmetic, the failure to reach any consensus on this issue has resulted from an inability to determine whether the surface form effects robustly identified in the literature result from encoding or fact retrieval mechanisms. The main aim of the third investigation was thus to address this shortcoming.

1.2.4 Problem Type Effects: Encoding or Fact Retrieval?

The possibility that simple arithmetic fact retrieval mechanisms are influenced by problem type is indicated in the varying patterns of reaction times observed for different problem types in production and verification tasks. For example, in comparison to standard problems (e.g., $2 \times 3 = 6$), verification of multiplication zero-problems is slow and error prone (e.g., $N \times 0 = 0$), whilst production of solutions to zero and one-problems (e.g., $N \times 1 = N$) occurs rapidly. These findings are often attributed to the use of a rule in solution retrieval (such as any number times one equals that number), an assumption supported by educational and clinical research (Stazyk et al., 1982; Sokol et al., 1991). However, in contrast to
this, self-report methodologies appear to indicate that rather than relying on rules, participants commonly directly retrieve solutions to these simple facts from memory (Campbell, & Xue, 2001; LeFevre et al., 1996a, 1996b). Furthermore, there are difficulties associated with claiming differences due to problem type on the basis of reaction time measures that are derived from production and verification tasks. Given that the overall reaction time taken to produce or verify a solution is dependent on encoding and fact retrieval mechanisms, any advantage in reaction time produced by a particular problem may be attributed to either process.

The topic of differences between the effects associated with problem encoding and fact retrieval mechanisms is particularly relevant to the investigation of tie problems, which involve the repetition of a single operand (e.g., $3 \times 3 = 9$ or $3 + 3 = 6$). Tie problems are generally solved more quickly and accurately than standard problems, are subject to greater interference effects, and produce minimal problem size effects (Blankenberger, 2001; Campbell & Gunter, 2002; Groen & Parkman, 1972; LeFevre & Kulak, 1994). One explanation for this effect is that it occurs at the retrieval stage of processing and results from greater exposure to tie problems in learning (Campbell & Gunter, 2002). Hence, stronger associations and greater activation between these problems and their correct solutions occurs in comparison to other problems. Alternatively, it has been suggested that this effect stems from the encoding stage of processing and results from the need to process the same operand twice, within the one problem (Blankenberger, 2001). At present, there is no method in the cognitive arithmetic literature that allows for a reliable distinction between the effects associated with each process to be made and consequently, these assumptions cannot be reliably tested. The fourth investigation in the present series of studies, thus aimed to address this limitation.
1.2.5 Split Effects in Priming Tasks

In verification tasks, the split effect refers to the finding that the reaction time taken to respond to false equations declines as the incorrect solution becomes more implausible (Ashcraft, 1992; Campbell, 1987). For example, it takes longer to verify the equation $3 + 5 = 7$ (i.e., split of 1) than it does to verify the equation $3 + 5 = 13$ (i.e., split of 5). However, to date, an investigation of just how the split between correct and incorrect solutions to simple arithmetic problems influences the distribution of reaction times that are produced within the context of simple arithmetic priming tasks has not been undertaken. Arithmetic priming tasks are different to verification tasks because they involve the successive presentation of problems and solutions, rather than simultaneous exposure to these stimuli. Consequently, the split effects that are produced in arithmetic priming tasks, and the processes leading to these effects, may be different to those found in verification tasks.

There is some evidence to suggest that split effects are task specific, and that they take on a different form in the context of arithmetic priming tasks. For example, in a priming investigation into the confusion product effect by Campbell (1987), participants were presented with solutions as primes and problems as targets in each of four conditions (true, baseline (#), table related, and unrelated conditions). They were then asked to produce the correct solutions to the presented target. As expected, the results revealed slower reaction times in the table related condition. However, interestingly, an increase in reaction time for larger splits i.e., a reversed split effect, was found. This finding was an anomaly in the literature for which Campbell offered two main explanations. Firstly, he suggested that it possibly resulted from the strategic use of predictive information provided by the prime about the approximate
magnitude of the correct answer. More than 85% of the trials in Campbell’s study involved splits that were less than 10. Secondly, Campbell suggested that magnitude may be a primeable arithmetic dimension, with the activation of evidence about the magnitude of a correct answer facilitating retrieval performance. Thus, Campbell suggested that there may be both strategic and automatic bases for the reversed split effect. Unfortunately, though, this finding was secondary to the main findings of Campbell’s research and he did not further investigate or elaborate on these explanations. Further research into the existence and nature of split effects in arithmetic priming tasks is therefore required. The fifth study aimed to provide this.

The following subsections provide an introduction to the single-word semantic priming paradigm and the interpretation of the data that it produces. Consideration is then given to the way in which this paradigm can be usefully applied to the investigation of the five main areas of research outlined above.

1.3 The Single-Word Semantic Priming Paradigm.

Described in Neely (1991), the typical form of the single-word semantic priming paradigm involves two sequentially presented events (also see Meyer & Schvaneveldt, 1971, for simultaneous presentation of events). The first event is the presentation of a prime, which is usually a single letter string. This letter string can either be a word that creates a semantic context (e.g., bread) or a neutral prime that does not create a semantic context (e.g., ‘blank’ or ‘XXXX’). The participant is not required to respond to the prime. Following a short time period, a second letter string, referred to as a target, is presented. The participant is required to respond to the target by performing a lexical decision task (i.e., deciding whether the target is a word or a non-word) or a pronunciation task (i.e., saying the word aloud). When the target is a word, it may be related (e.g., ‘bread’ and ‘butter’) or unrelated to the prime
(e.g., ‘doctor’ and ‘butter’). The effect of the related and unrelated conditions can then be assessed by subtracting the reaction time taken to respond to the target in each condition from the reaction time taken to respond to the same target following exposure to a neutral prime (i.e., the baseline condition). When the difference is positive, the effect is referred to as facilitation and when the difference is negative, it is referred to as inhibition. In word priming studies that do not employ neutral conditions (usually in investigations in which inhibitory processes are of little interest) the unrelated condition is used as the baseline. Here, the difference between the related and the unrelated conditions is referred to as the overall priming effect.

1.3.1 Priming Mechanisms

According to Neely (1991), three main mechanisms have been proposed to account for the facilitatory and inhibitory effects observed in the word priming literature (also see Balota & Lorch, 1986; Loftus, 1973; Neely, 1977). These include automatic spreading activation, expectancy based processing, and semantic matching. As noted earlier (see Section 1.1), automatic spreading activation is considered to occur rapidly and without conscious intention or awareness. Automatic spreading activation operates via exposure to the prime word, which activates its concept node in memory. Activation then spreads throughout the semantic network to the nodes of related words. Thus, target words related to the prime are already activated close to their recognition threshold upon presentation and consequently, processing of these targets is facilitated in comparison to other targets.

Expectancy based processing occurs where the participants use the prime strategically to generate an expectancy set of potential targets that are related to the prime (Neely, 1991). Targets that are included in the set are recognised more quickly than targets that are not included in the set. Consequently, the processing of related
targets is facilitated. In contrast, in the case of unrelated targets, a time consuming search through the expectancy set delays (i.e., inhibits) processing of the targets. Like automatic spreading activation, expectancy based processing is considered to be pre-lexical (i.e., it operates before access to the target’s lexical node). However, expectancy based processing is under the strategic control of the person and cannot occur without their intention or awareness.

In contrast to automatic spreading activation and expectancy based processing, semantic matching is thought to occur in lexical decision tasks after lexical access for the target and before a binary word/non-word decision is made (Neely, 1991). During this time, participants use information about whether the target is semantically related to the prime to speed up ‘word’ and ‘non-word’ responses. If a target is related to the prime, then it must be a word and a fast decision can be made. If the target is unrelated to the prime then it can either be a word or a non-word. In the latter case, non-word targets activate nodes corresponding to words that are visually similar to them (e.g., ‘bink’ may activate ‘pink’). However, in a typical lexical decision task, non-words are constructed so that they never look like words related to the preceding prime. Consequently, non-words activate nodes for words that are unrelated to the presented prime (e.g., ‘pink’ and the preceding prime ‘tree’). This, combined with a high non-word ratio (thus, increasing the likelihood that the correct response will be ‘non-word’), facilitates ‘non-word’ responses in comparison to the neutral condition, in which semantic matching does not operate.

1.3.2 Methodological Considerations.

A number of methodological considerations are important to the use of the single-word semantic priming paradigm for measuring facilitatory and inhibitory
effects, and distinguishing between automatic and controlled processes. These relate to choosing suitable neutral condition stimuli, SOAs, tasks (lexical decision vs. pronunciation), instructions, and an appropriate proportion of related trials. The following paragraphs outline each of these topics and the main recommendations that inform the methodology that is ultimately employed in this paradigm. Where few recommendations are offered on a particular topic, brief summaries of the main findings within the literature are provided.

According to Neely (1991), in studies that are designed to assess facilitatory and inhibitory effects, the choice of neutral condition stimuli is guided by three main principles. Firstly, neutral primes should be equivalent to other primes in terms of their value as a warning signal for the onset of a target. Secondly, neutral primes should be unrelated to the target so that they can be used as a baseline by which to compare spreading activation between related stimuli. Lastly, neutral primes should not offer any information about the semantic nature of the target to follow in order to provide a baseline by which to compare expectancy effects. Examples of the different types of neutral stimuli that have been employed in word priming research in the past are a string of XXXXs and the words *ready, blank* or *neutral*.

A major benefit of the single-word semantic priming paradigm is that the time period between the onset of the prime and presentation of the target (i.e., the stimulus onset asynchrony; SOA) can be varied to study the time course of semantic activation (Neely, 1991). In this paradigm, short SOAs in the order of 250 ms are commonly employed to assess automatic spreading activation (Neely, 1991; Perea & Rosa, 2002; Velmans, 1999). At such a brief SOA the time period between the onset of the prime and the presentation of the target is too short to allow for conscious awareness and strategic processing of the prime before exposure to the target (Libet,
1996; Velmans, 1999). At longer SOAs of more than 400 ms, the participants can use the primes strategically to generate sets of candidate targets and expectancy based processing can be measured (Perea & Rosa, 2002).

The choice of whether a lexical decision or pronunciation task is employed in an investigation is central to determining whether prelexical (i.e., automatic spreading activation and expectancy based processing) or postlexical processes (i.e., semantic matching) are measured. An important assumption underlying the notion of the semantic matching process is that it produces priming only in the lexical decision task (Neely, 1991). This is because knowledge that a target is related or unrelated to a prime provides relatively valid information about whether the correct response to a target is ‘word’ or ‘non-word.’ However, this same knowledge is not very informative of the correct sequence of phonemes that constitutes a response to the target in a pronunciation task. Consequently, Neely (1991) advises that investigations that focus on how a particular variable affects lexical access during reading (cf. an investigation into post lexical processing) should employ either a pronunciation task or a carefully controlled lexical decision task (i.e., with a low non-word ratio or GO-NOGO procedure in which the participant only responds to word targets).

A review of the literature by Neely (1991) indicates that the priming effects produced by each type of task are modulated both by SOA and by stimulus type. For example, differences in priming effects were produced in the context of lexical decision and pronunciation tasks in studies involving category name primes and exemplar targets (e.g., *bird* and *robin*, respectively). At short SOAs, lexical decision tasks were found to produce facilitation in the investigation of category primes and exemplar targets, and both facilitation and inhibition in the investigation of associatively related primes and targets. Pronunciation tasks, on the other hand, have
not been widely investigated at short SOAs. In lexical decision tasks, at SOAs greater than 500 ms, priming was found to be due to both facilitation and inhibition, whilst in pronunciation tasks it was due largely only to facilitation. In contrast, in a review of studies investigating associatively related primes and targets (e.g., rake and leaf) at SOAs greater than 1000 ms, both the lexical decision and pronunciation tasks mainly produced facilitation. Thus, in comparison to lexical decision tasks in which the findings appear mixed, pronunciation tasks produce a pattern of facilitation dominance at long SOAs.

The instructions given to participants can be manipulated in conjunction with SOA in order to distinguish between automatic and controlled processing (Neely, 1991). For example, at long SOAs, the instructions given to participants can explicitly draw their attention to the fact that the primes and targets are often related. They can then be encouraged to use this information strategically to help them to respond faster to the target. In contrast, at short SOAs, the instructions may deliberately avoid reference to any relationship between the primes and the targets.

The proportion of related prime-target trials can also be manipulated along with SOA in order to distinguish between automatic and controlled processing (Neely, 1991; Perea & Rosa, 2002). For example, at brief SOAs, when the proportion of trials in which the prime and target are related is low, the chance that the participants will either not notice or will ignore relationships between the primes and the targets increases. In contrast, at long SOAs, when the proportion of related trials is high and thus, the relationships between the primes and targets are somewhat more obvious, they may be employed strategically in responding (e.g., participants may try to anticipate the target or employ a process of context-verification checking; e.g., see Perea & Rosa, 2002).
However, relatedness proportion effects are also influenced by the task that is employed in an investigation. In lexical decision tasks, the evidence for relatedness proportion effects at short SOAs is relatively mixed. For example, in a study by de Groot (1984), relatedness proportions of 0.25 and 0.75 were compared at an SOA of 240 ms. A stronger priming effect of 74 ms was found in the high relatedness proportion condition than in the lower relatedness proportion condition (i.e., 58 ms), although this difference was not directly tested at this SOA. Similarly, Henik, Friedrich, Tzelgov and Tramer (1994) compared relatedness proportions of 0.20 and 0.80 at an SOA of 240 ms and found respective priming effects of 5 ms and 81 ms. Nevertheless, these effects may have been confounded by the requirement to either name the target or perform a letter search on the target following a lexical decision. Contrary to this, a study by Stolz and Neely (1995) that compared relatedness proportions of 0.25 and 0.50 at an SOA of 200 ms found no relatedness proportion effect. Similarly, a more recent investigation by Perea and Rosa (2002; Experiments 2, 4 and 5) that compared relatedness proportions of 0.18 and 0.82 at SOAs of 66, 83, 116, and 166 ms failed to find a relatedness portion effect. However, at an SOA of 800 ms, Stolz and Neely (1995) found a robust relatedness proportion effect (i.e., 37 ms in Experiment 1).

In contrast to lexical decision tasks, pronunciation tasks appear to produce slightly more consistent results. For example, an early investigation by Seidenberg, Waters, Sanders and Langer (1984) that compared relatedness proportions of 0.33 and 0.20 at an SOA of 500 ms failed to find a relatedness proportion effect. Similarly, Perea and Rosa (2002; Experiment 3) compared relatedness portions of 0.18 and 0.82 at SOAs of 66 and 116 ms, and failed to find a relatedness portion effect. In contrast, a study by Keefe and Neely (1990) that compared relatedness
proportions of 0.33 and 0.875 found an effect of 19 ms for high dominance exemplars (e.g., bird and ‘robin’ cf. ‘goose’) at an SOA of 1000 ms. Thus, relatedness proportion effects in pronunciation tasks do not appear to result at SOAs less than 500 ms for low relatedness proportions of up to 0.33 but do occur at a longer SOA of 1000 ms.

As is evident in the preceding discussion, the influence of the aforementioned key factors on the measurement and outcomes of the priming procedure, and consequently, the choices that are made surrounding their use, are not always mutually exclusive. Moreover, careful consideration of these key factors is needed in the development of an optimal procedure for the investigation of pre-lexical, automatic processes (cf. controlled processes). On the basis of the above discussion, this procedure would arguably include a brief SOA (in the order of 250 ms), a pronunciation task, instructions that do not draw attention to relationships between the prime and the target, and a relatedness proportion of no greater than 0.33. With these considerations in mind, the following paragraphs describe the application of the single word semantic priming paradigm to the present context i.e., the investigation of simple arithmetic processing.

1.4 The Present Priming Procedure

In the present studies, simple arithmetic problems were presented as primes (e.g., 2 + 3) and solutions were presented as targets (i.e., 5). In a design analogous to the single word semantic priming paradigm, three main prime-target relationship conditions were employed in the first four investigations, including a congruent (related) condition, an incongruent (unrelated) condition, and a neutral (baseline) condition. The same primes and targets were used in the congruent and incongruent conditions. In the congruent condition the target was the correct solution to the
problem presented as the prime. In the incongruent condition the target was an incorrect solution (e.g., 14) to the problem presented as the prime (i.e., 2 + 3). In this way, the congruent and incongruent conditions were balanced for the effects of problem size, split, and target magnitude. The targets employed in the neutral condition (see below for discussion of neutral prime stimuli) were the same as those employed in the congruent and incongruent conditions. The relatedness proportion employed in the first four investigations was thus, 0.33. In the final investigation, which examined split effects, two incongruent conditions were employed, including a close incongruent condition (e.g., 3 + 4 and 8) and a distant incongruent condition (e.g., 3 + 4 and 13). Therefore, the relatedness proportion employed in the final investigation was 0.25.

To date, a parallel form of the single word semantic priming paradigm has not been employed in the investigation of cognitive arithmetic. Correspondingly, neutral primes have not been widely employed in this area of research and the role of inhibitory mechanisms in simple arithmetic processing has remained largely unexplored. To address this in the current series of investigations, the utility and validity of two different neutral conditions were explored. In the first study, the neutral primes comprised the numerical symbol zero (i.e., 0 + 0 and 0 x 0). Consideration of the results of this experiment then lead to the use of alternative stimuli (i.e., X + Y and X x Y) in the remaining four investigations.

The investigation of automatic processes was accomplished using short SOAs (i.e., 120 and 240 ms in the first study, and 300 ms in the studies thereafter), whilst the use of a long SOA (i.e., 1500 ms in the first study and 1000 ms thereafter) enabled the measurement of expectancy effects. Importantly, the present procedure was also used in conjunction with a target-naming task. This task is comparable to
the word pronunciation task and simply requires that the participants respond by verbally naming target numbers as they appear on the computer screen. In this way, the risk of measuring post lexical semantic matching processes is reduced and processes that are essentially prelexical, i.e., spreading activation and expectancy, are measured (see recommendation by Neely, 1991, section 1.3.2 above).

The participants were asked to respond ‘quickly and accurately’ and no mention of the relationships between the primes and the targets was ever made. Instead, to ensure a level of familiarity with the stimuli and to draw attention the prime target relationships in the long SOA condition, the stimulus sets employed in the short SOA conditions were repeated at the long SOA.

1.4.1 The Benefits of Using the Priming Paradigm

An important benefit of employing the arithmetic based variant of the single word semantic priming paradigm is that it provides for the use of a neutral condition, and thus allows for the investigation of inhibitory mechanisms in simple arithmetic processing. To date, much of the research in the cognitive arithmetic area has focused on the investigation of automatic and strategic processes, at the expense of investigating inhibitory processes. However, theoretically, inhibitory mechanisms may play an important role in simple arithmetic fact retrieval. The simple arithmetic knowledge representation that is stored in memory is thought to be highly interrelated, i.e., both within and between operations. In the latter case, multiplication solutions are often incorrectly produced as answers to addition problems (e.g., the multiplication solution 6 is often produced to the addition problem 2 + 3), and vice versa (Ashcraft, 1992; Barrouillet & Lepine, 2005). Consequently a recent theory suggests that inhibitory mechanisms may work to suppress interference from
incorrect responses and so reduce errors in fact retrieval (Barrouillet & Lepine, 2005). Indeed, Barrouillet and Lepine (2005) suggest that the inability to inhibit interference from incorrect responses may actually underlie math’s difficulties. Substantial remedial benefits may therefore, be stood to be gained from a better understanding of the role of inhibitory mechanisms in simple arithmetic processing.

A further benefit of the present priming procedure is that it allows for a comparison of the facilitatory and inhibitory effects associated with each of the number and word knowledge domains. Such a comparison may have implications for models of cognitive processing and learning.

Finally, a major benefit of this priming procedure is that it allows for a reliable differentiation of the effects associated with problem encoding and fact retrieval mechanisms. As noted in relation to the investigation of surface form effects (section 1.2.3) and problem type (section 1.2.4), the inability to be able to distinguish between these two processes has beset progress in these areas of research for some time. The present priming procedure accomplishes this distinction in two main ways. Firstly, it examines patterns and magnitudes of facilitatory and inhibitory effects, rather than comparing overall reaction time measures. As mentioned earlier, facilitatory and inhibitory effects are difference scores created by subtracting the time taken to name congruent and incongruent targets from the time taken to name targets following exposure to the neutral primes. For example, in the study that examines surface form differences, in the digit condition, facilitation is calculated by subtracting the time taken to name the target ‘6’ following exposure to ‘3 x 2’ from the time taken to name this same target following exposure to the prime ‘0 x 0.’ This facilitation effect can then be compared to the effect produced in the investigation of number word stimuli. That is, in the given example, by subtracting the time taken to
name the target ‘6’ following exposure to ‘three x two’ from the time taken to name this same target following exposure to the prime ‘blank x blank.’ In this way, the time taken to encode each of the problems is effectively held constant.

Secondly, the present priming procedure allows for a distinction to be made between the effects due to encoding and fact retrieval mechanisms by comparing the patterns and magnitudes of the overall priming effects. As noted earlier, calculation of overall priming effects involves subtracting the congruent condition reaction times (e.g., $2 + 3$, presented with solution 5) from the incongruent condition reaction times (e.g., $2 + 3$, presented with the solution 14). Therefore, regardless of the surface form, the effect of encoding the problem is cancelled out. Where the stimulus set is balanced for target magnitude (i.e., by including the same problems and targets in both the congruent and incongruent conditions), only the portion of reaction time attributable to fact retrieval mechanisms remains.

1.5 Aims

The main aim of the present investigations was to determine whether the numerical variant of the single word semantic priming paradigm was able to identify facilitatory and/or inhibitory effects in the processing of simple addition and multiplication problems. Then, given that this methodology could be usefully applied in this context, the specific aim of the first study was to examine the circumstances under which arithmetic facts are automatically retrieved.

On the basis of the findings from the first study, the main aim of the second investigation was to examine individual differences in automaticity, using well-defined samples. Additionally, this study aimed to determine whether an alternative neutral condition, comprising the letters X and Y (i.e., $X + Y$ in the addition
condition and X x Y in the multiplication condition), could be usefully employed as a baseline, and it aimed to trial a 300 ms short SOA condition.

The aims of the third and fourth studies were to apply the new priming methodology to the questions of whether surface form and problem type influence simple arithmetic fact retrieval mechanisms. The inability of previous methodologies to distinguish between encoding and fact retrieval mechanisms has seen the answers to these questions evade researchers in the area of cognitive arithmetic for some time.

The main aim of the final study was to determine whether the reversed split effect observed in the context of Campbell’s (1987) study also characterises performance in the new arithmetic priming task. This finding would indicate that split effects are task specific, and may provide further insight into the cognitive processes underlying simple arithmetic performance.
2. THE PRESENT INVESTIGATIONS

2.1 The Question of Automaticity.

This article addressed the question of whether simple arithmetic facts are retrieved automatically from memory. Past research that has addressed this question has relied on tasks that effectively blur the line between automatic and conscious processing. This study employed a new priming methodology analogous to the single-word semantic priming paradigm. Participants were presented with problems as primes and solutions as targets, in one of three main conditions i.e., congruent (e.g., $2 + 3$ and $5$), incongruent (e.g., $2 + 3$ and $14$) and neutral conditions (e.g., $0 + 0$ and $5$). To minimise confounds of calculation and decisional processing, the participants were simply required to name the target numbers as they appeared on the computer screen. Additionally, SOAs of 120 and 240 ms were employed to assess automatic processing, and a long SOA of 1500 ms allowed for an investigation of expectancy effects.

The results of this study showed that correct solutions to simple addition problems are automatically activated from memory in individuals of all abilities. In contrast, only high ability arithmeticians demonstrated automaticity in the retrieval of multiplication solutions and were able to apply this knowledge toward superior performance in the naming task. Furthermore, the results indicated differences in processing due to problem size. In the multiplication condition, priming by small problems lead to greater levels of facilitation and inhibition than priming by large problems. In the addition condition, priming by small problems lead to greater levels of inhibition than priming by large problems, whilst the levels of facilitation were similar for both small and large problems.
Abstract

In adult simple arithmetic performance, it is commonly held that retrieval of solutions occurs automatically from a network of stored facts in memory. However, such an account of performance necessarily predicts a uniform reaction time for solution retrieval and is therefore not consistent with the robust finding that reaction time increases with problem size and difficulty. Additionally, past research into arithmetic performance has relied on tasks that may have actually induced and measured attentional processing, thereby possibly confounding previous results and conclusions pertaining to automaticity. The present study therefore, attempted to more reliably assess the influence of automatic processing in arithmetic performance by utilizing a variant of the well-established semantic word-priming procedure with a target-naming task. The overall results revealed significant facilitation in naming times at SOAs of 240 and 1500 ms for congruent targets i.e., targets that represented
the correct solutions to problems presented as primes (e.g., $6 + 8$ and 14). Significant inhibition in comparison to a neutral condition ($0 + 0$ and 17) was also observed at 120 and 240 ms SOAs in naming incongruent targets (e.g., $6 + 8$ and 17). Furthermore, response times were found to vary as a function of both arithmetic fluency and problem size. Differences in performance to addition and multiplication operations and implications for cognitive research and education are considered.

_PsycINFO classification: 2343; 2346_key Words:_ Arithmetic, Fluency, Automaticity, Priming, Naming
1. Introduction

How is simple arithmetic knowledge organised in and accessed from the adult human brain? Over the past three decades, most models of adult arithmetic processing have converged on the notion that adults solve single-digit addition and multiplication problems solely through automatic fact retrieval from memory (Ashcraft, 1992; LeFevre et al., 1996b). Foremost amongst these models has been Ashcraft’s (1992) Associative Network Retrieval model, which posits that arithmetic facts exist in a network of stored associations that are based on the operands and their related nodes. Retrieval of facts is thought to occur via automatic spreading activation, a process that is considered to be fast, accurate, obligatory, and requiring minimal cognitive load (LeFevre & Kulak, 1994).

Support for the associative network model and in particular the notion of obligatory (i.e., unintentional) activation of arithmetic knowledge derives from the presence of cross-operation confusion effects in the performance of production and verification tasks (LeFevre & Kulak, 1994). For example, in production tasks, cross-operation errors occur where the incorrect solution that is produced represents the correct solution to an alternative operation involving the same operands e.g., 2 + 3 = 6 (Ashcraft, 1992; Campbell, 1987; Cipolotti & Butterworth, 1995). Likewise, in verification tasks, it takes longer to determine that a cross-operation equation is false than it does to determine that an equation with an unrelated solution (e.g., 2 + 3 = 11) is false (Ashcraft, 1992; LeFevre & Kulak, 1994; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986). Thus, as opposed to being obtained through procedures and rules, arithmetic solutions appear to be directly retrieved from a highly organised and associated network in long-term memory (Ashcraft, 1992; LeFevre & Kulak, 1994).
Further support for this model and the notion of obligatory activation stems from a series of investigations undertaken by LeFevre and her colleagues that employed a number-matching task. In this task, participants were first presented with a pair of numbers (e.g., 3 + 4) and then following a given inter-stimulus interval, were simply required to decide whether a target number (e.g., 7) was one of the two numbers originally presented. The results of investigations involving both the addition (LeFevre, Bisanz & Mrkonjic, 1988; LeFevre & Kulak, 1994) and multiplication (Thibodeau, LeFevre & Bisanz, 1996) operations showed that in contrast to other unrelated numbers (e.g., 3 + 4 and 9), the presentation of the correct sum or product (respectively) led to lengthier decision times. Moreover, this interference effect occurred quickly, at very short SOAs, and was found regardless of the exclusion of the arithmetic operator. Importantly, consistent with the notion of automatic spreading activation, the solution was produced irrespective of the intentions of participants to simply match numerical symbols (LeFevre et al., 1988; LeFevre and Kulak, 1994; LeFevre, et al., 1996a, 1996b; Thibodeau et al., 1996).

In a similar study, Galfano, Rusconi and Umilta (2003) employed a number-matching task to determine whether multiplicatively related facts (e.g., 16) that either precede or follow the correct product (i.e., 24) could be automatically activated following the presentation of two numbers (i.e., 8 and 3). The results showed that decision times to target numbers that were adjacent to the correct product in the table (related to either of the operands within the prime) were increased in comparison to other unrelated targets. The authors again, concluded that multiplication facts are represented in a highly associated network, with automatic activation also spreading from the correct product to adjacent nodes.
Thus, it would seem that the evidence in support of the associative network model and obligatory activation is rather convincing. However, a disadvantage of this explanation of arithmetic processing arises from the fact that, in principal, it can not account for the problem size (or difficulty) effect. This refers to the apparently robust finding that it becomes more difficult and takes longer to process problems as they become larger in size (Ashcraft, 1992; Brysbaert, 1995). Such an effect is different to the uniform reaction time pattern that is predicted by automatic retrieval models.

Nonetheless, in keeping with the associative network model, explanations for the problem size effect have centred on structural rationales relating reaction times to numerical indices (such as the distance to be traversed within a network) or to the frequency of exposure to particular problems in early education (Ashcraft, 1992; Ashcraft, Donley, Halas & Vakali, 1992; LeFevre et al., 1996a). In the latter case, data taken from elementary textbooks, shows that smaller problems appear earlier in instruction and more frequently than do larger problems (Hamman & Ashcraft, 1986). Smaller numbers also appear more frequently than larger numbers in naturally occurring settings (Ashcraft, 1992). Accordingly, it is not inconceivable that greater exposure to, and more practice of, smaller problems will result in fact retrieval being increasingly automated in comparison to larger, more difficult problems (Ashcraft, 1992; Ashcraft et al., 1992; Koshmider & Ashcraft, 1991; Siegler, 1988; Siegler & Jenkins, 1989).

The notion that there may be differences in the way that particular problems are solved is not new to the cognitive arithmetic literature. In fact, this very idea serves as the basic premise underlying Siegler and Jenkins’ (1989) Distributions of Associations model, which posits that knowledge representations of particular
problems develop a set of associated solutions and a set of methods for their accurate retrieval. Furthermore, depending on the strength of associations between problems and their correct solutions (a factor influenced by frequency of exposure), retrieval of arithmetic facts may occur using either automatic or strategic processing mechanisms. Importantly, the inclusion of both mechanisms in performance allows for the prediction of reliable differences in reaction time and consequently a possible explanation of the problem size effect (Ashcraft et al., 1992; Koshmider & Ashcraft, 1991).

Empirical support for a difference in processing between problems of varying size and difficulty derives from a priming study conducted by Ashcraft et al. (1992). Simple multiplication problems and their solutions were first divided into three problem difficulty groups (i.e., low, medium and high). All problems were then presented twice, once neutrally primed by a line of two dashes (e.g., -- primed 6 x 5 = 30) and once primed by either the correct solution, a related solution or an unrelated solution (e.g., 30, 25, or 21 primed 6 x 5 = 30, respectively). Participants were simply required to decide whether the presented problem, was true or false. The results showed that correct primes had a positive effect on reaction time, although, for the high difficulty problems, this occurred only at a long SOA. Furthermore, related and unrelated (irrelevant) primes were found to yield negative effects, especially for the more difficult problems. This finding was deemed consistent with the notion that the incorrect problem led to confusion and that, in contrast to low and medium difficulty problems, the more difficult problems were solved using conscious processing.

Two main difficulties arose with the methodology employed in the Ashcraft et al. (1992) study. The first of these occurred in that the order of stimulus
presentation may have led to an overestimation of the levels of priming and inhibition that occur in normal arithmetic processing. For instance, in true trials, exposure to the correct solution before exposure to the problem would have led to prior activation of this number in memory, and consequently faster responses than would normally occur following simple exposure to an arithmetic problem. Similarly, in false trials, pre-exposure to an incorrect solution possibly created greater levels of confusion than would normally be encountered in arithmetic tasks.

The second difficulty with the Ashcraft et al. (1992) study occurred in relation to their choice of verification procedure. In such procedures, access to arithmetic processing may be confounded by responses to problems that are carried out using some sort of familiarity judgement, possibly involving comparison of the equation as a whole to information in memory (LeFevre et al., 1996). Additionally, Campbell (1987) argues that participants may rely on plausibility judgements in terms of approximate magnitude or on the odd-even status of the presented answer in relation to the problem’s operands. Moreover, for incorrect trials, previous research has shown that when the difference in magnitude between an incorrect and correct solution is large it is verified more quickly than if this difference is only small (commonly referred to as the ‘split effect’; Ashcraft, 1992; Campbell, 1987). Such an effect, whilst not distorting reaction times to correct trials, may confound other conditions and consequently influence the final outcomes of the study (Campbell, 1987). Finally, similar arguments to those levelled at verification procedures in the single-word semantic priming literature (i.e., lexical decision tasks) can be made with regard to those employed in studying simple arithmetic. Specifically, it has been suggested that ‘attentional’ decision processes, that occur after the simple matching of a stimulus with its lexical representation, may confound the overall reaction time
measured in the lexical decision task (Balota & Lorch, 1986; Friedrich, Henik & Tzelgov, 1991; Lorch, Balota & Stamm, 1986; Neely, 1991; Sereno, 1991; Slowiaczek, 1994; Smith, Besner & Myoshi, 1994). In the case of the aforementioned studies then, it could be argued that the requirement to actively make a binary decision as to the relationship between the prime and the target might interfere with the automatic processes, essentially thought to occur without intention or awareness, that they purport to measure.

Having acknowledged the difficulties inherent in verification tasks, Campbell (1987, 1991) resolved to employ a production task in the examination of differences in processing between multiplication problems of varying difficulty. In two studies (employing different SOAs of 300 and 200 ms, respectively), problems were first divided into easy and difficult categories, based on normative production error rates. Participants were then presented with one of four prime types: the correct product, a neutral prime (##), a related false prime (frequently occurring as an error for a given problem) and an unrelated false prime (occurring with low frequency as an error response to the problem). Following this, they were presented with a problem and required to produce the correct solution. In both studies, facilitation to more difficult problems was greater than to easy problems. According to Campbell, this showed that priming using the correct answer improved retrieval of less accessible (i.e., more difficult) answers in comparison to more automatic answers that had already reached a ceiling such that no appreciable effects on performance could be realised.

Interestingly, in Campbell’s (1987, 1991) studies, inhibition was found when a related but incorrect prime preceded each problem. As noted by Campbell, the presence of inhibitory effects suggests the use of attentional, conscious processes in performance and, in the context of these studies, may have reflected a deliberate
attempt by the participant to ignore interference caused by the prime. Furthermore, with the subject always intending to accurately perform arithmetic calculations, it could again be argued that the line between unintentional, automatic processing and conscious processing was blurred. Finally, the use of only short SOAs did not allow for the analysis of changes in facilitatory and inhibitory effects over time (Koshmider & Ashcraft, 1991).

More recently, LeFevre and colleagues addressed the issue of differences in processing in both addition and multiplication procedures using self-report measures. In two studies, samples of undergraduate students were first required to provide solutions to given problems and then to describe how they obtained them. In the addition study, the results indicated that an amazing 25% of all solutions from a ‘relatively skilled’ university sample were achieved through a strategic transformation \((6 + 5 = 6 + 4 + 1)\) or counting \((3 + 2 = 3, 4, 5)\) procedure (LeFevre et al., 1996a). This figure was again reflected in the multiplication study, with the use of such conscious retrieval methods as rules \((0 \times n = 0)\), repeated addition \((2 \times 3 = 3 + 3)\), number series \((3 \times 3 = 3, 6, 9)\) and derived facts \((3 \times 4 = [3 \times 3] + 3)\) reported on 20% of all trials (Lefevre et al., 1996b). In addition, examinations into individual differences revealed significant correlations between arithmetic fluency and the percentage use of retrieval in both operations. Thus, the authors concluded that learning and experience had a continuing influence on adult arithmetic performance and that solely automatic fact retrieval explanations of performance did not provide a complete account of adult processing.

Unfortunately, as noted by Lefevre and Colleagues (1996a, 1996b), the use of self-report as a valid and reliable measure of performance was critical to the interpretation of their data. The self-report methodology has nonetheless been
criticised on the grounds that, when asked to describe mental processing, people may change or be unable to accurately describe their behaviours (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003). Additionally, individual differences and instructional demands may bias verbal reports and the solution procedures that are reported (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003). Indeed, a recent study by Smith-Chant and LeFevre (2003) showed that low skill participants responded more slowly and accurately when asked to describe their solution procedures for large and very large problems. Furthermore, low skill participants exhibited greater variation in procedures and were more likely to alter their selection of retrieval method, with changes in instructional emphasis between speed and accuracy. Thus, the possibility of reactivity in the LeFevre et al (1996a, 1996b) study could not be ruled out, leading these authors to the call for the use of alternative, more reliable methods in the investigation of the role of automaticity in arithmetic performance.

In an attempt to address this, the present study borrowed from the well-established single word semantic priming paradigm and employed a procedure similar to that used in the previous Campbell (1987, 1991) studies. In contrast to the earlier research, however, the present study involved the presentation of a problem as the prime and a solution as the target, in the order that they would appear in a naturally occurring setting. Additionally, a naming task that simply required the subject to state the target number as it appeared and not to perform any verification, calculations, or relationship matching based on the prime, was utilised. This served to both minimise the possibility of decision-induced attentional processing and to reduce the influence of errors in production on subsequent trials (Campbell, 1991). Furthermore, as recommended in Koshmider and Ashcraft (1991), SOAs
representing both automatic and conscious processing conditions were employed. Problems were then randomly assigned to appear in all conditions and equally divided into problems containing both small and large numbers, and a mix of the two. This allowed for a comparison of processing between problem sizes (and difficulty) over time. Finally, to allow for an investigation into the influence of skill on arithmetic performance, the participants arithmetic fluency was measured using the arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) (Form C; 1984).

2. Method

2.1 Participants

Thirty-nine psychology students, including 16 males and 23 females, from Murdoch University participated in the present study. The participants’ ages ranged from 16 to 53 years, with a mean age of 27.

2.2 Design and stimulus materials

Three independent variables were examined in the present study. The first of these determined the arithmetic operation i.e., addition or multiplication. The second variable incorporated three prime-target relationships, including congruent (e.g., 2 + 4 = 6), incongruent (2 + 4 = 9) and neutral (0 + 0 = 6) conditions. The final independent variable was SOA with three levels: 120 ms, 240 ms and 1500 ms.

Four sets of primes were constructed for each of the addition and multiplication operations (see Appendix A). The first set for each operation consisted of 18 simple arithmetic facts selected from the 2s through 9s matrices (e.g., 2 + 3). The second set comprised the reverse operand placement equivalents of the first set (3 + 2). The third set for each operation contained a mix of problems taken from the
first and second sets, such that no two problems represented the same arithmetic fact (i.e., if 2 + 3 was already chosen in the third set, then 3 + 2 was not also selected). The final set consisted of the reverse operand placement equivalents of the third set.

Arithmetic facts resulting from ties (e.g., 3 + 3 and 3 x 3) were excluded from use as primes, because previous research indicates that these problems are solved more quickly than others (LeFevre et al., 1988). Additionally, to balance each prime set, half of the arithmetic facts were produced so that the smaller of the two operands in each problem was placed on the left-hand side and half with the smaller operands on the right hand side. Finally, to enable testing for the presence of the problem size effect, each stimulus set consisted of six smaller problems (i.e., with both operands of a magnitude less than or equal to five; 2 + 3), six larger problems (operands greater than or equal to six; 8 + 9), and six of mixed magnitude (2 + 9).

Target sets constructed for each of the congruent, incongruent and neutral conditions consisted solely of the correct solutions corresponding to the simple arithmetic facts investigated in this study. For the incongruent condition, these targets were simply paired with an alternative problem, making them mathematically erroneous. To guard against split effects in the multiplication condition, incongruent solutions were paired with problems so that they differed by at least 16 from the correct solutions to these problems. Similarly, for the addition condition, incongruent solutions differed by at least three from the correct solutions. Further constraints on the incongruent target sets addressed possible confounding relationships between the prime and the target. For example, incongruent targets were not permitted to be one of the operands or the numbers plus or minus one from the operands, used in the prime. Additionally, where possible, multiples or factors of the operands and number series relations were excluded. Finally, incongruent targets were such that they could
not be the correct solution to the prime using a different operation, a double-digit number containing the operand, or a number containing the correct solution (i.e., if the correct solution was 7, then numbers such as 17 and 70 were also excluded).

Neutral conditions have not been widely used in the study of arithmetic but have been useful in assessing facilitation and inhibition and hence distinguishing automatic from conscious processing in word priming research (Neely, 1991). As such, in the present study, the neutral stimuli (i.e., 0 + 0 for the addition condition and 0 x 0 for the multiplication condition) were developed in accordance with three main criteria that were outlined in a review of the word priming literature by Neely (1991). The first of these was that neutral primes should be equated with other primes in relation to their value as a warning signal that a target will soon appear. In the present study, the numerical prime 0 + 0 can be likened perceptually to the other primes such as 2 + 3, with both consisting of two numerical operands separated by an arithmetic operator. Secondly, according to Neely, neutral primes should be unassociated to the target so that they are a neutral baseline by which to assess spreading activation between related stimuli. The 0 + 0 and 0 x 0 stimuli were unrelated to the targets that were employed in the present study both in terms of arithmetic relatedness and distance along the number line. Lastly, Neely suggested that neutral primes should not offer any information as to the semantic nature of the target to follow in order to provide a baseline by which to compare expectancy effects. In the case of this last criterion, as with the use of neutral primes such as ready, neutral or blank in the word priming literature, it may be argued that the 0 + 0 and 0 x 0 stimuli are not necessarily semantically neutral (possibly leading to the expectation that 0 will be presented as the target). Importantly however, their repeated presentation ensures that semantic satiation is rapidly attained and that less
processing capacity is consumed, thereby enabling them to serve as an effective neutral baseline.

2.3 Psychometric testing

The arithmetic section of the ACER SCT incorporated 60 arithmetic problems that variously included the addition, subtraction, division and multiplication of single, two and three digit numbers (ACER, 1984). Participants were given five-minutes in which to accurately complete as many problems as they could. They were instructed to start from the first problem and to work through each in turn, without omitting any problems (ACER, 1984). Rough working out could be undertaken anywhere on the page and participants were informed that if they completed the first column, they should immediately go onto the second one (ACER, 1984).

The total number of problems solved correctly served as the participant’s fluency score. One participant did not return for this test. The remaining participants’ scores ranged between 10 and 47. A median split procedure was then used to allocate 19 participants who scored less than or equal to 17 to the low skilled group, and 18 participants scoring greater than 17 to the high skilled group. According to the ACER SCT manual, a score of 17 corresponds to a percentile rank of 2% in a normed sample of 124 candidates who had completed a three to four year degree or diploma in a tertiary institution. The mean correct score for the low skilled group was 14, which was lower than any score obtained by the normed sample. In contrast, the mean correct score for the high skilled group corresponded to a percentile rank of 14%, with the highest score in this group corresponding to a percentile rank of 92% in the normed sample.
2.4 Procedure

Participants were individually tested in a well-lit cubicle room containing an Amiga 1200 microcomputer with 1084S monitor, that controlled stimulus presentation, trial sequencing, timing and data collection. An additional monitor outside of the cubicle displayed reaction times and target stimuli so that accuracy could be monitored. All stimuli were centrally presented, white against an amber background. Individual operands within each problem did not exceed dimensions of 5 x 15 mm on the computer screen. Arithmetic operators (i.e., x and +) did not exceed 5 x 10 mm and a 5 mm gap separated operands from the operator within each problem. A chin rest was used to stabilise the participant’s head at a viewing distance of 60cm from the screen.

Each testing session began with 20 unique practice trials and thereafter comprised six blocks of 54 experimental trials (i.e., three for each of the addition and multiplication operations corresponding to each of the three SOA conditions). Addition and multiplication trials were separately blocked so as not to produce cross operation or relatedness errors. Half of the participants started with the addition block first and half started with the multiplication block first. Additionally, half of the participants were exposed to the first set in the 120 ms SOA condition, and half to the second set. The set not assigned to the 120 ms condition was then presented in the 240 ms condition. Participants were exposed to both sets in order to reduce repetition of the priming stimuli at the short SOA’s, whilst keeping the target stimuli the same. Half of the participants were then presented with the third set and half with the fourth set in the 1500 ms condition. Repetition of the first and second set trials at the longer SOA allowed for a level of familiarity with the stimuli, drawing attention to the prime-target relationship. Finally, the computer randomly generated the order.
of presentation of the individual congruent, incongruent and neutral trials within each block and exposure to all stimuli was counterbalanced across participants.

Prior to testing, participants were instructed on the need to respond both quickly and accurately. At the start of each trial participants were required to focus their gaze on a 1 x 1 mm blue central fixation dot, exposed for 600 ms. The screen then went blank for a period of 150 ms before the prime was presented for 100 ms. Following the given SOA, the target appeared and remained exposed until the participant identified the given number. An interval of two seconds separated the participant’s response and the onset of the next trial. A microphone connected to a headset, with padded ear guards preventing external noise intrusions, was used to detect participant vocal response sounds. The microphone amplifier triggered an electronic relay interfaced to the computer, which determined the time of relay closure using a hardware timer. The value of the timer, accurate to 1 millisecond, measured the participant’s vocal reaction time from the onset of the target.

On finishing the computer task participants completed the Arithmetic section of the ACER SCT. They were then debriefed, with the session having taken approximately 40 minutes to complete.

3. Results

3.1 Overall analysis

The correct mean response latencies were initially screened for outliers using a criterion of +/- 2.5 z-scores and replaced using mean substitution. This led to adjustment of less than 0.60% of all scores. The resulting reaction time data are presented in Table 1.
Table 1.
Mean Reaction Times (ms) and Standard Deviations (in parentheses) for all Prime-Target Relationships as a Function of SOA and Operation.

<table>
<thead>
<tr>
<th>Operation</th>
<th>SOA</th>
<th>120 ms</th>
<th>240 ms</th>
<th>1500 ms</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Congruent</td>
<td>120 ms</td>
<td>443 (52.6)</td>
<td>419 (50.0)</td>
<td>431 (52.1)</td>
</tr>
<tr>
<td>Incongruent</td>
<td>120 ms</td>
<td>446 (51.6)</td>
<td>433 (53.5)</td>
<td>451 (50.7)</td>
</tr>
<tr>
<td>Neutral</td>
<td>120 ms</td>
<td>435 (45.4)</td>
<td>430 (48.4)</td>
<td>447 (47.7)</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Congruent</td>
<td>120 ms</td>
<td>466 (49.0)</td>
<td>440 (49.0)</td>
<td>445 (61.8)</td>
</tr>
<tr>
<td>Incongruent</td>
<td>120 ms</td>
<td>470 (48.2)</td>
<td>456 (53.7)</td>
<td>470 (52.6)</td>
</tr>
<tr>
<td>Neutral</td>
<td>120 ms</td>
<td>464 (49.4)</td>
<td>447 (44.7)</td>
<td>465 (55.4)</td>
</tr>
</tbody>
</table>

An overall repeated measures analysis of variance, including operation, SOA and prime-target relationship as within group variables, was performed on these data. Significant main effects were found for all three variables. Firstly, reaction times to addition-related targets were 21 ms faster overall than to multiplication-related targets ($F(1, 37) = 46.1, MSe = 1,617.4, p < 0.001$). This difference in performance is best explained by differences in target magnitude. For example, in the present study, addition-related targets only ranged from 5 through 17 as compared to multiplication-related targets, which ranged from 6 through 72. Previous research has indicated that it takes longer to perform number naming tasks when numbers are large than when they are small (Ashcraft, 1992; Brysbaert, 1995). This finding was again supported in the problem size analysis below. Given this fundamental difference in processing, following the first analysis, the addition and multiplication operations were analysed separately.

Secondly, a significant main effect of SOA was found ($F(1.4, 52.7) = 10.7; MSe = 2,335.1, p = 0.001$). Violations of the assumption of compound symmetry were
corrected throughout the present analyses by adjusting the degrees of freedom using Huynh-Feldt epsilons). Reaction times to the 120 ms condition were 16 ms slower than to the 240 ms condition but were no different from those obtained to the 1500 ms condition. One possible explanation for the lengthier response times at the 120 ms SOA is that the short interval between the onset of the prime and the presentation of the target interfered with the effectiveness of the prime as a warning signal for the target (Posner, Klein, Summers & Buggie, 1973). Posner and colleagues showed that 200 ms is the optimal period for a warning stimulus to precede a target in a simple spatial choice reaction time task, with shorter or longer inter-stimulus intervals leading to progressively longer overall reaction times. Additionally, the advantage in response times at the 240 ms SOA may have in part reflected a speed accuracy trade off. For example, whilst very few errors in number naming were found (i.e., less than 0.50% of all trials), 49% of errors occurred at the 240 ms SOA in comparison to only 28% at the 120 ms SOA and 23 % at the 1500 ms SOA.

Thirdly, in the overall analysis, a significant main effect of prime-target relationship was found \( F(2, 74) = 27.9; MSe = 366.8, p < 0.001 \). Congruent trials had a 13 ms advantage over incongruent trials, and a 7 ms advantage over neutral trials. This overall pattern of performance was then further qualified by a significant interaction between SOA and prime-target relationship \( F(4, 148) = 9.2, MSe = 352.1, p < 0.001 \). The facilitatory (neutral – congruent) and inhibitory (incongruent – neutral) differences describing this interaction are presented in Fig. 1. All 95% confidence intervals were calculated using the \( MSe \) terms for individual one-factor repeated measures ANOVAs involving the difference scores representing each of the facilitation and inhibition functions (Loftus & Masson, 1994; Masson & Loftus, 2003).
Repeating measures t-test comparisons showed that the facilitation was significant at both the medium \((t = 4.2, df = 37, p < 0.001)\) and long SOAs \((t = 5.1, df = 37, p < 0.001)\). Furthermore, a one-way repeated measures ANOVA showed a significant increase in facilitation over time \((F(2, 74) = 17.7, MSe = 277.9, p < 0.001)\), with increments at both the 240 \((t = 4.3, df = 37, p < 0.001)\) and 1500 ms SOAs \((t = 2.3, df = 37, p = 0.027)\) reaching significance. In contrast, t-test comparisons showed that inhibition to incongruent targets reached significance only at the short \((t = 3.4, df = 37, p = 0.001)\) and medium SOAs \((t = 2.2, df = 37, p = 0.034)\) and a repeated measures ANOVA showed that it generally remained constant over time \((F(1.7, 61.6), = 0.9, MSe = 301.5, p = 0.402)\). The results of the present study, utilizing a naming task, are therefore generally consistent with those employing production and verification procedures in demonstrating positive effects of congruent primes on reaction times (Ashcraft et al., 1992; Campbell, 1987, 1991). Additionally, the
pattern of performance, with significant inhibition found to incongruent targets at short SOAs, is consistent with that previously found in number-matching and verification procedures (LeFevre et al., 1988; Zbrodoff & Logan, 1986).

Finally, in relation to the overall analysis, t-test comparisons revealed equivalent levels of facilitation and inhibition at the medium SOA (t = 0.99, df = 37, p = 0.324) and facilitation dominance at the long SOA (t = 2.7, df = 37, p = 0.010). The findings of the present study, employing the priming paradigm and arithmetic stimuli, are thus similar to those described in studies investigating the time course of facilitation and inhibition in the investigation of associatively related word primes and targets (Neely, 1991).

3.2 Arithmetic Fluency

A separate split plot analysis of variance for each of the addition and multiplication operations was used to explore the influence of the between group variable arithmetic fluency. For the multiplication operation, as in the overall analysis, significant main effects of SOA ($F(1.7, 59.0) = 7.2, MSe = 1,664.3, p = 0.003$) and prime target relationship ($F(2, 70) = 28.3, MSe = 210.1, p < 0.001$) and an interaction between SOA and prime-target relationship ($F(4, 140) = 6.1, MSe = 221.9, p < 0.001$) were again found. Furthermore, a significant two-way interaction between fluency and prime-target relationship ($F(2, 70) = 5.6, MSe = 210.1, p = 0.006$) and a three-way interaction between fluency, SOA and prime-target relationship ($F(2, 70) = 6.9, p = 0.002$) were found. Facilitation and inhibition differences underlying this interaction are presented in Fig. 2.
Fig. 2 Showing facilitation and inhibition for the multiplication and addition operations as a function of SOA and arithmetic fluency. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects.

For the high skilled group, a one-way repeated measures ANOVA showed that the facilitation to congruent targets increased significantly over time, across all SOAs ($F(2, 34) = 12.1, MSe = 372.6, p < 0.001$). Additionally, paired sample $t$-test comparisons showed that the level of facilitation at the medium SOA approached but did not quite reach significance ($t = 2, df = 17, p = 0.060$), whilst at the longest SOA it was highly significant ($t = 5.7, df = 17, p < 0.001$). The finding of such a strong advantage for the congruent condition, given a lengthy interval between the onset of the prime and the presentation of the target, possibly reflects the use of an expectancy strategy. Described in Neely (1991), this occurs where the participant deliberately generates a set of related targets that could be expected to follow a given
prime. Consequently, the processing of expected targets is facilitated, whilst the processing of unrelated targets is inhibited. In accord with this interpretation, examination of Fig. 2 suggests that the cost in response to incongruent targets was greatest at the longest SOA for this group and was significantly greater than that observed for the low skilled group \((F(1, 36) = 13.5, p = 0.001)\). However, this finding was not supported statistically, with a repeated measures ANOVA showing that the level of inhibition for the group remained constant over time \((F(2, 34) = 1.3, MSe = 282.9, p = 0.276)\). Within group t-test analyses revealed significant levels of inhibition at each of the 120 \((t = 3.2, df = 17, p = 0.006)\), 240 \((t = 2.2, df = 17, p = 0.040)\) and 1500 ms \((t = 3.9, df = 17, p < 0.001)\) SOAs for this group.

In contrast, for the low skilled group, no significant increase in facilitation was observed over time \((F(2, 36) = 1.4, MSe = 453.1, p = 0.249)\). Moreover, the only observable effects in the data were a significant level of facilitation reached at the long SOA \((t = 2.1, df = 18, p = 0.049)\) and a decrease in the level of inhibition observed between the 120 and 1500 ms SOAs \((t = 2.2, df = 18, p = 0.039)\). Finally, in relation to the multiplication analyses, a between groups ANOVA showed that the difference between high and low skilled facilitation levels at the long SOA approached significance \((F(1, 36) = 3.7, p = 0.063)\).

For the addition operation, as in the previous analysis, significant main effects of SOA \((F(1.8, 61.4) = 7.3, MSe = 1178.4, p = 0.002)\) and prime-target relationship \((F(2, 70) = 12.2, MSe = 348.5, p < 0.001)\), and a significant interaction between SOA and prime-target relationship were found \((F(4, 140) = 5.0, MSe = 290.5, p = 0.001)\). No main effect of arithmetic fluency was present and, in contrast to the multiplication operation, it was not involved in any interactions. Nevertheless, in line with the particular interests of the present study in the effects of arithmetic
fluency on processing, facilitatory and inhibitory differences were considered. For the high skilled group, a significant increase in facilitation over time was again observed ($F(1.5, 25.8) = 4.0, MSe = 818.0, p = 0.042$), occurring between the 120 and 240 ms SOAs ($t = 2.7, df = 17, p = 0.016$). Paired sample t-test comparisons showed that the level of facilitation was significant at both the 240 ($t = 4.2, df = 17, p = .001$) and 1500 ms ($t = 2.4, df = 17, p = 0.030$) SOAs. In contrast, the level of inhibition was significant only at the short SOA ($t = 2.4, df = 17, p = 0.026$) and, as in the multiplication condition, it remained constant over time. An ANOVA involving operation, SOA and prime-target relationship as within group variables, showed that although there was a main effect of operation (with responses to addition-related targets found to be 16 ms faster overall than to multiplication related targets; $F(1, 17) = 12.5, MSe = 1728.5, p = 0.003$), it was not involved in any interactions. High skilled performance in the addition and multiplication operations is therefore, generally the same.

As with the high skilled group performance, the low skilled results for the addition condition revealed a significant increase in facilitation over time ($F(2,36) = 6.0, MSe = 565.4, p = 0.006$), occurring between the 120 and 240 ms SOAs ($t = 2.6, df = 18, p = 0.018$). Furthermore, the level of facilitation approached significance at the 240 ms SOA ($t = 2.09, df = 18, p = 0.051$) and reached significance at the 1500 ms SOA ($t = 2.6, df = 18, p = 0.017$). No inhibitory differences reached significance in the data and they did not change significantly over time. Finally, no significant differences in the levels of facilitation or inhibition were observed between high and low skilled groups at any of the SOAs in the addition data.

In summary, the patterns of performance observed for both the high and low skilled groups in the addition condition were very similar, with increasing facilitation
over time and advantages in performance evident for both groups at the long SOA. At the 240 ms SOA, automaticity in processing typified high skilled performance and most likely also characterised low skilled performance, given that the level of facilitation so narrowly missed significance. Performance by high skilled participants did not vary statistically between operations, with the facilitation at the 240 ms SOA again approaching significance in the multiplication condition. However, unlike the addition condition, at the long SOA, high skilled participants appeared able to apply their multiplication fact knowledge strategically to advantage performance in the naming task. This was in direct contrast to the low skilled performance, with the facilitation at the long SOA barely reaching significance and no advantage evident at the 240 ms SOA.

3.3 Problem size analysis

In order to determine the influence of problem size on arithmetic processing a subset of the data that included only the reaction times to small (consisting of operands \( \leq 5 \)) and large problem sizes (operands \( > 5 \)) was selected. These data were initially screened for outliers using a cut off score of +/- 2.5 z-scores, leading to 1.24% of all scores being replaced using mean substitution.

In contrast to the earlier analyses in which each solution was presented in every condition, the use of only a subset of the data created a mis-match between the solutions in the congruent and incongruent conditions, and between problems and solutions of differing magnitudes. For example, in the multiplication condition congruent targets for small problems ranged between 6 and 20, whilst in the incongruent condition, except for the target 6, all other targets ranged between 30 and 63. Similarly, congruent targets for large problems ranged between 42 and 72, with incongruent targets mostly ranging between 15 and 24 (with the exception of
targets 56 and 72). This led to difficulties in making direct comparisons within and between problem sizes because previous research indicates that as number magnitude increases, reaction time increases (Brysbaert, 1995). The raw data for all problems within the original stimulus set were thus entered into correlation and regression analyses to first ascertain any effect of target magnitude and then to account for this variable in the obtained reaction times. Pearson correlation coefficients and the best fitting model between the mean overall reaction time and number magnitude are presented in Table 2.

<table>
<thead>
<tr>
<th>Table 2. Pearson Correlation Coefficients ($r$) and Models of Best Fit between Reaction Time and Number Magnitude.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SOA</strong></td>
</tr>
<tr>
<td><strong>Addition</strong></td>
</tr>
<tr>
<td><strong>120 ms</strong></td>
</tr>
<tr>
<td>Congruent</td>
</tr>
<tr>
<td>Incongruent</td>
</tr>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td><strong>240 ms</strong></td>
</tr>
<tr>
<td>Congruent</td>
</tr>
<tr>
<td>Incongruent</td>
</tr>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td><strong>1500 ms</strong></td>
</tr>
<tr>
<td>Congruent</td>
</tr>
<tr>
<td>Incongruent</td>
</tr>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td><strong>Model of Best Fit</strong></td>
</tr>
<tr>
<td>Reaction Time = 0.79(Number Magnitude) + 429**</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
</tr>
<tr>
<td><strong>120 ms</strong></td>
</tr>
<tr>
<td>Congruent</td>
</tr>
<tr>
<td>Incongruent</td>
</tr>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td><strong>240 ms</strong></td>
</tr>
<tr>
<td>Congruent</td>
</tr>
<tr>
<td>Incongruent</td>
</tr>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td><strong>1500 ms</strong></td>
</tr>
<tr>
<td>Congruent</td>
</tr>
<tr>
<td>Incongruent</td>
</tr>
<tr>
<td>Neutral</td>
</tr>
<tr>
<td><strong>Model of Best Fit</strong></td>
</tr>
<tr>
<td>Reaction Time = 0.48**(Number Magnitude) + 443**</td>
</tr>
</tbody>
</table>

*Note. *$p < 0.05$, two-tailed. **$p < 0.01$, two-tailed.*

The results again supported the previous findings of an increase in reaction time with number magnitude. Strong positive correlations were present across all prime-target relationships, over the extensive range of magnitudes covered in the
multiplication condition. Furthermore, number magnitude was shown to be a significant predictor of reaction time for this condition. For addition however, the association between number magnitude and reaction time was only evident at the longest SOA and number magnitude failed to reliably predict reaction time. The addition condition nonetheless, covered a much smaller range of magnitudes than that covered by the multiplication condition. Both models were thus employed to compute predicted reaction times scores for their respective operations. Following this, residual reaction time scores were calculated by subtracting the predicted reaction times from the observed ones. Mean residual reaction time scores were then produced for each of the small and large problem sizes, for all participants, by averaging the residual reaction times for the six smallest and the six largest problems, respectively. Analyses were then performed independently on both the raw and residual reaction time subsets. Both sets of data generally produced the same effects and so, only the residual analysis is reported here. The data for this analysis is presented in Table 3.
Table 3.
Mean Residual Scores (ms) and Standard Deviations (in parentheses) for all Prime-Target Relationships as a Function of SOA, Operation and Problem Size.

<table>
<thead>
<tr>
<th></th>
<th>SOA</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>120 ms</td>
</tr>
<tr>
<td><strong>Addition</strong></td>
<td></td>
</tr>
<tr>
<td>Small Congruent</td>
<td>0 (54.1)</td>
</tr>
<tr>
<td>Small Incongruent</td>
<td>12 (47.4)</td>
</tr>
<tr>
<td>Small Neutral</td>
<td>-7 (41.5)</td>
</tr>
<tr>
<td>Large Congruent</td>
<td>4 (51.0)</td>
</tr>
<tr>
<td>Large Incongruent</td>
<td>5 (51.1)</td>
</tr>
<tr>
<td>Large Neutral</td>
<td>1 (50.1)</td>
</tr>
<tr>
<td><strong>Multiplication</strong></td>
<td></td>
</tr>
<tr>
<td>Small Congruent</td>
<td>5 (49.1)</td>
</tr>
<tr>
<td>Small Incongruent</td>
<td>16 (51.9)</td>
</tr>
<tr>
<td>Small Neutral</td>
<td>4 (43.2)</td>
</tr>
<tr>
<td>Large Congruent</td>
<td>7 (53.7)</td>
</tr>
<tr>
<td>Large Incongruent</td>
<td>2 (45.0)</td>
</tr>
<tr>
<td>Large Neutral</td>
<td>1 (56.6)</td>
</tr>
</tbody>
</table>

A repeated measures ANOVA, including SOA, size and prime-target relationship was undertaken using the residual reaction time subset for each of the addition and multiplication operations. For the multiplication operation, significant main effects of SOA ($F(1.6, 60.9) = 8.2, MSe = 2598.2, p = 0.001$) and prime-target relationship ($F(2, 74) = 14.5, MSe = 780.6, p < 0.001$) and a significant interaction between SOA and prime-target relationship ($F(4, 148) = 5.4, MSe = 680.8, p < 0.001$) were again evident. Additionally, a significant interaction between size and prime-target relationship was found ($F(1.6, 58.6) = 5.2, p = 0.014$). Figure 3, showing the facilitation and inhibition evident in the residual reaction time subset for small and large problems, illustrates this interaction.
Fig. 3 Showing the facilitation and inhibition evident in residual reaction time scores (ms) for small and large problems i.e., after adjustment for target magnitude. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects.

For small problems, the facilitation to congruent targets \((t = 3.7, \, df = 37, \, p = 0.001)\) and inhibition to incongruent targets \((t = 3.0, \, df = 37, \, p = 0.005)\) both reached significance. Additionally, responses to small incongruent targets were significantly inhibited in comparison to large incongruent targets \((t = 2.5, \, df = 37, \, p = 0.016)\). No significant facilitation or inhibition was observed in response to large problems.

Similarly, in the addition condition, significant main effects of SOA \((F(1.8, \, 65.3) = 11.0, \, MSe = 2085.4, \, p < 0.001)\) and prime-target relationship \((F(2, \, 74) = 14.9, \, MSe = 819.5, \, p < 0.001)\) and a significant interaction between SOA and prime-target relationship \((F(3.7, \, 137.7) = 4.7, \, MSe = 671.6, \, p = 0.002)\) were again found. The interaction between size and prime target relationship approached but did not quite reach significance \((F(2, \, 74) = 3.0, \, MSe = 852.6, \, p = 0.058)\). Planned
comparisons showed that for small problems, the facilitation to congruent targets approached significance \((t = 1.9, df = 37, p = 0.061)\), whilst the inhibition to incongruent targets was highly significant \((t = 3.1, df = 37, p = 0.004)\). In contrast, for large problems, the facilitation reached significance \((t = 3.0, df = 37, p = 0.005)\), whilst no significant inhibition was observed in the data. No significant differences in the levels of facilitation and inhibition for small and large problems were observed between operations.

No interactions between SOA, size and prime-target relationship were found for either operation. Nevertheless, in view of the interest in the present study in facilitation and inhibition changes over time, planned comparisons were undertaken. With such large numbers of comparisons to be made (i.e., 6 comparisons at each problem size for each operation), a Bonferroni adjustment was used to reduce the alpha level to a more conservative level of 0.004 (i.e., \(0.05/12 = 0.004\)). In the multiplication condition, at the long SOA, significant facilitation of 25 ms was observed to small congruent targets \((t = 4.6, df = 37, p < 0.001)\) and 16 ms to large congruent targets \((t = 3.1, df = 37, p = 0.004)\). Inhibition of 13 ms to small incongruent targets approached significance at the short SOA \((t = 2.4, df = 37, p = 0.022)\). In the addition condition, at the long SOA, significant facilitation of 23 ms to small congruent targets was observed \((t = 3.8, df = 37, p < 0.001)\). Significant facilitation of 21 ms to large congruent targets occurred only at the medium SOA \((t = 3.1, df = 37, p = 0.003)\). Responses to small incongruent targets were significantly inhibited by 19 ms at the short SOA \((t = 5.0, df = 37, p < 0.001)\). Inhibition in responding to these targets of 16 ms approached significance at the medium SOA \((t = 2.2, df = 37, p = 0.031)\). No other significant effects were observed in the data.
although the earlier findings of constant inhibition and increasing facilitation over time were again supported.

In summary, when collapsed across SOA, responses to the small congruent multiplication condition were markedly facilitated in comparison to the small neutral condition. This facilitation was accompanied by significant interference to small incongruent targets, suggesting that in comparison to large problems, simple exposure to small problems led to the obligatory activation of their correct solution in memory. The results for the addition operation were similar, although less convincing. For example, although facilitation to small congruent targets approached significance, significant facilitation was observed for large incongruent targets only. As in the multiplication condition, the inhibition to small incongruent targets was highly significant and no inhibition was observed to large incongruent targets. Finally, inclusion of the SOA variable in analysis showed that the observed facilitatory effects occurred largely at the longest SOA for both problem sizes, whilst the inhibitory effects occurred only for the small problems at the shortest SOAs. Caution is advised in the interpretation of this data however, as the preceding adjustment for the effects of magnitude depended on the assumption that target magnitude is statistically additive with priming effects and SOA. Additionally, as noted previously, the present analysis involved only a subset of the data, possibly making it less reliable than the overall analyses reported earlier.

4. Discussion

The aim of the present study was to assess the influence of automatic processing on simple arithmetic performance. The overall results of a priming procedure employing a naming task revealed significant facilitation and inhibition effects consistent with those found in previous arithmetic and word-priming research.
That is, significant facilitation emerged at the 240 ms SOA and increased significantly over time, whilst inhibition was found at the two shortest SOAs and remained constant over time. Additionally, individual differences in multiplication performance were found, with highly skilled arithmeticians demonstrating stronger and more reliable advantages in naming at the 240 and 1500 ms SOAs. Increased facilitation and a significant inhibitory effect at the long SOA for this group indicated that they were able to apply their fact knowledge strategically to speed processing in the naming task. In contrast, advantages in performance were indicated for both groups at the 240 and 1500 ms SOAs in the addition condition. Finally, significant facilitation to small congruent targets in the multiplication condition was accompanied by significant costs in performance to small incongruent targets. Similar results were found in the addition condition, with responses to small incongruent targets found to be significantly inhibited and the level of facilitation to small congruent targets approaching significance. Facilitation to large congruent targets was evident only in the addition condition and no significant inhibition was observed for large incongruent targets in either operation.

A number of observations stem from the overall results. Firstly, the finding of significant facilitation to congruent targets at the 240 ms SOA is consistent with the notion that exposure to simple arithmetic problems results in the automatic activation of their correct solution in memory. Three main factors support this conclusion. Firstly, at the 240 ms SOA, the time period between the onset of the prime and the presentation of the target is too brief to permit conscious awareness and strategic processing of the prime before exposure to the target (Velmans, 1999). Secondly, if the benefit to congruent targets had resulted from processing that occurred after exposure to the target, then facilitation should also have been observed
at the shortest SOA. Finally, the use of a target naming procedure ensured that intentional processing of the prime and calculation were irrelevant to performance of the task. The function describing the time course of facilitation is thus indicative of the operation of an automatic spreading activation mechanism that arises at the 240 ms SOA and leads to marked facilitation at the long SOA, consistent with the use of expectancy (Neely, 1991). The results of the present study, therefore, improve on the findings of earlier arithmetic studies demonstrating obligatory activation and in doing so, highlight the utility of the word-priming procedure in the investigation of automatic processing in arithmetic research.

The second observation stemming from the present findings is that the facilitatory and inhibitory effects obtained in the naming task result from the operation of two independent mechanisms. Differences in the time course of the facilitation and inhibition functions support this notion. For instance, in contrast to the path described by the facilitation function, the inhibition function emerged at the shortest SOA and, regardless of changes in facilitation, remained constant over time. As noted earlier, SOAs of 120 and 240 ms are too short to allow for strategic processing to either speed or inhibit performance to the target. This, coupled with the finding of no increase in the level of inhibition at the long SOA, suggests that it must have resulted from processing that occurred after presentation of the target.

What mechanism is responsible for this inhibition? One possible explanation is that it arises due to a process of selective inhibition. Outlined in Tipper, Weaver and Houghton (1994), such a process in the context of the present study begins with exposure to a problem (i.e., the prime), which elicits a verbal response code for its correct solution. This response code then directly competes with the verbal naming response required to the incongruent target, thereby leading to inhibition. Support for
the operation of such a mechanism arises from the finding of significant inhibition in high skilled performance across all SOAs in the multiplication condition i.e., an operation that conventionally relies on verbal rote learning. However, on the basis of such an explanation, a comparable advantage in response time to the congruent condition might also be expected, a prediction that was not supported at the 120 ms SOA in the present study.

An alternative explanation is that the participants performed a self-regulatory validity check on their responses before vocalisation (see Siegler, 1988, for a discussion of similar mechanisms). That is, after exposure to the target and shortly before responding, the participants may have quickly compared the target to the correct solution evoked from memory. In the incongruent condition, in which the two did not match, this in turn would have led to hesitation in responding. Again, the finding of significant inhibition at all SOAs in the multiplication condition and at the shortest SOA in the addition condition for the high skilled group (who could be expected to show a greater tendency to engage in such a process) supports this conclusion. Additionally, such an explanation fits well with the 120 ms SOA findings in that it does not predict an advantage in response times to congruent targets. Furthermore, the notion of an obligatory self-regulatory mechanism operating toward exactness in arithmetic performance is compatible with the importance that is placed on accuracy in computation both in learning environments and in every day life (Smith-Chant & LeFevre, 2003). Certainly, the workings of such a mechanism, even at a voluntary level, could be seen to complement explanations of arithmetic performance such as Siegler and Jenkins (1989) Distribution of Associations model.
A third observation stemming from the overall analyses, relates to the clear parallels that can be drawn between performance in the present study and those investigating performance to associatively related word primes and targets (e.g., *rake* and *leaf*; Neely, 1991). This finding adds support to the notion that number knowledge may be represented in a similar form to word knowledge and accessed through similar mechanisms (Ashcraft, 1992; Dehaene, 1992; LeFevre et al., 1988). An assumption of this kind appears reasonable when one considers the mutual reliance of the two knowledge domains on the use of visual, written symbols in education, the fundamental dependence of numerical knowledge on language, and the use of verbal, mechanical word repetition in learning simple multiplication facts (Ashcraft, 1992). Yet, unlike word knowledge, arithmetic problems have only one correct solution and dozens of other relationships with differing degrees of association, built upon 10 basic symbols (Anderson, 1983). As such, the knowledge gained from the study of simple arithmetic performance may provide a valuable benchmark by which to compare information on word knowledge (Campbell & Clark, 1989).

Finally, in relation to the overall analysis, it is noteworthy that the average facilitation and inhibition effects observed in the present study are smaller than those evidenced in the previous arithmetic research. For example, the average levels of facilitation and inhibition observed in Ashcraft et al’s (1992) study were 30 ms and 75 ms, respectively. This finding possibly reflects differences in procedural and task requirements between the two studies, with the effects observed in Ashcraft et al’s verification study possibly enhanced due to prior exposure to, and activation of solutions, in memory. Additionally, it is noteworthy that a common finding in the word priming literature is that naming tasks produce smaller facilitatory effects than
lexical decision tasks (Neely, 1991). Nonetheless, the use of a naming task ensured that such factors as decision-induced attentional processing, retrieval of arithmetic solutions, and calculation were irrelevant to the task, thereby strengthening the present conclusions in relation to automaticity.

The findings of the arithmetic fluency analysis however, suggest that these conclusions need to be further qualified. For example, performance in the addition condition indicated automaticity in processing, regardless of fluency (although, the level of facilitation did not quite reach significance for the low skilled group at the 240 ms SOA; \( p = 0.051 \)). In the multiplication condition high skilled performance followed a similar pattern to that observed in the addition condition although again, it just failed to reach significance at the 240 ms SOA \( (p = 0.060) \). In contrast, low skilled facilitation at this SOA did not even approach significance, revealing no advantage in exposure to congruent targets whatsoever. In fact, only at a lengthy SOA did any evidence of an advantage in processing begin to emerge for this group and even then it was not strong. This in turn, contrasted with the performance of the high skilled group, who appeared able to use this initial advantage and apply their knowledge to speed processing in the congruent condition at the long SOA.

The results of the present study therefore revealed between group differences in multiplication performance and similarities in addition performance. In the former case, the findings possibly reflect a greater sensitivity of the word priming methodology to individual differences in multiplication performance. As noted earlier, in contrast to the addition operation, the multiplication operation is usually rote learnt and hinges on the development of verbal associations between words (Ashcraft, 1992; Butterworth, 1999; Dehaene, 1992). Consequently, high skilled participants who develop strong associations between problems and their correct
solutions may be more likely to stand apart from their counterparts in the present naming task. In the latter case, the findings are at odds with the results of a study by LeFevre and Kulak (1994) who found a significant between group difference in addition performance, with only skilled subjects demonstrating significant effects of obligatory activation in a matching task. In light of this, a replication of the present study employing a larger sample size and more distinctive groups than is accomplished via median split may be useful.

Alternatively, it may be the case that between group similarities are in fact, fundamental to addition performance. For instance, in most Western cultures, even before schooling, children begin to develop an understanding of simple addition through the use of finger counting (Butterworth, 1999). Once at school, they are formally taught addition through counting procedures and the use of concrete visual representations of numerosity involving small numbers (e.g., □□ + □ two squares added to one square equals three squares) (Butterworth, 1999). Only after this, is an understanding of the multiplication operation developed, again based on the notion of repeated addition (Butterworth, 1999; Swan, 1990). Multiplication facts, dealing with much greater quantities that are not easily visually or mentally portrayed, are then gradually rote learnt up to the age of approximately nine years (Butterworth, 1999). Thus, for the addition operation, earlier and greater exposure to simpler and more meaningful constructs may enable even the least fluent individual a comparable level of automaticity to other more skilled arithmeticians.

Tied in with the above explanation of between group similarities in addition performance is the notion that a central factor in both learning and performance is problem size. This notion is supported in the present study by differences in the patterns of facilitation and inhibition observed for each problem size between the
addition and multiplication operations. For example, in the multiplication condition, significant facilitation was found to small congruent targets (i.e., problems with small operands and their correct solutions) only. However, in the addition condition, facilitation to small congruent targets approached significance and facilitation to large congruent targets (i.e., problems consisting of large operands yet still involving much smaller target magnitudes than those in the multiplication condition) also reached significance. Moreover, a significant level of inhibition was found to small incongruent targets only in both operations. Thus, small correct solutions appear to be accessed from memory more quickly than large correct solutions and small incorrect solutions lead to greater levels of interference in naming. The results of the present study are therefore generally consistent with the notion that problems of differing size are processed differently (Ashcraft et al., 1992; Campbell, 1987, 1991; Koshmider & Ashcraft, 1991; LeFevre et al., 1996a, 1996b).

Finally, in relation to the problem size analyses, it is worth noting that the advantages to small congruent targets were not evidenced in the automatic processing conditions, instead occurring only at the long SOA in both operations. Furthermore, significant inhibition was observed to small incongruent targets in the short addition condition and inhibition to these targets approached significance in both the 240 ms addition condition and the 120 ms multiplication condition. These findings again suggest that two different mechanisms are responsible for the facilitation and inhibition observed in the data and provide support to the notion of the operation of an obligatory self-regulatory mechanism. However, the finding of an advantage to large congruent targets at the 240 ms SOA in the addition condition that did not persist to the 1500 ms SOA condition and that occurred in the absence of any comparable facilitation to small congruent targets, is difficult to explain. One
possibility is that this finding somehow reflects underlying differences in performance between fluency groups. Unfortunately, the data were too inconsistent in analysis at this level, suggesting again the use of greater numbers and better defined groups in testing.

A number of interesting educational implications stem from the present results. The first of these occurs in relation to the mathematical reform that has occurred over the last three decades in most major industrialised countries throughout the world. This reform, encouraged by rapid technological advance (i.e., with the inception of calculators) and shifting theoretical paradigms (such as the notion that children should think before they ‘fact;’ Lochhead, 1991, pp. 77) has seen the traditional role of rote learning be greatly undermined (Willis, 1990; Willoughby, 2000; Woodward & Montague, 2002). Such a situation is disturbing in light of the finding that greater access to multiplication facts and the ability to then apply this knowledge distinguishes performance between groups. Advantages of this kind possibly serve to free cognitive space and extend the number of functions that can be performed at once (Campbell, 1987; Koshmider & Ashcraft, 1991; Reed, 1988; Willoughby, 2000). This is important when one considers that students and programs, especially at the primary level, are still examined using standardised tests that are often speeded and rely on pencil and paper skills (Tsuruda, 1998). Even at the high school level, the ability to quickly approximate a solution and be confident that an answer obtained on a calculator is accurate can be seen to be an advantage in an exam situation (Meissner, 1980).

Secondly, the finding that in the addition condition, regardless of fluency, there was facilitation to the correct condition over both the 240 and 1500 ms SOA’s, raises serious doubts about the utility of verification tasks in the assessment of
addition competence (Campbell, 1987). As Campbell (1987) notes, with the presented solution to a given problem already priming the correct answer in memory, the probability of subsequently retrieving an error may be reduced. Consequently, the individual’s performance in a verification task may be an overestimate of their ability in a normal production task.

In summary, the present study demonstrated the utility of the word-priming paradigm (with naming task) in accessing facilitatory and inhibitory mechanisms associated with simple arithmetic performance. More specifically, it showed that brief exposure to simple addition problems leads to the automatic activation of correct solutions in memory in high skilled individuals and, most likely, in individuals of low skill also. Exposure to multiplication problems however, revealed individual differences in performance, with facilitatory and inhibitory effects at the longer SOA’s indicating that only the high skilled arithmeticians applied their multiplication fact knowledge toward superior performance in the naming task. Furthermore, the results indicated significant advantages in performance to small problems that are more frequently encountered in educational and natural settings than larger ones. The results of the present study therefore, demonstrate the need for further elaboration and revision of network retrieval models to account for differences between individuals, between problem sizes and between operations (LeFevre & Kulak, 1994; Lefevre et al., 1996). Furthermore, they highlight the importance of the continued use of the more traditional rote learning of simple arithmetic facts in mathematical education.

References


## Appendix A

### Prime Sets and Correct and Incorrect Targets for Multiplication Operation

<table>
<thead>
<tr>
<th>Set 1</th>
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### Prime Sets and Correct and Incorrect Targets for Addition Operation

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2.2. Individual Differences in Automaticity.

Intuitively, it would seem reasonable that like other cognitive abilities, persons having greater exposure to simple arithmetic problems and having additional practice at solution retrieval would become more fluent in performance (LeFevre & Kulak, 1994). The first study in this series of investigations appeared to support the notion that individual differences in automaticity exist. However, in the first study, the investigation of individual differences was subsidiary to the investigation of automaticity in fact retrieval. Consequently, the original sample, which comprised individuals of all abilities, was divided into skill groups using a median split and extreme groups were not chosen. The main aim of the second study was thus to address this shortcoming by re-examining individual differences in priming effects using a larger sample size and more distinguishable skill groups.

Additionally, the results of the first study revealed an influence of problem size on priming. Small problems produced greater facilitatory and inhibitory effects than large problems in the multiplication condition and greater inhibitory effects in the addition condition. Nevertheless, these results were produced from analyses that were collapsed across SOA. Analyses undertaken at each level of SOA produced somewhat inconsistent results. Thus, a secondary aim of the second study was to provide further investigation into the influence of problem size on processing.

Furthermore, in the first study, the Arabic digit numerical symbol for zero was used in the neutral prime condition (i.e., 0 + 0 and 0 x 0). However, given that there is a correct solution to these stimuli (i.e., 0), it can be argued that it is not necessarily semantically neutral. In the context of the initial study, the use of these stimuli may thus, have led to exaggerated facilitatory effects and reduced inhibitory effects. Having considered the original data, a further aim of the second study was
therefore, to examine the utility and validity of a set of different neutral condition stimuli (i.e., \( X + Y \) and \( X \times Y \)).

Finally, a short SOA condition of 300 ms (cf. the 120 and 240 ms SOAs) and a long SOA of 1000 ms were employed in this study. In the former case, the 300 ms SOA was chosen in an attempt to determine whether activation of simple facts from memory occurs for less skilled individuals but is delayed in comparison to high skilled individuals. Additionally, this enabled the examination of the effectiveness of a lengthier short SOA condition for demonstrating priming effects, with the subsequent study in this series of investigations utilising lengthier number word problems as primes. In the latter case, the 1000 ms SOA condition was selected in order to reduce the overall time needed to complete the experiment and hence, to guard against any effects due to fatigue.

Individual differences in target naming latencies were found. Priming using both multiplication and addition problems led to greater activation and earlier access to correct solutions for high skilled individuals. Moreover, significant inhibition in naming incongruent targets and an advantage in strategic access to multiplication solutions was found for the high skilled group alone. Individual differences in processing due to problem size and operation were also indicated.
Abstract

This study investigated individual differences in the ability to automatically access simple addition and multiplication facts from memory. It employed a target-naming task and a priming procedure similar to that utilized in the single word semantic-priming paradigm. In each trial, participants were first presented with a single digit arithmetic problem (e.g., 6 + 8) and were then presented with a target that was either congruent (e.g., 14) or incongruent (e.g., 17) with this prime. Response times for congruent and incongruent conditions were then compared to a neutral condition (e.g., X + Y, with target 14). For the high skilled group, significant facilitation in naming congruent multiplication and addition targets was found at SOAs of 300 and 1000 ms. In contrast, for the low skilled group, facilitation in naming congruent targets was only observed at 1000 ms. Significant inhibition in
naming incongruent multiplication and addition targets at 300 ms, and addition
targets at 1000 ms, was found for the high skilled group alone. This advantage in
access to simple facts for the high skilled group was then further supported in a
problem size analysis that revealed individual differences in access to small and large
problems that varied by operation. These findings support the notion that individual
differences in arithmetic skill stem from automaticity in solution retrieval and
additionally, that they also derive from strategic access to multiplication solutions.

PsycINFO classification: 2343; 2346
Key Words: Simple Arithmetic, Automaticity, Individual Differences, Priming,
Fluency
1. Introduction

Until recently, it was widely assumed that the majority of adults reached asymptotic performance on the retrieval of simple arithmetic facts such that they directly retrieved solutions from memory, most of the time (Ashcraft, 1992; Geary & Wiley, 1991; LeFevre et al., 1996; LeFevre & Kulak, 1994; LeFevre, Sadesky & Bisanz, 1996). However, a growing body of research suggests that the use of various solution procedures other than direct fact retrieval (e.g., counting or transformation procedures: \(9 + 7 = 9 + 1 + 6\)) may be far more widespread than was first considered and that this may vary with arithmetic fluency (LeFevre et al., 1996a; 1996b). That is, those who are fluent arithmeticians are assumed to be more likely to rely on automatic access to simple arithmetic facts than to rely on alternative solution procedures (LeFevre & Kulak, 1994).

Support for the influence of fluency on access to simple arithmetic facts is provided in two main studies by LeFevre and colleagues. In these studies, accessibility was indexed by unintentional sum activation produced in the performance of a number-matching task. Participants were first presented with a pair of numbers (e.g., \(3 + 6\)) and then following a short inter-stimulus interval, were required to decide if a target number (e.g., 3) was one of the original numbers presented. In the first study, by LeFevre, Kulak and Bisanz (1991), the presentation of the sum (i.e., 9) to a high skilled group led to significantly slowed processing in comparison to a neutral prime, at an SOA of 80 ms. In contrast, significant interference to the sum for low skilled participants was observed only at a lengthier SOA of 120 ms. In the second study, by LeFevre and Kulak (1994), the results again revealed significantly slower performance by high skilled participants in sum as opposed to neutral trials. This occurred at SOAs of 40 and 60 ms in the first
experiment and 60 ms in the second experiment. For the low skilled group, small non-significant interference effects were observed that were again, delayed in comparison to the high skilled group, being found at somewhat longer SOAs of 120 and 160 ms, respectively. Obligatory activation therefore appeared greater for high skilled individuals and occurred earlier in the processing sequence than it did for low skilled individuals. These findings, according to LeFevre and Kulak (1994), supported the hypothesis that individual differences in arithmetic skill may originate in automaticity of fact retrieval. Unfortunately, a comparable study involving the multiplication operation was not undertaken.

Further support for the notion that individuals with stronger arithmetic fluency are more likely to rely on direct solution retrieval stems from a series of investigations employing self report measures. In these investigations (Hecht, 1999; LeFevre et al., 1996a; 1996b; see also Geary & Wiley, 1991), participants were first required to solve simple addition or multiplication problems and then to report on a trial by trial basis the strategy that they employed to obtain their solution. The results were consistent across all studies in showing a significant positive correlation between a high level of fluency and the reported use of direct retrieval. Moreover, in the studies conducted by LeFevre and colleagues the results indicated that less skilled participants showed greater effects of problem size i.e., as problem size increased, solution latencies increased more for these individuals than they did for high skilled individuals. This, according to LeFevre et al (1996a), was a direct consequence of less skilled participants relying on solution strategies other than direct retrieval (e.g., counting or transformation procedures: $4 + 7 = 7 + 3 + 1$).

However, the veridicality of self report measures has been called into question due to the possibility that the instructions employed within this method may
lead to reactivity, which in turn, may be influenced by fluency (Kirk & Ashcraft, 2001; Smith-Chant & LeFevre, 2003). Support for individual differences in reactivity was provided in an investigation by Smith-Chant and LeFevre (2003). Low skilled individuals were found to be more affected by speed (vs. accuracy) biasing instructions and responded more slowly and accurately on large and very large problems when asked to provide self-reports of their solution procedures. High skilled participants, on the other hand, revealed smaller effects due to biasing instructions and were minimally reactive to the requirement to provide self-reports.

More recently, Jackson and Coney (2005) offered an alternative approach to the investigation of automaticity in multiplication and addition performance by employing numerical stimuli in a priming procedure analogous to that utilised in the single word semantic priming paradigm. Participants were first presented with either of two prime types: one representing a single digit arithmetic problem (e.g., 6 + 8), the other employed as a neutral condition (e.g., 0 + 0). Following a given SOA (i.e., of 120, 240 or 1000 ms), they were then presented with a target that was either congruent (e.g., 14) or incongruent (e.g., 17) with the prime. In the addition, 240 ms SOA condition, for high skilled participants, the time taken to name congruent targets was significantly facilitated in comparison to the neutral condition. For the low skilled group however, facilitation merely approached significance. At the longest SOA, facilitation was significant for both groups but appeared greater for the high skilled group. The trend in the addition data identified in the Jackson and Coney (2005) study was, therefore, generally consistent with the earlier findings of the LeFevre et al. (1991) and LeFevre and Kulak (1994) studies in revealing earlier and greater levels of activation for high skilled participants. Furthermore, this trend was also evident in the multiplication data, with the level of facilitation approaching
significance at the 240 ms SOA for the high skilled group, and reaching significance at 1000 ms. For the low skilled group, facilitation was not evident at the short SOA and barely reached significance at the long SOA.

In addition to the facilitatory effects, the Jackson and Coney (2005) study also revealed significant inhibition in naming incongruent targets for the high skilled group only. This was evident across all SOAs in the multiplication condition and at the shortest SOA in the addition condition. Furthermore, the inhibitory effect found at the long SOA was quite large and appeared to have increased in conjunction with an increase in facilitation, thereby, suggesting the use of expectancy in naming performance for this group alone. Unfortunately, the increase in inhibition at this SOA just failed to reach significance, a result possibly reflecting the use of a high skilled group who were not high enough in skill to be easily distinguished from the low skilled group. The investigation of individual differences in this study was a subsidiary aim. Hence, the sample was divided into skill groups on the basis of a median split and extreme groups were not selected.

The main aim of the present study was thus to re-examine individual differences in priming effects by replicating the earlier study using a larger sample size and more distinguishable skill groups. Additionally, unlike the earlier study, a lengthier short SOA condition of 300 ms was employed in an attempt to determine whether activation of multiplication facts also occurs for less skilled individuals (not previously found at the 240 ms SOA), but is delayed in comparison to that of high skilled individuals (LeFevre & Kulak, 1994). This study also employed neutral stimuli that differed from those used in the earlier Jackson and Coney (2005) study (i.e., $X + Y$ and $X \times Y$). This was done for two main reasons. Firstly, previous research indicates that the processing of zero stimuli may occur more slowly than
other numerical stimuli and therefore the use of the \(0 + 0\) and \(0 \times 0\) neutral stimuli in the earlier study potentially exaggerated the facilitatory effects that were identified (Stazyk, Ashcraft & Hamann, 1982). Secondly, the new neutral stimuli were employed to guard against artificial slowing of responses in this condition due to the incongruence between the prime and the target (e.g., \(0 + 0\) presented with 14). Finally, in view of the possibility that less skilled individuals show greater problem size effects because of their reliance on solution procedures other than direct retrieval (LeFevre et al., 1996a, 1996b) a second objective of the present study was to assess individual differences in access to small and large facts.

2. Method

2.1 Participants

Fifty-four undergraduate psychology and mathematics students, including 9 males and 45 females, from Murdoch University participated in this study. Participants either received credit toward partial fulfilment of course requirements or were reimbursed $10 for their time. The participants’ ages ranged from 17 to 52 years, with a mean age of 26.

2.2 Design and stimulus materials

Three within group variables were examined. The first of these determined the arithmetic operation i.e., addition or multiplication. The second variable incorporated three prime-target relationships, including congruent (e.g., \(2 + 4 = 6\)), incongruent (\(2 + 4 = 9\)) and neutral (\(X + Y = 6\)) conditions. The final within group variable was SOA with two levels: 300 ms and 1000 ms.

Two sets of primes originally utilised in the Jackson and Coney (2005) study were employed for each of the two operations (see Appendix A). The first set for
each operation comprised 18 simple arithmetic facts selected from the 2s through 9s matrices (e.g., $2 + 3$). The second set consisted of the reverse operand placement equivalents of the first set ($3 + 2$).

As in the previous research, arithmetic ties (e.g., $3 + 3$ and $3 \times 3$) were excluded from use as primes, as these problems have been shown to be solved more quickly than others (LeFevre et al., 1988). Additionally, to ensure that each prime set was balanced in terms of operand placement; half of the arithmetic facts were produced so that the smaller of the two operands in each problem was placed on the left-hand side and half with the smaller operands on the right hand side. Finally, each stimulus set consisted of six smaller problems (i.e., with both operands of a magnitude less than or equal to five; e.g., $2 + 3$), six larger problems (operands greater than or equal to six; e.g., $8 + 9$), and six of mixed magnitude (e.g., $2 + 9$). This enabled testing for the presence of the problem size effect.

The target sets for each of the congruent, incongruent and neutral conditions consisted solely of the correct solutions to the 18 simple arithmetic facts investigated in this study. These targets were then simply paired with an alternative problem for the incongruent condition. To guard against split effects in the multiplication condition, incongruent targets were paired with problems so that they differed by at least 16 from the correct solutions to these problems. For the addition condition, incongruent targets differed by at least three from the correct solutions. Further constraints on the incongruent target sets were included to address possible confounding relationships between the prime and the target. Firstly, incongruent targets were not permitted to be one of the operands or the numbers plus or minus one from those used in the prime. Secondly, where possible, multiples or factors of the operands and number series relations were excluded. Finally, incongruent targets were paired with primes in such a way that they could not be the correct solution.
using a different operation, a double-digit number containing the operand, or a number containing the correct solution (i.e., if the correct solution was 7, then numbers such as 17 and 70 were also excluded).

Neutral conditions have been useful in assessing facilitation and inhibition and hence distinguishing automatic from conscious processing in word priming research but to date have not been widely utilised in the study of arithmetic (Neely, 1991). The neutral condition stimuli (i.e., X + Y for the addition condition and X x Y for the multiplication condition) were thus chosen in accordance with three main recommendations outlined in a review of the word priming literature by Neely (1991). The first of these was that neutral primes should be equated with other primes in relation to their value as a warning signal that a target will soon appear. Secondly, neutral primes should be unassociated to the target so that they are a neutral baseline by which to assess spreading activation between related stimuli. Lastly, in order to provide a baseline by which to compare expectancy effects, neutral primes should not offer any information as to the semantic nature of the target to follow. In the present study the prime X + Y can be likened perceptually to the other numerical primes such as 2 + 3, with both consisting of two common individual symbols separated by an arithmetic operator. Additionally, with X and Y often used in the place of numbers to denote separate unknown quantities, the recommendations against any association between prime and target, and any indication of the semantic nature of the target, were also met. Unlike the previous Jackson and Coney (2005) study that employed a 0 + 0 and 0 x 0 neutral condition, the expectation of the target 0 being presented was avoided.

2.3 Psychometric testing
The arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) was used to identify two arithmetic fluency groups. This test incorporates 60 arithmetic problems that variously include the addition, subtraction, division and multiplication of single, two and three digit numbers (ACER, 1984). The participants were instructed that they had five minutes to answer as many questions as accurately as they could. They were instructed to begin with the first question and without omitting any, to work through each in turn (ACER, 1984). Rough working out could be undertaken anywhere on the page and participants were advised that if they finished the first column that they should immediately go onto the second one (ACER, 1984).

Participants were placed into high and low skilled groups based on the number of problems that they solved correctly. Twenty eight participants formed the low skilled group, with a mean correct score of 12 (SD = 1.73). This score corresponded to a percentile rank of 0 in a normative sample of 124 tertiary graduate/diplomates, 7 in a sample of 973 administrative officer or administrative assistant applicants, and 2 in a sample of 1270 bank trainees (ACER, 1984). Twenty six participants constituted the high skilled group, with a mean correct score of 31 (SD = 5.22). These scores corresponded to a percentile rank of 35 in the sample of tertiary graduate/diplomates, 83 in the administrative applicant sample, and 74 in the bank trainee sample (ACER, 1984).

2.4 Procedure

Participants were individually tested on the computer task in a well-lit cubicle room containing an Amiga 1200 microcomputer, with 1084S monitor. This system controlled stimulus presentation, trial sequencing, timing and data collection. Individual operands within each problem did not exceed dimensions of 5 x 15 mm on
the screen and were separated by 5 mm from the arithmetic operators (i.e., the x or + sign), which did not exceed 5 x 10 mm. Stimuli were presented centrally, white against an amber background. A chin rest stabilised the participant’s head at a viewing distance of 60cm from the screen.

Participants each completed four blocks of 54 experimental trials (i.e., two for each of the addition and multiplication operations corresponding to the two levels of SOA). Addition and multiplication trials were blocked separately so as not to produce cross operation or relatedness errors. Half of the participants started with the addition operation first and half started with multiplication. In the first 300 ms block, of the participants assigned to the addition condition first, half were exposed to addition Set 1, whilst half were exposed to addition Set 2 (see Appendix A). Similarly, half of the participants assigned to the multiplication condition first were exposed to multiplication Set 1, whilst the remaining half were exposed to multiplication Set 2. Participants were then exposed to the exact same set that they saw in the first block in the second 1000 ms block. Repetition of these trials at the longer SOA allowed for a level of familiarity with the stimuli, drawing attention to the prime-target relationship. This process was then repeated in the third and fourth blocks using the operation not tested in the first two blocks. Exposure to individual sets and all stimuli was counterbalanced across participants, with the computer randomly generating the order of presentation of the individual congruent, incongruent and neutral trials within each block.

Before testing, participants were advised to respond both quickly and accurately. Each trial began with the participants focussing their gaze on a 1 x 1 mm blue central fixation dot that was exposed for 600 ms. After a 150 ms period in which the screen remained blank, the prime was presented for 100 ms. The target number
appeared following the given SOA and remained exposed until the participant named the number. A two-second interval separated the participant’s response and the start of the next trial. Participants’ vocal responses were detected using a microphone connected to a headset. The microphone triggered an electronic relay that was interfaced to the computer and stopped a hardware timer. The value of the timer was accurate to 1 millisecond and measured the participant’s vocal reaction time from the onset of the target. Padded ear guards attached to the headset prevented external noise intrusions. The experimental session, including debriefing, lasted approximately 30 minutes.

3. Results

The mean response latency for each participant in each condition was recorded. These data were screened for outliers using an exclusion criterion of +/- 2.5 z-scores. This led to 0.77% of all scores being replaced using mean substitution. The resulting reaction time data are presented in Table 1. Due to the negligible error rates produced in target naming performance they were not considered in the present analysis.
Table 1. Mean Reaction Times (ms) for all Prime-Target Relationships as a Function of SOA, Operation and Fluency.

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<td>505(57)</td>
<td>464(48)</td>
<td>481(48)</td>
</tr>
<tr>
<td>Incongruent</td>
<td>487(47)</td>
<td>509(58)</td>
<td>476(54)</td>
<td>481(48)</td>
</tr>
</tbody>
</table>

*Note. Standard deviation in parentheses.*

These data were initially entered into an overall split plot analysis of variance used to assess the presence of operation differences. A significant main effect was found for operation, with reaction times to addition-related targets found to be 16 ms faster overall than to multiplication-related targets ($F(1, 52) = 10.6; MSe = 4088.6, p = 0.002$). This finding is consistent with operation differences recognised in earlier studies and possibly reflects differences in solution magnitudes between the two operations (ranging from 5 through 17 for addition and 6 through 72 for multiplication) (Jackson & Coney, 2005; Zbrodoff & Logan, 1986). Previous research indicates that it takes longer to perform number naming tasks when numbers are large than when they are small (Brysbaert, 1995; Jackson & Coney, 2005). In view of this difference in processing, the two operations were analysed separately.

3.1 Multiplication analysis
A split plot analysis of variance, involving SOA and prime-target relationship as within group variables and fluency as the between group variable, was used to analyse the multiplication data. Significant main effects were found for all three variables. Firstly, responses to the short SOA condition were 9 ms faster than to the long SOA condition ($F(1, 52) = 4.4; MSe = 1652.2, p = 0.041$). Secondly, responses to the congruent condition were 20 ms faster than to the incongruent condition and 17 ms faster than to the neutral condition ($F(1.8, 91.3) = 39.5; MSe = 357.9, p < 0.001$). Finally, high skilled participants responded 29 ms faster overall than did low skilled participants ($F(1, 52) = 4.9; MSe = 2321.3, p = 0.031$). These main effects were then further qualified by two significant two-way interactions. The first of these occurred between SOA and prime-target relationship ($F(2,104) = 4.7; MSe =348.4, p = 0.011$). Paired sample t-test comparisons involving the short SOA condition revealed significant facilitation (i.e., neutral – congruent) of 10 ms ($t(53) = 3.1, p = 0.003$) that increased to 24 ms at the long SOA ($t(53) = 5.9, p < 0.001$). The overall pattern of performance to multiplication-related targets was thus one of increasing facilitation over time.

The second and more important significant interaction in the context of the present study was that between prime-target relationship and fluency ($F(2, 104) = 10.7; MSe = 314.1, p < 0.001$). Paired sample t-test comparisons involving the low skilled results revealed significant facilitation of 10 ms ($t(27) = 4.6, p < 0.001$) but no inhibition. In contrast, analysis of the high skilled results revealed significant facilitation of 24 ms ($t(25) = 7.7, p < 0.001$) and a 6 ms inhibitory (incongruent - neutral) effect that approached but did not quite reach significance ($t(25) = 1.8, p = 0.088$). The advantage in facilitation for high skilled participants was significantly greater than that observed for low skilled participants ($t(43.9) = 3.9, p < 0.001$).
No significant three-way interaction was observed in the multiplication analysis however, in view of the particular interests of the present study in changes in facilitatory and inhibitory effects over time, planned comparisons between all prime-target relationships were undertaken for each group at both SOA’s. The facilitatory and inhibitory effects for these analyses are presented in Fig. 1.

Fig. 1 Showing facilitatory and inhibitory effects for high and low skilled groups as a function of SOA. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects for each group.

For the high skilled group, facilitation was found to be significant at both the short ($t(25) = 3.3, p = 0.003$) and long ($t(25) = 6.2, p < 0.001$) SOAs, and increased significantly over time ($t(25) = 2.8, p = 0.010$). Significant inhibition was evident at
the short SOA ($t(25) = 2.4, p = 0.022$) only. In contrast, for the low skilled group significant facilitation was observed only at the long SOA ($t(27) = 2.6, p = 0.014$). Thus, no obligatory activation of multiplication facts was identified for the low skilled group using the short 300 ms SOA employed in the present study.

3.2 Addition analysis

A split plot ANOVA was then performed on the addition data. Significant main effects were again found for all three variables. Firstly, a significant main effect of SOA was found, with responses to the 300 ms condition found to be 14 ms faster than to the 1000 ms condition ($F(1, 52) = 6.8; MSe = 2410.8, p = 0.012$). Secondly, a significant main effect of prime-target relationship was found ($F(1.6, 84.4) = 25.4; MSe = 417.7; p < 0.001$). Responses to the congruent condition were facilitated by 12 ms, with responses to the incongruent condition inhibited by 6 ms. Finally, a significant main effect of arithmetic fluency ($F(1, 52) = 8.4, MSe = 13235.9, p = 0.005$) was found. High skilled participants responded 37 ms faster overall than low skilled participants did. This finding was then further qualified by a significant interaction between prime-target relationship and fluency ($F(2, 104) = 4.6, MSe = 338.8, p = 0.013$). For the low skilled group, facilitation of 9 ms approached but did not quite reach significance ($t(27) = 1.9, p = 0.066$) and no inhibition was evident. In contrast, for the high skilled group, significant facilitation of 14 ms ($t(25) = 4.7, p < 0.001$) and inhibition of 11 ms ($t(25) = 4.6, p < 0.001$) was found.

No significant interaction between SOA and prime-target relationship was observed in the data ($F(2, 104) = 1.0; p = 0.364$) and, as in the multiplication analysis, no significant three-way interaction involving fluency was found. Nevertheless, planned comparisons of changes in facilitation and inhibition effects for each group (see Fig. 1) over time were again undertaken. For the high skilled
group, significant facilitation ($t(25) = 3.6, p = 0.002$) and inhibition ($t(25) = 3.4, p = 0.002$) were found at the short SOA. These effects then persisted over time with similar facilitation ($t(25) = 2.6, p = 0.015$) and inhibition ($t(25) = 2.6, p = 0.016$) effects found at the long SOA. The only significant effect observed for the low skilled group was facilitation that again occurred only at the long SOA ($t(27) = 2.8, p = 0.009$).

In summary, the findings of the present study demonstrated individual differences in target naming latencies as a function of arithmetic fluency. Priming using both multiplication and addition problems led to earlier access to correct solutions for high skilled participants than it did for low skilled participants. Moreover, it produced significant inhibition in naming incongruent targets at 300 ms in both the addition and multiplication conditions, and at 1000 ms in the addition condition, for the high skilled group alone. The present results therefore extend the previous findings of the LeFevre et al (1991) and LeFevre and Kulak (1994) studies that demonstrated interference effects in a number matching task, involving the addition operation only.

3.3 Problem size analysis

A subset of the data including reaction times to small and large problems (consisting of operands $\leq 5$ or $> 5$, respectively) was selected for use in determining the influence of problem size on arithmetic processing. These data were initially screened for outliers using a cut off score of +/- 2.5 z-scores. This led to mean substitution of 1.47% of all scores.

With the selection of only a subset of the data in this analysis, a mis-match was created between the solutions in the congruent and incongruent conditions, and between problems and solutions of differing magnitudes. For example, congruent
targets for small multiplication problems ranged between 6 and 20, whilst incongruent targets for these problems largely ranged between 30 and 63 (i.e., except for the incorrect solution ‘6’). Thus, any differences found in direct comparisons between the two problem sizes may have resulted from a confound of target magnitude. In order to remove any confounding influence of this kind, the raw data for all problems within the original stimulus set were entered into regression analyses to first ascertain any effect of magnitude and then to adjust for it in the obtained reaction times. Pearson correlation coefficients for each group and the best fitting model between the mean overall reaction time and number magnitude are presented in Table 2.

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<tr>
<td>Reaction Time</td>
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<tr>
<td>Reaction Time</td>
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*Note: *p < 0.05, two-tailed. **p < 0.01, two-tailed.*
A strong positive correlation between naming latencies and number magnitude was found in the congruent addition condition for the low skilled group only. This finding is consistent with previous research indicating greater increases in solution latencies with problem size for low skilled individuals than for high skilled individuals in addition performance (LeFevre et al., 1996a, 1996b). However, a negative relationship between fluency and the problem size effect was not indicated in the present multiplication data, with similar patterns of correlations found for both fluency groups.

As in the earlier Jackson and Coney (2005) study, both models were employed to compute predicted reaction times scores for their respective operations. Residual reaction time scores were then computed by subtracting observed reaction times from predicted ones. Following this, mean residual reaction time scores were determined for each of the small and large problem sizes, for all participants. These were calculated for each operation by averaging the residual reaction times for the six smallest and the six largest problems. These data were then entered into separate split plot analyses of variance for both the multiplication and addition conditions.

3.3.1 Multiplication analysis

As in the previous analyses, the split plot ANOVA involving the residual multiplication data revealed significant main effects of SOA ($F(1, 52) = 6.3$, $MSe = 2895.0$, $p = 0.015$) and prime-target relationship ($F(2, 104) = 16.8$, $MSe = 1595.6$, $p < 0.001$) and a significant interaction between these two variables ($F(2, 104) = 4.5$; $MSe = 906.8$, $p = 0.014$). Additionally, a significant two-way interaction between size and prime target relationship ($F(2, 104) = 4.4$, $MSe = 1405.7$, $p = 0.015$) and a significant three-way interaction between SOA, size and prime target relationship ($F(1.8, 95.5) = 4.6$, $MSe = 1059.8$, $p = 0.015$) were found. For small problems at the
short SOA, significant facilitation of 13 ms ($t(53) = 2.1, p = 0.037$) and inhibition of 16 ms ($t(53) = 2.2, p = 0.030$) was found. At the long SOA, significant facilitation of 30 ms ($t(53) = 5.5, p < 0.001$) was found. In contrast, for large problems, significant inhibition of 15 ms was observed at the short SOA ($t(53) = 2.4, p = 0.022$) and significant facilitation of 19 ms was observed at the long SOA ($t(53) = 3.0, p = 0.004$).

No four-way interaction involving fluency was found in the data. Nevertheless, in the interest of locating individual differences in facilitatory and inhibitory effects over time, planned paired sample $t$-test comparisons were undertaken at each SOA, for both problem sizes. The facilitatory and inhibitory effects for these comparisons are presented in Fig. 2.

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![Graphs showing facilitation and inhibition effects for small and large multiplication problems at short and long stimulus-onset asynchronies (SOAs).](image-url)
Fig. 2 Showing facilitatory and inhibitory effects for high and low skilled groups as a function of problem size and SOA. The 95% confidence intervals were calculated based on the MSe term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects for each group, at each problem size.

For the high skilled group, in the small problem condition, facilitation approached significance at the 300 ms SOA ($t(25) = 2.0$, $p = 0.052$) and reached significance at the 1000 ms SOA ($t(25) = 5.3$, $p < 0.001$). In the large problem condition, facilitation was observed only at the long SOA ($t(25) = 2.9$, $p = 0.007$).

For the low skilled group, in the small problem condition, significant facilitation was observed at the long SOA ($t(27) = 2.7$, $p = 0.013$) and a marginally significant inhibitory effect was observed at the short SOA ($t(27) = 2.1$, $p = 0.048$). The facilitation found for the high skilled group for small problems at the long SOA was
significantly greater than that observed for the low skilled group ($t(52) = 2.1, p = 0.037$).

### 3.3.2 Addition analysis

The split plot analysis of variance involving the addition data revealed significant main effects of SOA ($F(1, 52) = 9.5, MSe = 3977.9, p = 0.003$), prime-target relationship ($F(2, 104) = 14.9, MSe = 1215.5, p < 0.001$) and fluency ($F(1, 52) = 7.6, MSe = 24775.6, p = 0.008$), and a significant interaction between prime-target relationship and fluency ($F(2, 104) = 4.1, MSe = 1215.5, p = 0.020$). As in the multiplication analysis, a significant interaction between size and prime-target relationship was found ($F(2, 104) = 13.1, MSe = 1107.8, p < 0.001$). In the small problem condition, significant facilitation of 23 ms ($t(53) = 4.8, p < 0.001$) and inhibition of 10 ms ($t(53) = 2.2, p = 0.030$) was found. No significant facilitation or inhibition was observed in the large problem condition.

Planned paired sample $t$-test comparisons examining facilitatory and inhibitory effects for each group were again undertaken at each SOA, for both problem sizes (see Fig. 2). For the high skilled group, in the small problem condition, significant facilitation ($t(25) = 4.1, p < 0.001$) and inhibition ($t(25) = 2.9, p = 0.007$) was observed at the short SOA. These effects again reached significance at the long SOA (with $t(25) = 2.8, p = 0.010$; and $t(25) = 2.7, p = 0.011$, respectively). In contrast, for the low skilled group, facilitation reached significance at the long SOA only ($t(27) = 3.4, p = 0.002$). In the large problem condition, no facilitatory or inhibitory effects were observed for either group.

In summary, the problem size analysis revealed differences in access to solutions to small and large problems, as a function of arithmetic fluency. In the
multiplication condition at the long SOA, facilitation observed in the small problem condition for the high skilled group was significantly greater than that observed for the low skilled group. Moreover, significant facilitation was observed in the large problem condition at this SOA, for the high skilled group alone. In the addition condition, significant facilitation and inhibition was observed in the small problem condition, at both SOAs, for the high skilled group only. In contrast, facilitation was only observed in the small problem condition at the long SOA for the low skilled group. Finally, pre-exposure to large addition problems resulted in no priming effects for either group.

4. Discussion

The present study employed a priming procedure and naming task to determine whether arithmetic fluency influences the ability to automatically access simple arithmetic facts from memory. The overall results showed that high skilled individuals access simple arithmetic facts earlier in the processing sequence than low skilled individuals do. At 300 ms, significant facilitatory and inhibitory effects in target-naming performance following exposure to multiplication and addition problems were observed for the high skilled group alone. At 1000 ms, significant facilitation was observed for both groups, in both operations. For the high skilled group, in the multiplication condition facilitation increased significantly over time. In the addition condition, significant inhibition was observed for the high skilled group only. Further analyses revealed individual differences in access to small and large problems that varied by operation. In the multiplication condition at 1000 ms, facilitation observed in the small problem condition was significantly greater for the high skilled group than for the low skilled group. Furthermore, significant facilitation was observed in the large problem condition at this SOA for the high skilled group.
alone. In the addition small problem condition, significant facilitation and inhibition was observed at both SOAs for the high skilled group, whilst facilitation only was observed at 1000 ms for the low skilled group. No priming effects were observed for either group in the addition large problem condition.

The findings of the present study are consistent with previous research involving number matching and priming procedures in demonstrating that high skilled individuals have earlier and, in some cases, greater access to simple addition and multiplication facts than low skilled individuals (Jackson & Coney, 2005; LeFevre et al, 1991; LeFevre & Kulak, 1994). Furthermore, given the use of a brief SOA and a task in which solution retrieval was not explicitly required, the findings of the present study support the hypothesis that individual differences in arithmetic skill stem from automaticity in solution retrieval (Galfano, 2003; LeFevre & Kulak’s, 1994; Velmans, 1999). Additionally, the finding of a large and significant increase in facilitation over time for high skilled individuals in the multiplication condition indicates that individual differences in arithmetic skill also derive from strategic access to multiplication solutions.

For the high skilled group, the finding of equivalent levels of facilitation and inhibition at the short SOA, and facilitation dominance at the long SOA in the multiplication condition, is similar to the pattern of performance observed in the investigation of associatively related word primes and targets (Neely, 1991). This finding supports the notion that, in skilled arithmeticians, multiplication knowledge is represented in memory in a similar form to word knowledge and accessed through similar mechanisms (Ashcraft, 1992; Dehaene, 1992; LeFevre, Bisanz & Mrkonjic, 1988). Such a finding is not surprising when the reliance on verbal rote learning of
associations between words in the acquisition of multiplication knowledge is considered (Jackson & Coney, 2005).

The absence of an inhibitory effect at the long SOA in the multiplication condition is notably at odds with the Jackson and Coney (2005) finding of a significant 16 ms inhibitory effect for this group. This result possibly occurred due to differences between the skilled samples used in each study. The scores obtained by the previous skilled sample on the ACER SCT ranged between 18 and 47, with a mean correct score of 25. In contrast, the scores obtained by the present skilled sample varied less, ranging between 24 and 47, with a mean correct score of 31. Thus, it may be the case that the more skilled sample employed in the present study were able to suppress interference to incongruent targets before responding at the lengthier SOA (LeFevre et al., 1988).

Alternatively, the lack of an inhibitory effect at the long SOA in the multiplication condition may have resulted from lengthier response times in the neutral condition created by the use of the letter stimuli (i.e., X + Y and X x Y). That is, it may be the case that numerical stimuli, such as the 0 + 0 and 0 x 0 neutral stimuli employed in the earlier Jackson and Coney (2005) investigation, actually primes responses to like numerical stimuli (i.e., cf. letter stimuli priming numerical stimuli). In support of this, a comparison of neutral condition reaction times at the 1000 ms SOA, which was employed in both studies, reveals a significant advantage ($p < 0.001$) in responding to the zero stimuli in both the addition and multiplication conditions, for both groups. Nevertheless, given the differences between the samples mentioned earlier, further research into the priming effects produced as a result of the use of the different number and letter stimuli, involving comparable samples, would be beneficial.
Tied in with the above interpretation of the lack of inhibition in the long multiplication SOA condition is the notion that responses to congruent targets might also be speeded by priming using like stimuli. Whilst this possibility cannot be ruled out, the influence of such a confound appears minimal when considered in light of the similar facilitation effects observed between studies. For example, the facilitation effects of 10 and 26 ms observed at the 240 and 1000 ms SOAs (respectively) in the previous study are comparable to the 14 and 35 ms facilitation effects observed for the more skilled participants in the two SOA conditions employed in the present study. Similarly, the inhibition of 10 ms observed at the 240 ms SOA for this group in the previous study is comparable to the 12 ms inhibitory effect observed at the 300 ms SOA in the present study.

The results of the addition analysis in the present study, employing a larger sample size and more distinct fluency groups, differ from those of the previous Jackson and Coney (2005) study that found similar patterns of performance, irrespective of fluency level. Furthermore, in contrast to the previous research, the patterns of facilitation and inhibition for the high skilled group were found to be both significant and constant over time. This difference between studies may have resulted from the use of a longer short SOA condition in the present study (i.e., 300 ms as compared to 240 ms), possibly leading to the use of strategic processing by high skilled participants who had already reached a ceiling in activation due to priming earlier in the addition processing sequence. In such a scenario, with the use of a short (300 ms) SOA condition that was too brief to allow for strategic processing to influence responses to the target, this processing would have to have taken place after presentation of the target. However, given the finding of the same pattern of performance for these participants in the 300 ms multiplication condition, an
appreciable increase in facilitation might also be expected in the 1000 ms addition condition. Furthermore, such an explanation is at odds with the results of the previous Jackson and Coney (2005) study that indicated that the facilitation and inhibition effects observed in the number naming task derived from the workings of two independent mechanisms. That is, the facilitation appeared to result from the automatic activation of correct solutions that occurred prior to exposure to the target. In contrast, given that automatic spreading activation does not produce inhibition and that expectancy does not operate at SOAs of 240 ms or less, the inhibition appeared to result from processing that occurred after exposure to the target (Neely, 1991). Consequently, the inhibition in this study was explained in terms of the operation of an obligatory response validity checking mechanism that involves the comparison of a given target to the correct solution in memory and hence, hesitation in responding to the incongruent condition where the two do not match. In the present study, only the high skilled group, who might be more inclined to engage in such a process, demonstrated significant inhibition at 300 ms in both operations and at 1000 ms in the addition condition. Thus, the findings of the present study support and extend those of the earlier Jackson and Coney (2005) study in demonstrating the operation of this inhibitory mechanism in high skilled multiplication and addition performance.

Consistent with self-report data obtained by LeFevre et al. (1996a), a correlational analysis revealed a negative relationship between fluency and problem size effects in the addition condition. However, strong positive correlations between naming latencies and number magnitude in the multiplication analysis were found that were comparable between groups. These results, given that the participants were not required to retrieve solutions in the present naming task, suggest that explanations of the problem size effect based on the differential selection of solution
procedures between groups are incomplete (LeFevre et al., 1996a, 1996b). Moreover, the absence of positive correlations between problem size and reaction time in the neutral condition for the addition operation, suggest that explanations of this effect based on the time taken to articulate solutions containing various numbers of syllables are equally inadequate (Brysbaert, 1995). At the very least, models of the problem size effect in adult performance should be revised to incorporate the operation differences identified in the present study.

The problem size analysis revealed greater access to solutions to small and large multiplication problems and earlier access to small addition problems for the high skilled group. Interestingly, the large addition problem condition was the only one in which no significant priming effects were observed for either group. This finding potentially results from a disparity in the frequency of exposure to small and large addition problems. Small numbers occur more frequently than large numbers in naturally occurring settings, and small problems are presented earlier in instruction and with far greater frequency than large problems (Ashcraft, 1992; Hamman & Ashcraft, 1986). What is more, given that rote learning is commonly employed in the learning of multiplication tables, it could reasonably be assumed that large multiplication problems are verbally practiced to a greater extent than large addition problems. Consequently, performance on large addition problems may be at a permanent disadvantage and, given the lack of priming effects observed in the present study (even for relatively skilled individuals), may rely on strategic processing in solution retrieval.

The present study revealed significant advantages in access to correct addition and multiplication solutions for high skilled arithmeticians that varied as a function of arithmetic operation, SOA and problem size. Furthermore, it extended the
results of the earlier Jackson and Coney (2005) study by demonstrating the operation of an inhibitory response validity checking mechanism in addition performance. Finally, the present study showed that individual differences in arithmetic skill originate not only in automaticity of solution retrieval but also in strategic access to correct multiplication solutions (Galfano, 2003; LeFevre & Kulak’s, 1994; Velmans, 1999).

References


## Appendix A

### Prime Sets and Congruent and Incongruent Targets for Multiplication Operation

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<td>9 + 5</td>
<td>14</td>
<td>7</td>
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<tr>
<td>6 + 8</td>
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<td>14</td>
<td>17</td>
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<td>9 + 7</td>
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<td>2 + 3</td>
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<td>9 + 6</td>
<td>6 + 9</td>
<td>15</td>
<td>11</td>
</tr>
</tbody>
</table>
2.3. Surface Form Effects in a Priming Task.

This study addressed the question of whether simple addition and multiplication problems represented in different surface forms (Arabic digits or written number words) are processed along separate or common pathways. The main failure of previous research in this area has been an inability to determine whether the surface form effects identified in production, verification and repetition priming tasks, have resulted from encoding or fact retrieval mechanisms.

The arithmetic variant of the single word semantic priming paradigm was again employed. The priming effects produced in naming targets following exposure to Arabic digit primes (e.g., 2 + 3) were compared to the priming effects produced in naming targets following exposure to word primes (e.g., two + three). In the Arabic Digit condition, the neutral condition stimuli comprised ‘X + Y’ and ‘X x Y,’ whilst in the written word neutral condition, the stimuli comprised ‘blank + blank’ and ‘blank x blank.’ SOAs of 300 and 1000 ms were employed and, based on the findings of the second study, only a sample of high skilled arithmeticians participated in this study.

The results provided support to separate pathway accounts of arithmetic processing for problems represented in different surface forms. For example, they revealed facilitatory effects in responding to congruent digit stimuli at SOAs of 300 and 1000 ms, in both operations. In contrast to this, inhibitory effects in response to incongruent word stimuli in both the addition and multiplication operations at the short SOA, and in the addition operation at the long SOA, were found. These surface form effects were explained in terms of differences in access to visual and phonological representations.
Abstract

Models of numerical processing vary on whether they assume common or separate processing pathways for problems represented in different surface forms. The present study employed a priming procedure, with target naming task, in an investigation of surface form effects in simple addition and multiplication operations. Participants were presented with Arabic digit and number word problems in one of three prime-target relationships, including congruent (e.g., ‘2 + 3’ and ‘5’), incongruent (e.g., ‘9 + 7’ and ‘5’) and neutral (e.g., ‘X + Y’ and ‘5’) conditions. The results revealed significant facilitatory effects in response to congruent digit stimuli at SOAs of 300 and 1000 ms, in both operations. In contrast, inhibitory effects were observed in response to incongruent word stimuli in both the addition and multiplication operations at 300 ms, and in the addition operation at 1000 ms. The overall priming effects observed in the digit condition were significantly greater than in the word condition at 1000 ms in the multiplication operation and at 300 ms in the addition
operation. The results provide support to separate pathway accounts of simple arithmetic processing for problems represented in different surface forms. An explanation for variation in processing due to differences in access to visual and phonological representations is provided.

PsycINFO classification: 2343; 2346
Key Words: Arithmetic, Arabic Digits, Number Words, Priming, Surface Form
1. Introduction

Do the surface characteristics of arithmetic problems (e.g., Arabic digits: $2 + 3$; written number words: two + three) influence cognitive processing? This question is central to much of the research undertaken in the past three decades in the cognitive arithmetic area, having implications for models describing the componential architecture of numerical knowledge and the access to this information in the brain (Ashcraft, 1992; Campbell, 1994; Campbell, 1999; Dehaene, 1992; Noel, Fias & Brysbaert, 1997). Four main models of numerical processing are prominent in the literature, including the abstract-modular model (McCloskey, Caramazza & Basili, 1985), the triple code model (Dehaene, 1992), the preferred entry code model (Noel & Seron, 1993), and the encoding complex hypothesis (Campbell & Clark, 1988. See Noel et al., 1997, for a review of these models). Importantly, all of the numerical processing models assume that problems represented in different surface forms can be converted to the same mental representation and then processed along a common pathway. However, the encoding complex hypothesis differs from the other models in that it also assumes that problems represented in different surface forms can remain different and can be individually processed along separate pathways i.e., as specific codes (Campbell & Clark, 1988).

Empirical support for the notion that separate pathways can be used to process numbers represented in different surface forms is provided in a series of investigations into simple arithmetic fact retrieval. In the first of these, Campbell and Clark (1992) tested one of the main assumptions underlying McCloskey et al.’s (1985) abstract modular model, which suggests that fact retrieval is achieved through the operation of an independent calculation module and therefore, is a process that is not sensitive to the initial form of a problem. The participants in this study were
asked to retrieve solutions to simple multiplication problems represented as either Arabic digits or written number words. The results revealed an interaction between problem size and surface form, with a greater increase in reaction times and error rates for larger problems following presentation of the word stimuli. Furthermore, a regression analysis showed that variables that were theoretically related to retrieval difficulty and interference (i.e., problem size – where reaction time and errors increase with problem magnitude, and fan - problems that share solutions produce greater reaction times) predicted word-digit differences. These findings were considered not easily reconcilable with the abstract modular model’s assumption that number fact retrieval is mediated by a single, format independent, abstract representation.

In response to this, McCloskey et al. (1992) argued that the digit and word form differences identified by Campbell and Clark (1992) were possibly the result of encoding differences, with fact retrieval for word problems being carried out under greater speed pressure than for digit problems. According to McCloskey et al. (1992), this occurred for two main reasons. Firstly, the encoding of words requires the processing of several characters spread over a greater physical length than digits, thereby necessitating longer encoding times for word problems. Secondly, substantial frequency differences occur not only between the words *two* and *nine* but also between words and digits, a factor that Campbell and Clark (1992) had failed to consider. In support of their argument, McCloskey et al. (1992) repeated Campbell and Clark’s (1992) regression analysis, with predictor variables that included the number of characters comprising each problem and frequency, and found that the problem size and fan effects disappeared. Furthermore, in view of these encoding effects, McCloskey et al. (1992) argued that if participants were to adopt a response
deadline that limited the amount of time between exposure to the problem and responding, word problems would be subject to less processing in the retrieval stage, potentially increasing error rates for larger problems and the incidence of numerically distant errors. Nevertheless, in their study, Campbell and Clark (1992) concluded that the surface form effects had ‘emerged over and above encoding effects’ and further supported their claim with an in depth analysis of errors in performance that suggested an interaction between number-reading processes and number fact retrieval (pp. 478; but see Noel et al., 1997, for a critical review of the interpretation of error data). As noted by Campbell (1994), such a finding was inconsistent with the abstract-modular model, which holds that these two processes should not interact.

In a subsequent study by Campbell (1994) that also included an addition condition, the problem size and surface form variables were again shown to interact. In addition to this, the results revealed word format costs in reaction time that were greater for the larger, more difficult problems in the addition condition than in the multiplication condition. With the same operands utilised for both operations, the finding of an operation-by-format-by-size interaction was difficult to explain in terms of encoding processes (Campbell, 1994; Noel et al., 1997). However, as Campbell (1994) himself noted, given the possibility that the effects of problem size vary as a function of operation, it is plausible that the processing of attention-demanding larger problems (e.g., \(9 + 5 = 10 + 5 - 1\)) would be interfered with more by the encoding of problems that required greater attentional resources i.e., the encoding of problems represented in a word format.

Following the initial suggestion by McCloskey et al. (1992) that Campbell and Clark’s (1992) findings might be explained in terms of encoding processes and
the acknowledgement of this possibility in Campbell’s (1994) study, a number of studies were undertaken that attempted to separate the effects of encoding from fact retrieval processes. In one such study, Noel et al. (1997) reasoned that if the interaction obtained in the multiplication task was due mainly to encoding processes then a similar interaction should be found in a non-arithmetic task that involved similar encoding processes. Participants in this study were first asked to produce the solutions to multiplication problems represented in digit and word format and then to perform a number matching task on the same pairs of digits and words. In the latter case, participants were first exposed to two canonical dot patterns and then were presented with either a pair of digits or a pair of number words. Their task was simply to indicate whether the digits or words represented the same numerosities as those expressed by the dots. The results revealed a similar format-by-size interaction in both the fact retrieval and the number matching tasks, thereby supporting an encoding based account of Campbell’s (1994) findings.

However, the possibility exists that the number matching task employed by Noel et al. (1997) may have unintentionally confounded encoding processes with obligatory fact retrieval processes (which are also shown to produce problem size effects e.g., see Jackson & Coney, 2005, 2006). For example, in a study by LeFevre, Bisanz and MrKonjic (1988), participants were presented with two numbers (e.g., 3 + 2) and were then required to decide if a target number (e.g., 5) was one of the original numbers presented. Lengthier decision times in responding to the correct sum following the presentation of simple addition problems were found. Moreover, this effect was found even without the presence of the arithmetic operator (e.g., 3 2) showing that the obligatory activation of simple arithmetic facts occurs simply as the result of exposure to a pair of numbers. This finding was later supported in a similar
study of the multiplication operation by Thibodeau, LeFevre and Bisanz (1996), although in this case, the arithmetic operator was included in all conditions. It is at least possible therefore, that the number matching task employed by Noel et al. (1997) may have inadvertently accessed fact retrieval processes, hence producing the same format-by-size interaction as that in their multiplication task.

In another study by Campbell (1999), the influence of encoding in the format-by-size interaction was investigated using simple addition stimuli and the simultaneous or sequential presentation of operands (also see Blankenberger & Vorberg, 1997, who employed a similar methodology). In the simultaneous condition, i.e., the standard method of stimulus presentation, the usual interaction was predicted by Campbell (1999). However, in the sequential condition, the right operand was presented 800 ms after the left operand, thereby allowing time for the left operand to be processed before the right one was presented. Campbell (1999) argued that the encoding differences should therefore arise only in connection with the second operand and if the format-by-size interaction occurred mainly at the encoding stage, its magnitude should be reduced by half when compared to the simultaneous condition. The results showed that the interaction did not differ between simultaneous and sequential conditions leading Campbell to conclude that it did not occur at the encoding stage but instead arose during calculation or production.

Nevertheless, it is questionable as to whether the simplistic interpretation of the encoding process in the sequential condition described by Campbell (1999) is what actually occurs. For example, if access to a correct arithmetic solution requires the encoding of the problem as a whole (e.g., see Blankenberger & Vorberg, 1997, or Campbell, 1987, and Campbell & Graham’s, 1985, Network Interference model of
arithmetic processing) then potentially, the encoding process in this condition will be more complex, requiring the integration of the numerical representation of the right operand with the left operand and operator held in short term memory. Then, with both methods of presentation ultimately requiring whole problem encoding, the same format-by-size interaction should be found. Whatever the case may be, the issue is that any assumptions made regarding the encoding and fact retrieval stages associated with each condition, at this point, are speculative at best.

More recently, Campbell and Fugelsang (2001) investigated the format-by-size interaction by exploring the notion that surface form effects could arise from differences in the choice of strategy employed to access arithmetic solutions. According to Campbell and Fugelsang, because simple arithmetic problems are rarely encountered as words, visual familiarity with these problems will be low. This, together with the robust finding of greater problem difficulty with word stimuli, may promote the use of calculation strategies (e.g., counting or transformation: \(6 + 7 = 6 + 6 + 1\)) and discourage the use of direct memory retrieval, which is possibly more likely to be used with the more familiar digit stimuli. To test this hypothesis, a verification procedure that required participants to indicate whether addition problems presented as digits (\(3 + 4 = 8\)) or words (three + four = eight) were true or false was employed in conjunction with self report measures of the participants’ solution strategies. The results revealed the same format-by-size interaction in reaction times that was recognised in earlier production and matching tasks. Furthermore, the reported use of calculation strategies was found to be much greater for word stimuli than digit stimuli, a difference that was exaggerated for larger problems. Accordingly, the findings were again interpreted as evidence for surface form effects in central, rather than encoding stages of processing.
However, a recent study by Smith-Chant and LeFevre (2003) showed that in simple arithmetic processing, individual differences in arithmetic fluency and instructional demands can bias self reports and the solution procedures that are described. In this study, participants were asked to solve single digit multiplication problems under both speed and accuracy instructions and then half of the participants provided self reports of their solutions to the problems. Low skilled participants were shown to respond more slowly and accurately when asked to describe their solution procedures for large and very large problems. Moreover, they were more likely to use a greater variety of procedures, altering these with changes in emphasis on instructions between speed and accuracy. Unfortunately, Campbell and Fugelsang (2001) did not consider skill level at the time that they conducted their study.

Thus, regardless of ‘considerable experimental effort,’ the question of just what influence encoding processes have in producing the format-by-size interaction remains largely unanswered (Campbell, 1999, pp. B26). As noted by McCloskey et al. (1992), unless subjective size differences between large and small stimuli are made equivalent for each format, size incongruity effects cannot meaningfully be compared between formats. Possibly as a consequence of this, in the final example of a study that addressed the issue of surface form in numerical processing and that attempted to isolate the effects of encoding from fact retrieval processes, the influence of problem size in processing was not considered.

In Experiment 1 of a repetition priming investigation, Sciama, Semenza & Butterworth (1999) presented participants with addition problems represented as Arabic digits and number words. In Experiment 2, the addition problems were represented as Arabic digits and dot configurations. In each experiment, one third of the problems were preexposed in the same notation, one third were preexposed in a
different notation, and one third were not preexposed. Participants were simply asked to sum the numbers. The results indicated that preexposure to the same number pair represented in the same form produced greater benefits in reaction time for word and dot stimuli than did preexposure of the same number pair in digit form. With addition problems seldom ever represented using number words or dots, the authors concluded that the influence of surface form on repetition priming was dependent on the typicality of the surface form for that task. However, in addition to this, the results also revealed priming effects across surface form. That is, preexposure to the same number pair represented as digits, words or dots led to the same amount of priming in digit stimuli. Such a finding is consistent with models that assume that after encoding, processing involves a common representation. The results of the Sciama et al (1999) study therefore, supported the encoding complex hypothesis and the notion that both common and form specific codes co-exist together.

Nonetheless, as noted by Sciama et al. (1999), it is possible that the surface form effects observed for the word and dot stimuli in their first two experiments resulted from facilitated encoding processes, due simply to exposure to atypical stimuli. Consequently, in Experiment 3 of their study, the authors reasoned that if this was the case, priming should be found for the same numbers presented in different operations (e.g., $2 + 3$ and $2 \times 3$) for the word and dot stimuli alone. To test this, the same method as that employed in the first two experiments was utilised but this time, the surface form was maintained across repetitions. Additionally, three study phases were employed, the first of which, required participants to perform multiplication on the prime instead of addition. Of the remaining study trials, one third of the items were not presented at study (i.e., they were new in the test phase) and the other third were presented for addition. The results suggested priming for
number pairs that had been multiplied in the study phase, and priming reached significance when the number pairs had to be added at study. Furthermore, this trend for cross operation priming was apparent for all surface forms, and was more reliable with the digit stimuli. The findings were thus deemed inconsistent with models that explain effects of surface form in terms of encoding processes.

1.1 The Present Study

In the cognitive arithmetic literature, models of numerical processing differ on the fundamental issue of whether the surface characteristics of arithmetic problems influence later cognitive processing. That is, there is disagreement as to whether problems represented in different surface forms are first converted to a single representation before processing along a common pathway or remain unique, and are processed individually as specific codes. Underlying this disagreement, there appears to be an inability to reliably determine whether the surface form effects (e.g., the format-by-size interaction) that are robustly identified in simple arithmetic tasks result from encoding or fact retrieval mechanisms.

The aim of the present study was thus to resolve this problem by utilising an arithmetic based variant of the single word semantic priming paradigm in the investigation of multiplication and addition processing (e.g., see Jackson & Coney, 2005, 2006). This priming procedure differed from earlier cognitive arithmetic priming investigations (e.g., see Campbell, 1987, 1991) in that it involved the presentation of problems as primes (e.g., 2 + 3) and solutions as targets (e.g., 5), in the order that they occur in natural settings. Moreover, the time period between the onset of the prime and presentation of the target (i.e., the stimulus onset asynchrony; SOA) was varied in order to assess automatic and strategic processing. In line with the single word semantic priming paradigm in which automatic effects are measured
at SOAs in the order of 250 ms and strategic effects are measured at SOAs of greater
than 400 ms, the present study employed SOAs of 300 and 1000 ms (Perea & Rosa,
2002; Velmans, 1999). When used in conjunction with a target naming (i.e.,
pronunciation) task, this procedure allowed for a more valid investigation into
automaticity in arithmetic fact retrieval than occurs with verification or production
tasks. This is because, in both verification and production tasks, faster responses and
greater accuracy are attributed to automatic processing. However, there is little basis
for determining where the boundary is in the range of reaction time and error rate
measures that separates the operation of automatic and strategic fact retrieval
mechanisms. Furthermore, verification tasks may induce attentional processing
through the requirement to make a binary decision about the relationship between the
prime and the target, and may be accomplished via processes other than fact
retrieval, including familiarity, plausibility and odd/even judgements (Campbell,
1987). Thus, by simply requiring that participants’ verbally identify target numbers
as they appeared on a computer screen, the naming task minimised the possibility of
calculation and decision induced attentional processing.

Importantly, in the context of the present study, the use of this priming
procedure allowed for a comparison of the priming effects produced by exposure to
each surface form (i.e., rather than making direct comparisons of reaction times
between digits and words). To do this, simple addition and multiplication problems
represented in each surface form were assigned to three prime-target relationship
conditions i.e., congruent (‘2 + 3’ and ‘5’), incongruent (‘7 + 9’ and ‘5’) and neutral
(‘X + Y’ and ‘5’) conditions. Consistent with Neely (1991), the effects of the
congruent and incongruent prime-target relationships were then assessed
independently for each surface form by subtracting the reaction time taken to name
the targets in each of these conditions from the reaction time taken to name the target following exposure to the neutral condition. Positive differences were referred to as facilitation and negative differences were referred to as inhibition. Additionally, by subtracting the reaction time taken to name the targets in the congruent condition (e.g., ‘2 + 3’ and ’5) from the reaction time taken to name the targets in the incongruent condition, in which the same prime was presented (i.e., ‘2 + 3’ and ‘14’), an overall priming effect that was independent of encoding times was produced for each surface form. Accordingly, it was assumed that if problems represented as digits and words are accessed via common pathways, then the patterns of priming effects that they each produce would not differ.

2. Method

2.1 Participants

Twenty-nine undergraduate psychology students, including 9 males and 20 females, from Murdoch University participated in this study. The participants’ ages ranged from 17 to 52 years, with a mean age of 26. The participants scores on the arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) indicated that they were a relatively skilled sample. The mean correct score of 23 (SD = 6.06) for this sample corresponded to a percentile rank of 68% in a normed sample of 124 candidates who had completed a three or four year diploma at a tertiary institution, and 93% in a normed group of administrative officer or assistant applicants (ACER, 1984). All participants received credit toward partial fulfilment of course requirements for their time.

2.2 Design and stimulus materials
Four within group variables were examined in the present study. The first of these was arithmetic operation with two levels i.e., addition and multiplication. The second variable was surface form and included two levels: digits (e.g., $2 + 4 = 6$) and words (e.g., two + four = 6). The third variable was prime-target relationship, with three levels: congruent (e.g., $2 + 4 = 6$) incongruent ($8 + 9 = 6$) and neutral ($X + Y = 6$) conditions; and the fourth variable was SOA, with two levels: 300 ms and 1000 ms.

Two sets of primes (Sets 1 and 2 employed in Jackson & Coney, 2005, 2006) addressing each operation and represented in both of the digit and word formats were utilised in the present study (see Appendix A for the stimulus set represented in digit form). The first set consisted of 18 simple arithmetic facts selected from the 2s through 9s matrices (e.g., $2 + 3$) and the second set comprised the reverse operand placement equivalents of the first set ($3 + 2$). Arithmetic ties (e.g., $3 + 3$ and $3 \times 3$) were excluded from use as primes, as research by LeFevre et al. (1988) showed that these problems are solved more quickly than standard problems. Each set was balanced in terms of operand placement, with half of the arithmetic facts produced so that the smaller of the two operands was placed on the left-hand side and half with the smallest operand on the right hand side. Each set consisted of six smaller problems (i.e., with both operands of a magnitude less than or equal to five; $2 + 3$), six larger problems (operands greater than or equal to six; $8 + 9$), and six of mixed magnitude ($2 + 9$), to allow for the investigation of problem size effects.

The correct solutions corresponding to the 18 simple arithmetic facts were employed as targets in each of the congruent, incongruent and neutral conditions. In the incongruent condition, the correct solutions were paired with an alternative problem so that they were mathematically incorrect. Constraints on the pairing of
stimuli for this condition were included to guard against split effects and to address any confounding relationships. In the former case, multiplication targets were paired with problems so that they differed by at least 16 from their correct solutions and addition targets differed by at least three from their correct solutions. In the latter case, incongruent targets were not permitted to be one of the operands or their near neighbours (i.e., a number ± 1 from an operand), a multiple or factor of the operands, the correct solution using a different operation, or a double-digit number containing the operands or correct solution. The use of the same primes and targets in each of the congruent and incongruent conditions balanced for the effects of problem size, split and target magnitude, between conditions.

The neutral stimuli employed in the digit condition of the present study was the same as that utilised in the Jackson and Coney (2006) study i.e., ‘X + Y’ for the addition operation and ‘X x Y’ for the multiplication operation. For the word condition, the neutral stimuli consisted of ‘blank + blank’ and ‘blank x blank,’ respectively. The choice of these stimuli was informed by recommendations made by Neely (1991) in the context of word priming research. Specifically, Neely suggested that neutral stimuli should be equivalent to other primes in terms of their alerting properties that a target is soon to be presented. Additionally, neutral primes should be completely unrelated to the targets to enable them to serve as a neutral baseline to which to compare expectancy effects and performance to related stimuli. The X + Y and X x Y stimuli are particularly suited to the purposes of the present study as they are perceptually similar to the numerical primes and are semantically unrelated to the target stimuli, with the X and Y symbols often used to denote separate unknown quantities (Jackson & Coney, 2006). Similar observations can be made in relation to
the neutral word stimuli, with their utility further evidenced in the relatively common use of the term *blank* in the word priming research (de Groot, 1982; Neely, 1991).

### 2.3 Procedure

Participants were individually tested on the computer task in a well-lit cubicle. This task was completed on an Amiga 1200 microcomputer, with 1084S monitor that controlled stimulus presentation, trial sequencing, timing and data collection. Digit operands and individual letters in number words did not exceed dimensions of 5 x 15 mm. Digit operands and number words were placed 5 mm either side of the arithmetic operator (i.e., the x or + sign), which did not exceed dimensions of 5 x 10 mm. The stimuli were presented centrally, white against an amber background and a chin rest was used to stabilise the participant’s head 60 cm directly in front of the screen.

Participants each completed eight blocks of 54 experimental trials (i.e., four blocks for each of the digit and word conditions, with two of the four blocks addressing the addition operation and two addressing the multiplication operation, at each of the levels of SOA). Trials were blocked separately by surface format and arithmetic operation. Exposure to all stimuli was counterbalanced across participants. That is, half of the participants completed the digit condition first and half completed the word condition first. Half started with the addition operation first and half started with multiplication first. At the short SOA, for each operation, half of the participants were exposed to Set 1 and half were exposed to Set 2. Each participant was then exposed to the same set at the long SOA to enable a level of familiarity with the stimuli and draw attention to the prime-target relationship. This process was repeated in the third and fourth blocks using the operation not tested in the first two
blocks. The computer randomly generated the order of presentation of congruent, incongruent and neutral trials in each block.

Participants were instructed to respond both quickly and accurately. Trials began with participants focusing their gaze on a 1 x 1 mm blue central fixation dot. The fixation dot was exposed for 600 ms and then the screen went blank for 150 ms before the prime was presented for a duration of 100 ms. Following the SOA of either 300 or 1000 ms, the target number was presented and this remained exposed until the participant verbally identified the number. A two-second interval ensued before the start of the next trial. A microphone connected to a headset was used to detect vocal response sounds, with reaction time measured from the onset of the target. To accomplish this, the microphone amplifier triggered an electronic relay interfaced to the computer and the time of relay closure was determined using a hardware timer that was accurate to 1 millisecond. Padded ear guards helped to block out external noise intrusions and the experimental session took approximately 45 minutes to complete.

3. Results

3.1 Overall Analyses

The mean naming latencies were initially screened for outliers using a criterion of +/- 2.5 z-scores. Only 0.72% of all scores exceeded this criterion and were replaced using mean substitution. The resulting data are presented in Table 1.
Table 1.
Mean Naming Times (ms) and Standard Deviations (in parentheses) for all Prime-Target Relationships as a Function of Surface Form, SOA and Operation.

<table>
<thead>
<tr>
<th></th>
<th>Digits</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>300 ms</td>
<td>1000 ms</td>
</tr>
<tr>
<td>Multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Congruent</td>
<td>457 (48)</td>
<td>453 (49)</td>
</tr>
<tr>
<td>Incongruent</td>
<td>476 (61)</td>
<td>481 (50)</td>
</tr>
<tr>
<td>Neutral</td>
<td>468 (55)</td>
<td>480 (55)</td>
</tr>
<tr>
<td>Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Congruent</td>
<td>440 (62)</td>
<td>440 (48)</td>
</tr>
<tr>
<td>Incongruent</td>
<td>459 (65)</td>
<td>465 (52)</td>
</tr>
<tr>
<td>Neutral</td>
<td>448 (60)</td>
<td>461 (56)</td>
</tr>
</tbody>
</table>

The data for the addition and multiplication operations were analysed separately.

3.1.1 Multiplication Analysis

The multiplication data were entered into a repeated measures analysis of variance (ANOVA) involving surface form, SOA and prime-target relationship as within group variables. A significant main effect of prime-target relationship was found \(F(2, 56) = 18.2; MSe = 467.9, p < 0.001\). Responses in the congruent condition were 11 ms faster than in the neutral condition \(t(28) = 3.9, p = 0.001\) and responses in the incongruent condition were 6 ms slower than in the neutral condition \(t(28) = 2.6, p = 0.013\). No main effect of surface form was found in the data \(F(1, 28) = 0.112, MSe = 2742.8, p = 0.740\), a finding that differs from previous research involving production tasks (e.g., Campbell, 1999).

The main effect of prime target relationship was qualified by a significant interaction between surface form and prime-target relationship \(F(2, 56) = 6.4, MSe = 334.2, p = 0.003\). Paired sample t-test comparisons revealed significant facilitation (i.e., neutral – congruent) in naming congruent targets in the digit condition \(t(28) = 5.9, p < 0.001\) and inhibition (i.e., incongruent – neutral) in naming incongruent
targets in the word condition ($t(28) = 2.2$, $p = 0.032$). Significant overall priming effects (i.e., incongruent – congruent) were identified in both the word ($t(28) = 2.5$, $p = 0.019$) and digit conditions ($t(28) = 5.9$, $p < 0.001$). The overall priming effect observed in the digit condition was significantly greater than that observed in the word condition ($t(28) = 2.6$, $p = 0.014$).

No significant three-way interaction was observed in the data. However, in view of an interest in changes in priming effects over time, planned comparisons between all prime-target relationships were undertaken for each surface form, at both SOAs. The facilitatory, inhibitory, and overall priming effects observed in these analyses are illustrated in Fig. 1.
Fig. 1 Showing facilitation, inhibition, and overall priming effects as a function of SOA, surface form and operation. The 95% confidence intervals for each of the reaction time differences were calculated based on a pooled estimate of $MSe$ for individual two-factor (SOA and surface form) repeated measures ANOVAs.

In the digit condition, significant facilitation was observed at both the short ($t(28) = 2.3, p = 0.031$) and the long SOAs ($t(28) = 4.8, p < 0.001$). Additionally, significant overall priming effects were observed at each SOA (with ($t(28) = 3.7, p = 0.001$) and ($t(28) = 6.9, p < 0.001$), respectively). In the word condition, significant inhibition was found at the short SOA ($t(28) = 2.6, p = 0.016$) and a significant overall priming effect was observed at the long SOA ($t(28) = 2.2, p = 0.035$). The overall priming effect observed in the digit condition at the long SOA was significantly greater than that observed in the word condition ($t(28) = 3.2, p = 0.003$).

### 3.1.2 Addition Analysis

A repeated measures ANOVA on the addition data revealed a significant main effect of prime target relationship ($F(1.7, 46.7) = 23.0, MSe = 495.8, p < 0.001$). Significant facilitation of 8 ms ($t(28) = 4.2, p < 0.001$) and inhibition of 10 ms ($t(28) = 3.5, p = 0.002$) was observed. No significant main effect of surface form was evident in the data ($F(1, 28) = .10, MSe = 2838.2, p = 0.760$), a finding that
again, differs from previous production (e.g., Campbell, 1994) and verification research (e.g., Campbell & Fugelsang, 2001).

Two significant two-way interactions were identified in the addition analysis. Firstly, a significant two-way interaction was found between SOA and prime target relationship ($F(2.56) = 3.3, MSe = 227.1, p = 0.043$). Paired sample t-test comparisons revealed inhibition of 12 ms at the short SOA ($t(28) = 3.1, p = 0.005$).

At the long SOA, facilitation of 13 ms ($t(28) = 4.2, p < 0.001$) and inhibition of 9 ms ($t(28) = 2.8, p = 0.009$) was found. Secondly, and more importantly in the context of the present study, a significant interaction between surface form and prime target relationship was again found ($F(2, 56) = 6.5, MSe = 186.6, p = 0.003$) (See Fig. 1 for facilitatory and inhibitory effects). In the digit condition, significant facilitation was observed ($t(28) = 5.5, p < 0.001$), whilst in the word condition, only a significant inhibitory effect was evident ($t(28) = 4.2, p < 0.001$). Significant overall priming effects were again identified in both the word ($t(28) = 3.9, p = 0.001$) and digit conditions ($t(28) = 6.1, p < 0.001$).

No significant three-way interaction involving surface form was found in the addition analysis. Nevertheless, planned comparisons of changes in priming effects over time were again undertaken for each surface form. In the digit condition, significant facilitation was found at both the short SOA ($t(28) = 2.2, p = 0.034$) and the long SOA ($t(28) = 3.9, p < 0.001$). Additionally, significant overall priming effects were identified at the short ($t(28) = 5.2, p < 0.001$) and the long SOAs ($t(28) = 4.3, p < 0.001$). In the word condition, significant inhibitory effects were found at both the short ($t(28) = 3, p = 0.006$) and the long ($t(28) = 3.5, p = 0.002$) SOAs, and significant overall priming effects were found at each SOA (with ($t(28) = 2.7, p = 0.013$) and ($t(28) = 3.9, p = 0.001$) respectively). At the short SOA, the overall
priming effect observed in the digit condition was significantly greater than that observed in the word condition ($t(28) = 2.2, p = 0.036$).

In summary, the general pattern of digit performance in both of the addition and multiplication operations was one of significant facilitation in naming congruent targets. In contrast, in the word condition, inhibition was found in naming incongruent targets at the short SOA for both operations, and at the long SOA for the addition operation. The overall priming effects observed in the digit condition were significantly greater than in the word condition at the long SOA in the multiplication operation and at the short SOA in the addition operation. The results of the overall analyses are thus consistent with models of numerical processing that assume that, after encoding, problems represented in different surface forms are processed along separate pathways.

### 3.2 Problem Size Analyses

In order to determine any influence of surface form in the processing of problems of differing size, a subset of the data that included naming times for small and large problems (consisting of operands $\leq 5$ or $> 5$, respectively) only was selected. Unfortunately, this created a mis-match between the solutions in the congruent and incongruent conditions, and between problems and solutions of differing magnitudes (e.g., small congruent multiplication targets ranged between 6 and 20, whilst the majority of small incongruent multiplication targets ranged between 30 and 63). Thus, any differences resulting from direct comparisons between the two problem sizes may have been attributable to a confound of target magnitude. To avoid this possibility, the raw data for all problems within the original data set were first entered into regression analyses to determine any effect of target
magnitude. The regression equations for both the digit and word surface forms were then used to adjust for magnitude in the obtained naming times for each of the multiplication (Digit: \( \text{naming time} = (0.45** \times \text{number magnitude}) + 454** \); Word: \( \text{naming time} = (0.47** \times \text{number magnitude}) + 452** \)) and addition operations (Digit: \( \text{naming time} = 0.13 \times \text{number magnitude}) + 443** \); Word: \( \text{naming time} = 2.02*(\text{number magnitude}) + 428** \))(note. \(*p < 0.05, **p < 0.01\).

The resulting data were entered into an overall repeated measures ANOVA to test for the presence of an operation-by-format-by-size interaction. No significant interaction was found between these three variables (\(F(1, 28) = 1.9, MSe = 1480.9, p = 0.181\)) and they did not significantly interact with SOA (\(F(1, 28) = 1.1, MSe = 918.1, p = 0.310\)). As in the overall analyses, repeated measures ANOVAs, with surface form, SOA, size and prime target relationship as within group factors, were again undertaken independently for each of the multiplication and addition operations.

### 3.2.1 Multiplication Analysis

In the multiplication condition a significant main effect of prime target relationship (\(F(2, 56) = 5.7, MSe = 1863.8, p = 0.005\)) and a significant two-way interaction between surface form and prime target relationship (\(F(2, 56) = 3.3, MSe = 1055.9, p = 0.044\)) were again found. No significant two-way interaction between surface form and size was indicated (\(F(1, 28) = 1.1, MSe = 1148.3, p = 0.295\)) and no other significant effects were observed in the multiplication data. Nonetheless, planned comparisons of changes in priming effects due to problem size and surface form were investigated at each SOA. These priming effects are illustrated in Figure 2.
Multiplication

<table>
<thead>
<tr>
<th>Facilitation</th>
<th>Inhibition</th>
<th>Overall Priming</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Small</strong></td>
<td><strong>Small</strong></td>
<td><strong>Small</strong></td>
</tr>
<tr>
<td><strong>Large</strong></td>
<td><strong>Large</strong></td>
<td><strong>Large</strong></td>
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</tbody>
</table>

Addition

<table>
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</tr>
</thead>
<tbody>
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<td><strong>Small</strong></td>
</tr>
<tr>
<td><strong>Large</strong></td>
<td><strong>Large</strong></td>
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</tbody>
</table>
### Overall Priming

<table>
<thead>
<tr>
<th>Digits</th>
<th>Words</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Fig. 2 Showing facilitation, inhibition, and overall priming effects as a function of operation, SOA, surface form and problem size. The 95% confidence intervals for each of the reaction time differences were calculated based on a pooled estimate of $MSe$ for individual two-factor (SOA and surface form) repeated measures ANOVAs.

With such a large number of comparisons, a Bonferroni adjustment was used to reduce the alpha level to a more conservative level of 0.004 (i.e., 0.05/12). Significant facilitation of 30 ms was observed at the long SOA for both the small ($t(28) = 3.1, p = 0.004$) and large ($t(28) = 3.6, p = 0.001$) digit problems. Additionally, an overall priming effect of 24 ms for small digit problems approached significance ($t(28) = 2.9, p = 0.008$) and an overall priming effect of 22 ms for large digit problems reached significance ($t(28) = 3.2, p = 0.003$) at the long SOA. No other comparisons in either the digit or word conditions reached significance.

#### 3.2.2 Addition Analysis

In the addition condition, the significant main effect of prime target relationship ($F(2, 56) = 12.4, MSe = 1503.8, p < 0.001$) and a significant interaction between surface form and prime target relationship were again evident ($F(2, 56) = 6.3, MSe = 916.1, p = 0.003$). Additionally, unlike the multiplication analysis, a significant two-way interaction between size and prime-target relationship was found ($F(2, 56) = 12.5, MSe = 843.6, p < 0.001$). Significant facilitation of 19 ms ($t(28) = 4.2, p < 0.001$) and inhibition of 12 ms ($t(28) = 2.9, p < 0.008$) was observed for
small problems, whilst no facilitatory or inhibitory effects were observed for large problems. These findings are consistent with those previously observed in Jackson and Coney (2006), who found similar effects of 23 ms and 10 ms, respectively.

As in the multiplication analysis, the results again failed to show a significant interaction between surface form and problem size ($F(1, 28) = 1.0, MSe = 1546.7, p = 0.336$). However, a significant four way interaction between surface form, SOA, size and prime target relationship was found ($F(2, 56) = 4.0, MSe = 819.2, p = 0.023$) (see Figure 2). When tested at an adjusted alpha level of 0.004, significant facilitation of 35 ms was observed in the small digit condition at the long SOA ($t(28) = 4.6, p < 0.001$). An inhibitory effect in the small digit condition of 24 ms was found at the short SOA ($t(28) = 3.1, p = 0.004$). Significant overall priming effects of 38 ms ($t(28) = 5.0, p < 0.001$) and 40 ms ($t(28) = 4.5, p < 0.001$) were observed for small digit problems at the short and long SOAs, respectively. In the word condition, a significant overall priming effect of 28 ms was observed at the long SOA for small problems only ($t(28) = 4.1, p < 0.001$). No other effects reached significance.

In summary, examination of the problem size data revealed processing differences that varied by surface form, with facilitatory and inhibitory effects observed for digit stimuli only. The only significant priming effect found in the word condition was an overall priming effect that was observed for small word problems at the long SOA. The results of the problem size analyses are therefore, consistent with separate pathway models of arithmetic processing.

4. Discussion

The present study aimed to determine whether the surface form of a problem influences cognitive processing. The overall analyses suggest that this is the case. In
the digit condition, significant facilitation in naming congruent targets was observed in both the addition and multiplication conditions, at both SOAs. In contrast, in the word condition, inhibitory effects were observed in naming incongruent word targets in both the addition and multiplication conditions at 300 ms, and in the addition condition at 1000 ms. Furthermore, the overall priming effects (incongruent – congruent condition naming times i.e., the effects after encoding) observed in the digit condition were significantly greater than that observed in the word condition at the long SOA in the multiplication condition and at the short SOA in the addition condition. In the problem size analysis, at 1000 ms, facilitation was observed in naming congruent digit targets following exposure to small addition and multiplication problems and large multiplication problems. At 300 ms, inhibition was found in naming incongruent digit targets following exposure to small addition problems only. An overall priming effect in naming congruent digit targets approached significance at 1000 ms in the small multiplication condition and reached significance in the large multiplication condition. Overall priming effects were observed at both SOAs in the small addition digit condition. No facilitatory or inhibitory effects were identified in naming either small or large targets in the word condition. In fact, the only significant priming effect observed for the word stimuli was an overall priming effect following exposure to small word problems in the addition condition, at 1000 ms. The results of the present study thus provide partial support to the encoding complex hypothesis and the notion that problems represented in different surface forms are indeed processed differently.

What mechanisms are responsible for the facilitatory and inhibitory effects observed in the present study? The results of the investigation by Jackson and Coney (2005), which revealed very similar results to the present study, are instructive in this
regard. This study utilised the same priming technique, the same proportions of congruent, incongruent and neutral trials, and almost exactly the same stimulus set as that used in the present digit condition (two problems were excluded from use in the present set). Facilitatory and inhibitory effects that differed as a function of time and that could thus be attributed to the operation of two independent mechanisms were identified. In the case of the facilitatory effect, three sources of evidence suggested that it resulted from the operation of an automatic spreading activation mechanism. Firstly, the facilitation arose at an SOA of 240 ms, a time period between the onset of the prime and presentation of the target that was too short to allow for conscious processing. Secondly, no facilitation was observed at an SOA of 120 ms. Had the facilitation resulted from conscious processing that occurred after presentation of the target, then it should have been present at this SOA. Thirdly, calculation was not necessary to performance of the naming task. Thus, with the same procedure and a more skilled sample employed in the present study, it is likely that the pattern of facilitation observed in the digit condition of the present study reflected the operation of an automatic mechanism that arose at 300 ms and lead to marked facilitation at the long SOA.

In contrast, examination of the inhibition function in Jackson and Coney’s (2005) study suggested the workings of a mechanism that operates independently of the facilitation mechanism. Support for this position was provided by the finding of inhibitory effects at the shortest SOAs of 120 and 240 ms, time periods too short to allow for strategic processing of the prime before exposure to the target. Furthermore, the inhibitory effect remained constant over time, occurred even though calculation was not necessary to performance of the task, and was found only in the performance of the skilled group. Accordingly, the inhibitory effect was explained in
terms of the operation of a self regulatory, response validity checking mechanism. This mechanism operates after exposure to the target and before vocal responding, and involves the comparison of the just presented target to the correct solution from memory. In the incongruent condition, when the correct solution and target do not match, hesitation in responding occurs. Again, with the use of the same procedure in the present study and the finding of constant inhibition over time, it is likely that a similar mechanism was employed. The finding that the inhibitory effect occurred only in the word condition, involving problem stimuli that the participants had probably never previously encountered (and hence, that would be more likely to benefit from such a process), is consistent with this assumption.

Given the likelihood that two independent mechanisms were responsible for the facilitatory and inhibitory effects observed in the present study, a further question is of what type of representation these mechanisms act upon? In relation to the facilitatory mechanism, two possibilities exist. Firstly, it is reasonable to assume that the activation of the solution from memory occurred directly via visual representations of the digit stimuli (Campbell & Clark, 1992). Such an explanation is compatible with the high frequency of exposure to visual representations of addition and multiplication problems represented as digits in formal learning procedures such as mental mathematics. Secondly, consistent with models that assume the existence of autonomous asemantic transcoding routes (e.g., the encoding complex hypothesis and the triple code model) it is possible that the visual representations of the digit stimuli were automatically converted into phonological representations that then elicited the automatic activation of solutions from memory. However, the sequential nature of phonological representations (cf. simultaneous visual representations) and the theoretical inefficiency of such a conversion process, together make this latter
position appear less likely. Additionally, as the results show, the digit and word stimuli were processed differently (cf. Dehaene’s, 1992, triple code model) and the conversion of visual representations to phonological representations appears a more obvious choice in the context of the word stimuli.

Support for the notion that word problems are solved via phonological representations stems from the improbability that correct solutions would be activated from a stable semantic network of arithmetic problems represented in visual word form in memory (Sciama et al., 1999). This improbability is supported in the present results by the finding of no facilitatory effects following visual exposure to congruent word stimuli. Had this information been represented in a network in memory, then activation of word problems resulting from exposure to the prime should have lead to spreading activation along the paths of this network to the associated correct solution, consequently leading to facilitation in naming congruent targets in this condition (Neely, 1991; Reed, 1988). Moreover, in contrast to the present word findings, previous research involving the same methodology shows that even low skilled performance involving digit stimuli produces facilitation effects at long SOAs that are consistent with the existence of some knowledge representation in memory (Jackson & Coney, 2005, 2006). Thus, with written numerals more commonly encountered in reading contexts, it would seem more feasible that in the present word condition, correct solutions would be activated through strong, verbal, reading based mechanisms (possibly via subvocalisation) (Campbell, 1994; MacLeod, 1991). The activation of phonological representations would, in turn, activate correct solutions that are then acted upon by the obligatory validity checking mechanism to produce the observed inhibitory effects.
In view of this interpretation, the differing trend in the pattern of inhibition found between the addition and multiplication word conditions at the long SOA (see Fig. 1) could be explained in terms of differences in exposure to phonological representations between the two operations in educational practices. For example, in formal schooling, the development of multiplication fact knowledge can rely quite heavily on verbal rote learning, thereby producing strong phonological associations between multiplication problems and their correct solutions. Accordingly, at the long SOA in the present study, when participants had ample time to process the multiplication prime before presentation of the target, a pattern of facilitation approaching that observed for the well practiced Arabic digit stimuli was found. However, the need for the operation of an obligatory validity response checking mechanism at this SOA may have been minimal in comparison to its requirement at shorter SOAs, when there was little time to process the prime stimuli. In contrast, addition facts are not generally rote learnt and any phonological representations possibly develop whilst addition problems are practiced through methods employing visual exposure. As such, only weak verbal associations may develop between addition problems and their correct solutions that are enough to enable the recognition of inaccuracy but are not strong enough to speed processing. Hence, the observed inhibitory effects at both SOAs for the addition operation.

The interpretation of digit processing in terms of visual codes and word processing in terms of phonological codes appears at odds with the assumption of the triple code model that access to stored simple arithmetic facts occurs solely via phonological representations. However, the intuitive appeal of the preceding interpretation is demonstrated by its recognition over a decade ago by Campbell and Clark (1992), who noted that “visual codes may be especially salient with digit
stimuli, whereas activation of phonological codes may be more salient with number words” (pp. 461). Furthermore, the notion that the role of phonological and visual processing depends upon the presentation format of arithmetic stimuli was recently supported in an empirical investigation by Trbovich and LeFevre (2003). In this study, participants were required to solve multidigit problems (e.g., 52 + 3) that were presented in either a vertical (i.e., the standard visual format used in pencil and paper tasks) or horizontal format. At the same time, participants were also asked to retain a phonological load (consisting of pronounceable consonant-vowel-consonant nonwords such as *nof*), a visual load (i.e., a pattern of asterisks) or no load in memory. Any mutual interference observed between the performance of the arithmetic task presented in different formats and the memory load task was theoretically assumed to indicate that the two tasks relied upon the same processing resources or codes. Consistent with the present interpretation, the results showed that performance was worse in the phonological load task in the atypical horizontal condition, whilst performance was worse in the visual load task in the vertical condition.

Interestingly, the results of the present study together with those of Trbovich and LeFevre (2003) imply that, when confronted with problems represented in an unusual visual form, the fact retrieval process reverts to a reliance on more familiar phonological representations. What is more, given that fact retrieval was completely unnecessary for accurate performance in the present study, it would seem that the dependence on this representation was obligatory. Such a process may be likened to the operation of a ‘backup’ procedure that enables a faster and more accurate fact retrieval approach (Siegler, 1988; Siegler & Jenkins, 1989; Siegler & Shipley, 1995).
In the present study, participants named digits that were preceded by arithmetic problems represented in either digit or word form. This procedure effectively allowed for the removal of encoding influences in performance and enabled a comparison of the priming effects associated with each surface form over time. The results revealed facilitatory effects in target naming performance following exposure to digit primes. Based on previous research by Jackson and Coney (2005), these effects were explained in terms of a spreading activation mechanism elicited via a stable semantic network of visual representations in memory. In contrast, inhibitory effects were revealed following exposure to word primes in all except the long SOA multiplication condition. Consistent with Jackson and Coney (2005) these effects were explained in terms of the operation of an obligatory response validity checking mechanism acting upon phonological representations, due to the novelty of the word problem stimuli. Additionally, the results of the present study revealed differences in overall priming effects between problems represented in different surface forms. The present results are therefore inconsistent with common pathway models of numerical processing (i.e., the abstract modular model, the preferred entry code model and the triple code model) that assume that after encoding, all surface forms are processed in the same way. Furthermore, they partially disconfirm number processing models that assume both common and form-specific processing pathways (i.e., the encoding complex hypothesis and Sciama et al.’s (1999) common and form-specific co-existence approach). A revision of number processing models that includes acknowledgement of the influence of stimulus novelty on cognitive processing is advised.

References


## Appendix A

<table>
<thead>
<tr>
<th>Digit Prime Sets and Congruent (C) and Incongruent (I) Targets for Each Operation</th>
<th>Multiplication</th>
<th>Addition</th>
</tr>
</thead>
<tbody>
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<tr>
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</table>
2.4. Does Problem Type Influence Fact Retrieval Mechanisms?

This study addressed the problem of how different simple addition and multiplication problem types are organised in and accessed from memory. Unfortunately, to date, using production and verification tasks, it has been impossible to discriminate between the influences of fact retrieval mechanisms and encoding processes on reaction time measures.

To address this, the new arithmetic variant of the single word semantic priming paradigm was employed to measure priming in target naming following exposure to four problem types, in each operation. These included: standard problems (e.g., \(2 \times 3 = 6\) and \(2 + 3 = 5\)), tie problems (e.g., \(3 \times 3 = 9\) and \(3 + 3 = 6\)), one-problems (e.g., \(N \times 1 = N\) and \(N + 1 = \) next number in counting sequence), and zero-problems (\(N \times 0 = 0\) and \(N + 0 = N\)). As in the third investigation in this series, this methodology effectively enabled the encoding process to be held constant, with comparisons made between the overall priming effects, and the facilitatory and inhibitory effects, produced for each problem type. Again, SOAs of 300 and 1000 ms were employed.

The following paragraphs present an overall analysis of the data obtained in this study (i.e., collapsed across problem type). This analysis was conducted to determine whether differences in priming effects result in this paradigm when additional problem types (i.e., ties, ones and zero-problems) are employed in the stimulus set (cf. standard problems only). The analysis of problem type effects is then briefly summarised before presentation of the fourth manuscript.

2.4.1 Analysis of Data Collapsed Across Problem Type
In the previous three Jackson and Coney (2005, 2006a, 2006b) investigations, the time course of facilitation and inhibition effects was investigated using standard problems only. The effects identified for the skilled participants employed in these three studies are presented in Table 1.

<table>
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<tr>
<th>Effect</th>
<th>Facilitation: Difference (D) = Neutral – Congruent.</th>
<th>Inhibition: Difference (D) = Incongruent – Neutral.</th>
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<td>Addition</td>
<td>11 19</td>
<td>7 6</td>
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</table>

Note. *p < 0.05, **p < 0.01.

As shown in Table 1, for both the multiplication and addition operations, the pattern of facilitation in the congruent standard problem condition is one that emerges at the short SOA and increases somewhat at the long SOA. The inhibition effects observed in the incongruent standard condition at the short SOA appear to be generally similar in magnitude to the facilitation effects observed at this SOA, and are relatively reduced at the long SOA.
The mean reaction times scores for each of the addition and multiplication operations (i.e., collapsed over problem type) in the problem type investigation, are presented in Table 2.

<table>
<thead>
<tr>
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<td>488(54)</td>
<td></td>
</tr>
<tr>
<td>Neutral</td>
<td>473(53)</td>
<td>484(51)</td>
<td></td>
</tr>
</tbody>
</table>

In order to test for an influence of operation on performance, these data were initially entered into an overall repeated measures analysis of variance (ANOVA) involving operation, SOA and prime-target relationship as within group variables. Unlike the earlier investigations utilising this procedure (Jackson & Coney, 2005, 2006a, 2006b), no significant main effect of operation was found in this study ($F(1, 26) = 1.8, MSe = 2827.8, p = 0.190$). This finding possibly reflects the greater proportion of multiplication solutions that were small in magnitude in the present study. That is, 61% of multiplication solutions were less than the highest score in the addition condition (i.e., 18), as compared to 32% in previous research.

As in previous research, the operation variable was not involved in any interactions. Nevertheless, due to the difference in the range of target magnitudes between the addition (i.e., ranging between 2 and 18) and multiplication (ranging between 0 and 81) operations and previous research indicating an increase in naming time with target magnitude, each operation was analysed separately (Brysbaert, 1995; Jackson & Coney, 2005a, 2005b, 2005c; Zbrodoff & Logan, 1986).
In the multiplication analysis, a repeated measures ANOVA revealed a significant main effect of prime-target relationship, with facilitation of 27 ms in naming congruent targets and inhibition of 8 ms in naming incongruent targets ($F(2, 52) = 41.7, MSe = 252.8, p < 0.001$). The interaction between SOA and prime-target relationship approached but did not quite reach significance ($F(2, 52) = 2.8, MSe = 135.9, p = 0.072$). Planned repeated measures $t$-test comparisons indicated that there were no significant differences between the levels of facilitation and inhibition observed in the present study and those observed in the earlier investigations (i.e., Jackson & Coney, 2005, 2006a, 2006b).

For the addition operation, a significant main effect of prime-target relationship ($F(2, 52) = 33.9, MSe = 344.2, p < 0.001$) and a significant interaction between SOA and prime-target relationship ($F(2, 52) = 8.7, MSe = 150.3, p = 0.001$) were found. Repeated measures $t$-test comparisons revealed significant facilitation of 10 ms ($t = 4.1, df = 26, p < 0.001$) and inhibition of 17 ms ($t = 5.8, df = 26, p < 0.001$) at the 300 ms SOA. At the long SOA, significant facilitation of 29 ms was found ($t = 5.6, df = 26, p < 0.001$) and the increase in facilitation over time also reached significance ($t = 3.7, df = 26, p = 0.001$). A one-sample $t$-test comparison showed that the 300 ms SOA level of inhibition was significantly greater than that observed in the earlier studies that investigated standard problems only (i.e., 7 ms; $t = 3.4, df = 26, p = 0.002$).

In summary, in the multiplication condition, similar patterns of facilitation and inhibition to those identified in the earlier Jackson and Coney (2005, 2006a, 2006b) studies were found at both SOAs. In contrast, in the addition condition, whilst a similar level of facilitation to that obtained in the previous research was found, the level of inhibition at the short SOA was significantly greater than the
mean level of inhibition obtained in these studies. This latter finding should be considered when deciding whether to include or exclude other problem types (i.e., tie, one and zero-problem types) from addition stimulus sets in future research employing the present paradigm.

2.4.2 Problem Type Analysis

The fourth manuscript focused specifically on an analysis of problem type effects. To enable this, the original naming time measures were first adjusted to control for a possible confound of target magnitude. For example, congruent targets in the multiplication one condition ranged between 2 and 9, whilst congruent targets in the standard condition ranged between 8 and 63 (see Appendix A). Similarly, in the multiplication zero condition the correct target was always 0, whilst the incorrect targets ranged between 4 and 56. The raw data were first entered into regression analyses to produce a model of best fit between naming time and number magnitude for each operation. The multiplication model was then used to calculate predicted reaction time scores. Residuals were computed by subtracting the predicted reaction times from the observed reaction times and the multiplication analyses were conducted on these data. The addition model suggested a weak negative relationship between number magnitude and reaction time. Therefore, it was not necessary to adjust for the effects of target magnitude and the addition analyses were conducted on the raw data.

The results revealed similar patterns of priming effects for all problem types (except multiplication zero-problems) to that found in standard problem performance. Thus, tie, one and addition zero-problems appear to be stored in memory as facts and accessed directly. However, a zero-target naming disadvantage was found in the multiplication zero-problem condition that made interpretation of
the results for this problem type difficult. This finding was explained in terms of competition or interference in verbal responding due the large number of terms that may be activated in memory following exposure to the target ‘0’ (e.g., ‘zero,’ ‘nil,’ ‘nought,’ or ‘oh’).
Abstract

This study investigated the notion that different problem types (e.g., tie: $4 + 4$, zero: $4 + 0$, one: $1 + 4$, and standard problems: $3 + 4$) may be accessed from memory differently. University students participated in a priming procedure, with target naming task. Participants were presented with addition and multiplication problems represented in one of three prime-target relationships, including congruent (e.g., ‘$2 + 3$’ and ‘$5$’), incongruent (e.g., ‘$9 + 7$’ and ‘$5$’) and neutral (e.g., ‘$X + Y$’ and ‘$5$’) conditions. The results indicated that solutions to the different problem types (except for multiplication zero-problems) are accessed from memory in much the same way. These findings show that one and addition zero-problems are stored in memory as facts and are accessed directly, and suggest that the advantage in access to tie problems identified in earlier investigations resulted from encoding processes.

PsycINFO classification: 2343; 2346
Key Words: Arithmetic, Problem Type, Priming, Ties, Zero-rule
1. Introduction

In the cognitive arithmetic literature the single digit operands describing simple arithmetic problems can be used to divide them into various problem types. For example, *standard problems* generally comprise two different operands that are greater than or equal to 2 and less than or equal to 9 (e.g., \(2 \times 3 = 6\) and \(2 + 3 = 5\)). In contrast, *one-problems* and *zero-problems* are defined by their inclusion of the single digit operands 1 (e.g., \(3 \times 1 = 3\) and \(3 + 1 = 4\)) and 0 (e.g., \(3 \times 0 = 0\) and \(3 + 0 = 3\)) and *tie problems* involve the repetition of a single operand such as \(3 \times 3 = 9\) and \(3 + 3 = 6\) (see also Campbell & Oliphant, 1992, and LeFevre et al., 1996a, for reference to *five-times*, *nines rule* and *sum to ten* problems). Also within this literature, it is generally recognised that a number of different retrieval methods can be employed to access the solutions to single digit problems. Examples of these include direct retrieval from memory (e.g., automatically knowing and stating the answer 4 to a problem such as \(2 \times 2\)), calculation procedures such as transformations and counting (e.g., \(7 + 4 = 7 + 3 + 1\)), and rule based retrieval methods (e.g., \(N \times 0 = 0\)) (LeFevre, Sadesky & Bisanz, 1996; LeFevre et al, 1996a). Evident in the preceding examples, is the implication that the problem’s type may influence the retrieval method that is ultimately employed to access its solution.

Support for the notion that problem type can influence the fact retrieval process stems from studies that investigate the processing of addition and multiplication zero and one-problems (Ashcraft, 1992). Previous research shows that the verification of multiplication zero-problems is a slow and error prone process, whilst production research indicates that zero and one-problems are solved relatively quickly (Miller, Perlmutter & Keating, 1984; Stazyk, Ashcraft & Hamann, 1982). These findings are generally attributed to a briefer learning period than standard
problems in which a rule (i.e., \(N \times 0 = 0, N + 0 = N, N \times 1 = N\)) is learnt in place of a fact (see below for solution of \(N + 1\) problems) (Stazyk et al., 1982). Support for this conclusion stems from the finding of a much lower frequency of presentation of zero and one-problems in arithmetic texts (Hamann & Ashcraft, 1986). Furthermore, clinical research indicates a dissociation between rule-based processing of zero and one-problems, and the solution of standard problems via direct fact retrieval from memory (e.g., see Sokol, McCloskey, Cohen & Aliminosa’s, 1991, case study of patient PS).

However, self report research involving non-clinical samples suggests that zero and one-problems are solved in much the same way as standard problems. Research by LeFevre et al. (1996a, 1996b) and Campbell and Xue (2001) reveals an almost exclusive reliance on direct fact retrieval in the solution of all zero and one-problems, except for addition one-problems (which are possibly solved using the verbal counting sequence). Thus, it may be the case that the learning of rules, together with practice, leads to zero and one-facts being stored in memory and accessed directly (Sokol et al., 1991). Unfortunately, such a proposition is difficult to test using the standard production and verification tasks in which accuracy and reaction time measures are dependent on both encoding and fact retrieval mechanisms. That is, any advantage in the performance of zero and one-problems could actually begin at the encoding stage of processing through the earlier activation of correct solutions by their presentation within the problem itself (e.g., \(3 \times 1 = 3\) cf. \(3 \times 7 = 21\)).

The difficulty associated with determining whether encoding or fact retrieval mechanisms are responsible for performance differences between problem types is perhaps, best exemplified in the case of tie problems. Tie problems are perhaps, the
most widely investigated of all of the problem types. Like standard problems, they can not be solved via a general rule, and the relationship between pairs of operands and their solution needs to be known for each different pair. Nevertheless, tie problems lead to slower responses in number matching tasks in which participants are first presented with a pair of numbers (e.g., 4 + 4) and then following a given inter-stimulus interval, are required to decide whether a target number (e.g., 8) was one of the two numbers originally presented (LeFevre & Kulak, 1994). Furthermore, tie problems produce minimal problem size effects, and are generally solved more quickly and accurately than standard problems (Blankenberger, 2001; Campbell & Gunter, 2002; Groen & Parkman, 1972). Consequently, three main fact retrieval accounts of the tie-advantage have been offered. Firstly, Campbell and Gunter (2002) suggest that ties may receive more practice during initial learning, thereby leading to stronger associations and possibly greater activation between tie problems and their correct solutions. Secondly, they note that the two different operands in non-tie problems may activate two families of answers in memory, whilst a single repeated operand in tie problems will only activate one family, leading to less interference in retrieval. Thirdly, Campbell and Oliphant (1992) propose that the tie-advantage results from a partial dissociation between ties and non-ties in memory, such that they are categorically distinct. Consequently, ties may lead to only weak activation of non-ties and less interference in performance.

In contrast, two main encoding accounts of the tie-advantage exist. In the first of these, Gallistel and Gelman (1992) suggest that the processing of arithmetic problems first entails the mapping of the problem’s operands onto a mental magnitude representation. In tie problems, given that the same mental magnitude is represented twice, this mapping process would only have to be undertaken once and
would thus be less time consuming and produce fewer errors. Secondly, Blankenberger (2001) offered a much simpler encoding explanation for the tie-advantage i.e., that it arises due to the fast encoding of perceptually identical operands. To test this hypothesis, Blankenberger (2001) asked participants to produce solutions to tie and non tie addition and multiplication problems represented in number pairs that were either homogeneous (3 + 4, four x four) or heterogeneous (three + 4 or 4 x four) in surface form. The authors reasoned that if the tie effect resulted from fast fact retrieval processes then a faster response time should be found for heterogeneous ties than heterogeneous non-ties. No such effect was found, with significant tie effects observed for homogeneous number pairs only. Blankenberger concluded therefore, that tie effects are not fact retrieval based but instead result from the faster encoding of perceptually identical stimuli.

However, a partial replication of the Blankenberger (2002) study by Campbell and Gunter (2002) indicated that the surface form mis-match in the heterogeneous number pairs had interfered with the processing of the two operands as the same numerosity in such a way that it offset an already reduced tie-advantage (Cambell & Gunter, 2002). To address this Campbell and Gunter (2002; Experiment 1) re-examined the tie effect using Arabic digits only. In this study, Asian Chinese and non-Asian Canadian participants were first asked to solve small and large tie and non-tie problems represented in all four basic operations and then were required to report their solution strategy. The authors reasoned that if tie effects are due to an encoding advantage then they should be found for problems of both sizes, in participants of all skill levels, and they should be equivalent for the addition and multiplication operations, which require encoding of exactly the same operands. Additionally, they noted that tie effects should not be found for subtraction and
division operations because there is no repetition of operands in these problems (e.g., 16 – 8 and 49 ÷ 7). In contrast to this, the reaction time results revealed no tie effect for small problems, for either group in the addition and multiplication conditions. Furthermore, the more skilled Asian Chinese group showed no tie effect for small or large addition problems, a finding consistent with the notion that these participants had highly developed direct memory access to addition solutions. Finally, a greater tie effect was found for large multiplication problems than large addition problems, and a large tie effect was found for the subtraction and division operations. Self report data reinforced these findings. Consistent with faster fact retrieval accounts of tie effects, participants reported substantially less use of time consuming calculation strategies for ties than non-ties in the addition, subtraction, and division operations.

The Campbell and Gunter (2002) investigation therefore, appeared to offer comprehensive support for the notion that tie effects result from an advantage in direct memory access to tie solutions. However, a number of methodological issues raise serious doubts as to the validity of this conclusion. Firstly, the assumption that the magnitude of the tie effect should be the same for the addition and multiplication operations does not allow for differences in fact retrieval and production processes between operations (e.g., due to solution magnitudes) that may occur after encoding. Secondly, an unavoidable confound of reaction times for subtraction and division tie problems is that the correct solution to each problem is contained within the actual problem itself (e.g., 49 ÷ 7 contains the correct solution 7). This may lead to earlier activation and hence, an unfair advantage in access to this solution. Thirdly, the self report measures employed in this investigation may have lead to reactivity. Smith-Chant and LeFevre (2003) found that low skilled participants respond more slowly and accurately when asked to report their solution strategies for large problems,
exhibit wider variation in their choice of solution strategies, and are more likely to change their selection with changes in instructions. Finally, in relation to the Campbell and Gunter (2002) study, it is questionable as to whether a distinction between ‘slow’ calculation of non-tie problems and ‘fast’ direct fact retrieval of tie solutions provides an effective account of the tie-advantage. For instance, the self report measures indicated that multiplication trials were solved mainly via direct fact retrieval. Nonetheless, the reaction time data produced strong tie effects. Moreover, this finding was robust for all other operations when calculation trials were excluded from the analyses. At the very least, these results indicate that factors other than the method of solution retrieval may be responsible for the tie problem advantage and thus, add weight to the argument that it originates at the encoding stage of processing.

From the previous discussion, it is clear that what is needed in the investigation of problem type differences in fact retrieval mechanisms is a procedure that removes the influence of problem encoding from reaction time measures. One such procedure is the priming procedure recently utilised in a series of investigations by Jackson and Coney (2005, 2006a, 2006b). In this procedure, simple addition and multiplication problems are assigned to three prime-target relationship conditions i.e., congruent (‘2 + 3’ and ‘5’), incongruent (‘2 + 3’ and ‘14) and neutral (‘X + Y’ and ‘5’) conditions. In each trial, participants are presented with a prime and then following a brief inter-stimulus interval, are presented with the target number and required to name it. The effects of the congruent and incongruent conditions are assessed by subtracting the reaction time taken to name the targets in each of these conditions from the reaction time taken to name the target following exposure to the neutral prime. Consistent with Neely (1991), if the resulting difference is positive it
is referred to as facilitation and if it is negative it is referred to as inhibition. Furthermore, by subtracting the reaction time taken to name the targets in the congruent condition (e.g., ‘2 + 3’ and '5) from the reaction time taken to name the targets in the incongruent condition (i.e., ‘2 + 3’ and ‘14’) an overall priming effect that is independent of encoding times can be produced for each problem type. Finally, by adjusting the stimulus onset asynchrony (SOA i.e., the time period between the onset of the prime and the presentation of the target), time differences in the activation of the solutions corresponding to the different problem types can be investigated. An SOA of 300 ms is too short to enable strategic processing of the prime (e.g., via the use of rules or transformations) before presentation of the target (Velmans, 1999). This, coupled with the use of the target naming procedure instead of calculation, ensures that any facilitation observed at this SOA reflects the direct activation of facts from memory. In contrast, at 1000 ms, there is ample time to apply a strategy for correct solution retrieval, leading to facilitation when a congruent target is presented and inhibition when an unexpected incongruent target is presented.

In the previous three Jackson and Coney (2005, 2006a, 2006b) studies, the facilitation and inhibition effects produced following exposure to standard problems were investigated. In both the multiplication and addition operations, the general pattern of facilitation observed in the congruent standard problem condition was one that emerged at brief SOAs of 240 ms and 300 ms, and increased at 1000 ms. The inhibition effects observed in the incongruent condition at the short SOAs were similar in magnitude to the facilitation effects observed at these SOAs, and were relatively reduced at the long SOA. Thus, given the use of the same methodology and a similar sample, the present study first predicts a pattern of standard problem
performance that is similar to that observed in the earlier studies. Secondly, given that tie problems cannot be solved via a rule and therefore, the strong possibility that the tie-advantage in previous research originated at the encoding stage of processing, it is predicted that an overall pattern of priming effects similar to that observed for standard problems will be found. Finally, if zero and one-problems are solved via conscious processing strategies (i.e., via rules or the verbal counting sequence), no significant facilitation in congruent target naming should be observed at the short SOA, and a pattern of increased facilitation in naming congruent targets and increased inhibition in naming unexpected incongruent targets should be observed at the long SOA.

2.0 Method

2.1 Participants

Twenty seven psychology students from Murdoch University, including five males and 22 females, participated in the present study. The participants’ ages ranged from 19 to 54, with a mean age of 31 years. The participants’ mean correct score on the arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) was 24 (SD = 5.00). This score was equivalent to a percentile rank of 61% in a normed group of 973 administrative officer or administrative assistant applicants that had completed year 11 and 12. All students received course credit for their participation.

2.2 Design and stimulus materials

Four repeated measures variables were investigated in the present study. The first three included arithmetic operation (addition and multiplication), SOA (300 ms and 1000 ms) and prime-target relationship (congruent e.g., 2 x 4 = 8, incongruent
e.g., 2 x 4 = 0 and neutral e.g., X x Y = 8 conditions). The fourth variable was problem type, with four levels including tie (e.g., 2 x 2 = 4), one (e.g., 2 x 1 = 2), zero (e.g., 2 x 0 = 0) and standard (e.g., 2 x 4 = 8) problems.

Two sets of primes containing 36 problems (i.e., 8 from each problem type) were constructed for each operation. The first of these for each operation is presented in Appendix A. The second set comprised the reverse operand placement equivalent problems to the first set. Each prime set and problem type was balanced for operand size so that half of the non-tie problems had the smallest number as the left operand and half had the smallest number as the right operand. Standard problems were balanced for problem size with half of the problems comprising operands less than or equal to 5 and half comprising operands greater than 5. The digits 2 through 9 were used equivalently to construct one-problem in each of the tie, one and zero sets. The correct solutions corresponding to the 36 primes were employed as targets in each of the congruent, incongruent and neutral conditions. In the incongruent condition, the correct solutions were paired with an alternative problem so that they were mathematically erroneous (see Appendix A for incongruent prime-target pairings). The neutral primes employed in the present study were ‘X + Y’ and ‘X x Y’ stimuli. The reaction times taken to name target numbers following exposure to these stimuli have previously provided a useful baseline to which to compare congruent and incongruent reaction times in the demonstration of facilitatory and inhibitory effects in simple arithmetic processing (e.g., see Jackson & Coney, 2006a, 2006b).

2.4 Procedure

Participants were individually tested in a well-lit cubicle room containing an Amiga 1200 microcomputer, with 1084S monitor. This system controlled stimulus presentation, trial sequencing, timing and data collection. Stimuli were presented
centrally on the screen, white against an amber background. Individual operands did not exceed dimensions of 5 x 15 mm and were placed 5 mm either side of the arithmetic operator (i.e., the x or + sign), which did not exceed dimensions of 5 x 10 mm. A chin rest situated 60 cm directly in front of the screen was used to stabilise the participant’s head during testing.

Each participant completed four blocks of 108 experimental trials, i.e., one block for each of the addition and multiplication operations, represented at each of the 300 and 1000 ms SOAs. Addition and multiplication trials were blocked separately so as not to induce cross operation errors and exposure to all stimuli was counterbalanced across participants. That is, half of the participants completed the addition condition first and half completed the multiplication condition first. At the short SOA, for each operation, half of the participants were exposed to the first set and half were exposed to their reverse operand placement equivalents. To enable a level of familiarity with the stimuli and to draw attention to the prime-target relationship, each participant was then exposed to the exact same set at the long SOA. This process was repeated in the third and fourth blocks using the operation not tested in the first two blocks. Within each block, the computer randomly generated the order of presentation of all problem types and all prime target relationships.

Instructions to the participants placed equal emphasis on responding both quickly and accurately. Trials began with participants focussing their gaze on a 1 x 1 mm blue central fixation dot that was exposed for 600 ms. The screen then went blank for a duration of 150 ms after which, the prime was presented for 100 ms. The target number was presented after the given SOA (i.e., either 300 or 1000 ms) and remained exposed until the participant verbally named the number. The time taken to
verbally respond following the onset of the target was recorded via a microphone connected to a headset. The microphone amplifier triggered an electronic relay that was interfaced to the computer and determined the time of relay closure using a hardware timer that was accurate to 1 millisecond. Ear defenders were used to block out external noise intrusions and the experimental session took approximately 25 minutes to complete.

3.0 Results

To enable an analysis of problem type effects, the original naming time measures were first adjusted to control for a possible confound of target magnitude. For example, congruent targets in the multiplication one-problem condition ranged between 2 and 9, whilst congruent targets in the standard condition ranged between 8 and 63 (see Appendix A). Similarly, in the multiplication zero-problem condition the correct target was always 0, whilst the incorrect targets ranged between 4 and 56. Thus, the raw data were entered into regression analyses and the best fitting model between the overall mean naming time and number magnitude for each operation was produced.

3.1 Multiplication Analysis

The model of best fit together for the multiplication operation was: \( \text{Naming Time} = 0.233(\text{Number Magnitude}) + 468 \). To adjust the naming time scores for the effect of target magnitude, predicted naming time scores were computed and then residuals were calculated by subtracting the predicted naming times from the observed naming times. The problem type analyses were then undertaken on both the raw data (i.e., unadjusted for target magnitude) and the residual data (adjusted for
target magnitude). Both sets of data generally produced equivalent effects and so,
only the residual analysis is reported here.

The residual multiplication data were initially screened for outliers using a
cut off criterion of +/- 2.5 z-scores. Consequently, 0.008% of these scores were
replaced using mean substitution. The resulting residual naming time scores are
presented in Table 1.

| Table 1. |
| Raw Mean Naming Time and Residual Naming Time Scores (ms) for the Multiplication Operation, and Raw Mean Naming Times (ms) for the Addition Operation (with Standard Deviations in Parentheses) for all Prime-Target Relationships at each SOA. |

### Raw Mean Naming Times for Multiplication

<table>
<thead>
<tr>
<th>SOA:</th>
<th>Problem Type</th>
<th>300 ms</th>
<th>1000 ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tie</td>
<td>Congruent</td>
<td>453(49.3)</td>
<td>469(47.3)</td>
</tr>
<tr>
<td></td>
<td>Incongruent</td>
<td>483(61.6)</td>
<td>459(73.6)</td>
</tr>
<tr>
<td></td>
<td>Neutral</td>
<td>469(47.3)</td>
<td>483(65.6)</td>
</tr>
<tr>
<td>One</td>
<td>Congruent</td>
<td>452(59.5)</td>
<td>462(49.4)</td>
</tr>
<tr>
<td></td>
<td>Incongruent</td>
<td>492(69.4)</td>
<td>452(65.1)</td>
</tr>
<tr>
<td></td>
<td>Neutral</td>
<td>462(49.4)</td>
<td>497(66.7)</td>
</tr>
<tr>
<td>Zero</td>
<td>Congruent</td>
<td>479(78.4)</td>
<td>491(67.6)</td>
</tr>
<tr>
<td></td>
<td>Incongruent</td>
<td>474(39.8)</td>
<td>475(72.1)</td>
</tr>
<tr>
<td></td>
<td>Neutral</td>
<td>491(67.6)</td>
<td>475(58.6)</td>
</tr>
<tr>
<td>Standard</td>
<td>Congruent</td>
<td>459(56.9)</td>
<td>472(52.2)</td>
</tr>
<tr>
<td></td>
<td>Incongruent</td>
<td>480(63.4)</td>
<td>462(65.2)</td>
</tr>
<tr>
<td></td>
<td>Neutral</td>
<td>472(52.2)</td>
<td>488(61.7)</td>
</tr>
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</table>

### Residual Naming Time Scores for Multiplication

<table>
<thead>
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<th>Tie</th>
<th>One</th>
<th>Zero</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>-23(49.3)</td>
<td>-18(59.1)</td>
<td>6(71.9)</td>
<td>-17(56.8)</td>
</tr>
<tr>
<td>9(61.6)</td>
<td>22(69.6)</td>
<td>1(39.8)</td>
<td>4(56.9)</td>
</tr>
<tr>
<td>-6(48.3)</td>
<td>-7(49.4)</td>
<td>16(56.2)</td>
<td>-3(52.1)</td>
</tr>
<tr>
<td>-18(73.5)</td>
<td>-17(65.0)</td>
<td>7(72.2)</td>
<td>-14(65.2)</td>
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<tr>
<td>9(65.5)</td>
<td>26(66.8)</td>
<td>2(58.0)</td>
<td>12(52.5)</td>
</tr>
<tr>
<td>12(53.6)</td>
<td>4(52.5)</td>
<td>28(63.7)</td>
<td>18(58.8)</td>
</tr>
</tbody>
</table>

### Raw Mean Naming Times for Addition

<table>
<thead>
<tr>
<th>Tie</th>
<th>One</th>
<th>Zero</th>
<th>Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>453(53.0)</td>
<td>440(54.0)</td>
<td>452(63.5)</td>
<td>444(59.4)</td>
</tr>
<tr>
<td>482(60.8)</td>
<td>478(58.4)</td>
<td>479(57.5)</td>
<td>475(51.6)</td>
</tr>
<tr>
<td>463(60.1)</td>
<td>458(55.7)</td>
<td>459(57.0)</td>
<td>461(58.6)</td>
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<tr>
<td>447(68.2)</td>
<td>449(62.5)</td>
<td>449(70.4)</td>
<td>456(60.2)</td>
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<tr>
<td>474(60.1)</td>
<td>494(63.6)</td>
<td>482(67.5)</td>
<td>478(53.2)</td>
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<tr>
<td>473(63.8)</td>
<td>475(62.0)</td>
<td>482(63.0)</td>
<td>474(68.4)</td>
</tr>
</tbody>
</table>

A repeated measures ANOVA involving problem type, SOA and prime-target
relationship was carried out on the residual multiplication data. Significant main
effects of problem type ($F(3, 78) = 3.2, MSe = 1586.7, p = 0.029$) and prime-target
relationship were found ($F(2, 52) = 23.3, MSe = 1374.2, p < 0.001$). These effects
were qualified by a significant interaction between problem type and prime target
relationship \((F(3.5, 90.2) = 5.1, \text{MSe} = 2353.0, p = 0.002)\). One-way repeated measures ANOVAs involving prime-target relationship as the within group variable were undertaken for each problem type. Significant facilitation in congruent target naming (i.e., as compared to the neutral condition) and significant overall priming effects (i.e., incongruent – congruent naming times) were found in the tie \((F(2, 52) = 18.8, \text{MSe} = 344.5, p < 0.001)\) and standard problem conditions \((F(2, 52) = 5.8, \text{MSe} = 838.2, p = 0.005)\). For one-problems, significant facilitation, inhibition (i.e., incongruent – neutral condition naming times) and overall priming effects \((F(2, 52) = 25.7, \text{MSe} = 462.9, p < 0.001)\) were found. In the zero-problem condition, facilitation in naming both congruent and incongruent targets was found \((F(2, 25) = 4.9, p = 0.016)\). A one-way repeated measures ANOVA indicated that the levels of facilitation did not vary significantly between problem types \((F(3, 78) = 0.5, \text{MSe} = 970.3, p = 0.688)\). However, the level of inhibition observed for one-problems was significantly greater than for all other problem types \((F(2.3, 59.8) = 7.6, \text{MSe} = 1672.3, p = 0.001)\) and the overall priming effect observed in the zero-problem condition was significantly less than that observed in the one and tie problem conditions \((F(1.8, 47.4) = 5.8, \text{MSe} = 3012.2, p = 0.007)\).

No significant three way interaction between problem type, SOA and prime-target relationship was identified in the multiplication analysis. Nevertheless, in view of an interest in problem type differences in priming effects over time, repeated measures t-test analyses were undertaken at each SOA. Due to the large number of comparisons undertaken, the alpha level was reduced to a more conservative level of 0.008, using a Bonferroni adjustment (i.e., 0.005/6). The facilitation, inhibition and overall priming effects for each problem type, at each SOA, are presented in Fig 1.
Fig. 1. Showing the priming effects observed for each problem type in the multiplication and addition conditions at each SOA (facilitation = neutral – congruent condition naming times, inhibition = incongruent – neutral condition naming times, and overall priming = incongruent – congruent condition naming times).
At the short SOA, facilitation in congruent target naming was observed for tie problems only \((t(26) = 2.9, p = 0.008)\). At the long SOA, facilitation in congruent target naming was observed for ties \((t(26) = 3.4, p = 0.002)\), ones \((t(26) = 3.4, p = 0.002)\) and standard problems \((t(26) = 3.8, p = 0.001)\). Nevertheless, no significant differences in the levels of facilitation were observed between problem types at either SOA. Significant inhibition in incongruent target naming was observed for one-problems only, at both the short \((t(26) = 3.5, p = 0.002)\) and the long SOAs \((t(26) = 3.0, p = 0.006)\). However, no significant differences in the levels of inhibition were observed between tie, one and standard problems at either SOA. No inhibitory effect was observed in the zero problem condition. Instead, a reverse effect occurred with target naming in the neutral condition found to be inhibited in comparison to the incongruent condition at the long SOA (i.e., tested at alpha = 0.05; \((t(26) = 2.3, p = 0.028)\). Notably, the zero-problem neutral condition involved naming the target ‘0’ only, thereby suggesting a zero-target naming disadvantage in performance. Significant overall priming effects were observed for tie problems at both the short \((t(26) = 4.3, p < 0.001)\) and long SOAs \((t(26) = 3.2, p < 0.003)\). Similarly, significant overall priming effects were observed for one problems at both the short \((t(26) = 5.4, p < 0.001)\) and the long SOAs \((t(26) = 6.3, p < 0.003)\). The overall priming effects for one problems were significantly greater than those observed for zero-problems at the short \((t(26) = 3.4, p = 0.002)\) and the long SOAs \((t(26) = 2.9, p = 0.008)\). No significant differences in overall priming were observed between tie, one and standard problems at either SOA.

To further explore the possibility of a zero-target naming disadvantage, one way repeated measures ANOVAs involving only the neutral condition data were undertaken at each SOA. At the short SOA, target naming in the zero-problem
neutral condition was significantly slower than in the tie-problem condition by 22 ms ($t(26) = 3.2, p = 0.003$), significantly slower than in the one-problem condition by 23 ms ($t(26) = 3.0, p = 0.006$), and significantly slower than in the standard problem condition by 19 ms ($t(26) = 2.1, p = 0.044$). At the long SOA, target naming in the standard problem neutral condition was significantly slower than in the one problem condition by 13 ms ($t(26) = 2.4, p = 0.025$) and target naming in the zero-problem neutral condition was significantly slower than in the one-problem condition by 24 ms ($t(26) = 2.3, p = 0.026$).

Given that the zero-problem congruent condition was the only other condition in which the solution was always zero, one way repeated measures ANOVAs involving these data were also undertaken. At the short SOA, target naming in the tie problem and one-problem conditions was 29 ms ($t(26) = 2.6, p = 0.015$) and 24 ms ($t(26) = 2.3, p = 0.031$) faster than in the zero-problem condition, respectively. Similarly, at the long SOA, target naming in the tie problem and one-problem conditions was 25 ms ($t(26) = 2.3, p = 0.028$) and 24 ms ($t(26) = 2.6, p = 0.017$) faster than in the zero-problem condition, respectively. No other differences in the naming times of congruent solutions to the different problem types were found.

Finally, single-sample $t$-test comparisons comparing the facilitatory effects found in standard problem performance in the present study to that found in the earlier studies revealed no differences at the short (previous $M = 12$; $t(26) = -0.9, p = 0.399$) or long SOA (previous $M = 29$; $t(26) = -0.1, p = 0.940$). Similarly, single-sample $t$-test comparisons comparing the inhibitory effects found in standard problem performance in the present study to that found in the earlier studies revealed no differences at the short (previous $M = 11$; $t(26) = 0.6, p = 0.554$) or long SOA (previous $M = 7$; $t(26) = -1.4, p = 0.170$).
In summary, similar patterns of priming effects were observed in naming solutions to tie, one and standard problems in the multiplication analysis. This suggests that solutions to these multiplication problems are accessed from memory in the same way. However, an analysis of the multiplication zero-problem data revealed a zero-target naming disadvantage.

### 3.2 Addition Analysis

The model of best fit for the addition analysis was: \( \text{Naming Time} = -0.070(\text{Number Magnitude}) + 467 \). This model suggests a weak negative relationship between number magnitude and reaction time. Thus, it was not necessary to adjust for the effects of target magnitude in this analysis. Instead, the addition analyses were performed on the raw data, the means for which are presented in the bottom half of Table 1.

A repeated measures ANOVA involving problem type, SOA and prime-target relationship was carried out on the addition data. A significant main effect of prime-target relationship \( (F(2, 52) = 40.3, MSe = 1362.2, p < 0.001) \) and a significant interaction between SOA and prime-target relationship \( (F(2, 52) = 4.0, MSe = 698.0, p = 0.024) \) were found. At the short SOA, target naming in the congruent condition was significantly faster than in the neutral condition by 13 ms \( (t(26) = 3.9, p = 0.001) \) and target naming in the incongruent condition was significantly slower than in the neutral condition by 18 ms \( (t(26) = 5.9, p < 0.001) \). At the long SOA, target naming in the congruent condition was significantly faster than in the neutral condition by 26 ms \( (t(26) = 5.5, p < 0.001) \), with no significant difference between naming in the incongruent and neutral conditions.
A significant interaction between problem type and SOA was also identified \( (F(3, 78) = 3.1, MSe = 487.8, p = 0.033) \). At the short SOA, a one way repeated measures ANOVA showed that target naming in the tie problem condition was significantly slower than in the one-problem condition by 7 ms, and significantly slower than in the standard problem condition by 6 ms \( (F(3, 78) = 3.2, MSe = 86.6, p = 0.026) \). At the long SOA, no significant difference in response times was observed.

A repeated measures \( t \)-test analysis of the one-problem data revealed a significant increase in response times between the short and long SOAs of 14 ms \( (t = 2.2, df = 26, p = 0.037) \). No other change in response times was observed for the other three problem types over time.

No other significant effects were observed in the addition data. Nevertheless, in view of an interest in priming effects for each problem type over time, repeated measures \( t \)-test analyses were again undertaken at each SOA (see priming effects illustrated in Fig 1). As in the multiplication analysis, a more conservative alpha level of 0.008 was employed. At the short SOA, significant facilitation in congruent target naming was observed in the one \( (t(26) = 4.2, p = 0.001) \) and standard problem \( (t(26) = 3.0, p = 0.006) \) conditions only. At the long SOA, significant facilitation in congruent target naming was observed for tie \( (t(26) = 3.9, p = 0.001) \), one \( (t(26) = 4.9, p < 0.001) \) and zero problems \( (t(26) = 3.3, p = 0.003) \). However, no significant differences in the levels of facilitation were observed between problem types at either SOA. Significant inhibition in incongruent target naming was observed at the short SOA for all problem types i.e., for tie \( (t(26) = 3.2, p = 0.004) \), one \( (t(26) = 4.2, p < 0.001) \), zero \( (t(26) = 3.4, p = 0.002) \), and standard problems \( (t(26) = 3.1, p = 0.005) \). No significant difference in the levels of inhibition was observed between problem types and no significant inhibitory effects were observed at the long SOA.
Significant overall priming effects were observed at the short SOA for all problem types i.e., for tie ($t(26) = 5.0, p < 0.001$), one ($t(26) = 6.0, p < 0.001$), zero ($t(26) = 3.2, p = 0.003$) and standard ($t(26) = 6.2, p < 0.001$) problems. At the long SOA, significant overall priming effects were observed for one ($t(26) = 5.2, p < 0.001$), zero ($t(26) = 3.1, p = 0.004$) and standard problems ($t(26) = 3.2, p = 0.004$). The overall priming effect for tie problems approached significance ($t(26) = 2.8, p = 0.009$). No significant differences in the levels of overall priming were observed between problem types at either SOA.

Single-sample $t$-test comparisons comparing the facilitatory effects found in standard problem performance in the present study to that found in the earlier studies revealed no differences at the short (previous $M = 11; t(26) = -1.1, p = 0.300$) or long SOA (previous $M = 19; t(26) = 0.2, p = 0.857$). Similarly, single-sample $t$-test comparisons comparing the inhibitory effects found in standard problem performance in the present study to that found in the earlier studies revealed no differences at the short (previous $M = 7; t(26) = 1.5, p = 0.133$) or long SOA (previous $M = 6; t(26) = -0.3, p = 0.768$).

The results of the addition analysis reveal similar patterns of priming effects in naming solutions to tie, one, zero and standard problems. This suggests that solutions to the different addition problem types are accessed from memory in the same way.

4.0 Discussion

The present study investigated priming effects on target naming following exposure to tie, one, zero and standard problems, to determine whether problem type influences the solution retrieval process. This study addressed three main predictions. Firstly, in view of the similarities between the methodologies employed in the
present and the previous Jackson and Coney investigations (2005, 2006a, 2006b), a pattern of facilitation and inhibition in target naming similar to that previously found in standard problem performance was expected. Comparisons of the levels of facilitation and inhibition produced in each study produced no significant differences in standard problem priming effects between studies, in either the multiplication or the addition analysis.

Consistent with encoding explanations of the tie-advantage found in previous research, the third prediction made in the present study was that solutions to tie and standard problems would be accessed in the same way. Consequently, similar patterns of priming effects were expected for both problem types. The results of the present study confirmed this hypothesis. Comparisons of the levels of facilitation, inhibition and overall priming produced in the tie and standard problem conditions revealed no differences in either operation.

The final hypothesis in the present study related to the solution of zero and one-problems. Given the possibility that zero and one-problems are solved via conscious processing strategies, no significant facilitation was expected at the short SOA. In contrast, a pattern of increased facilitation in naming congruent targets, and increased inhibition in naming incongruent targets, was expected at the long SOA. In the multiplication one-problem and the addition zero-problem conditions, the patterns of facilitation were consistent with this hypothesis, with the facilitation failing to reach significance at the short SOA and appearing to increase over time. Nevertheless, the patterns of facilitation observed for these problems did not differ markedly to that observed in the tie and standard problem conditions (i.e., problems that cannot be solved via rules). Moreover, the levels of inhibition did not increase at the 1000 ms SOA. This latter finding is important because if the participants used
rules (i.e., \( N \times 1 = N \) or \( N + 0 = N \)) strategically to activate the correct solutions in memory and thus to speed the processing of congruent targets, the presentation of unexpected (incongruent) targets should have led to greater levels of inhibition than was observed. Instead, the levels of inhibition either remained relatively constant over time or were reduced somewhat, indicating that strategic processing was not employed before presentation of the targets. Furthermore, it is noteworthy that performance in the multiplication one-problem condition was very similar to that observed in the addition one-problem condition. Like tie and standard problems, addition one-problems can not be solved via a general rule. Indeed, the finding of significant facilitation in solution retrieval at the short SOA, a time period too short to allow for strategic processing of the prime in order to speed processing of the target, suggests that the solutions to the addition one-problems were actually retrieved automatically. The results of the present study therefore, did not support the final hypothesis, instead suggesting a direct access route to solutions of addition and multiplication one-problems, and addition zero-problems.

The finding of a zero-target naming disadvantage in the congruent and neutral zero-problem conditions was surprising given that the zero-problem stimuli (i.e., the neutral prime and the target ‘0’) were presented to participants with far greater frequency than any other stimuli in this study. Consequently, rather than slowed responses, faster responses due to early semantic satiation and the use of less processing capacity in performance would have been expected. In explanation of this result, it is noteworthy that although most participants responded with ‘zero’ in the majority of congruent and neutral zero-problem trials, various other terms such as ‘nil’, ‘nought’ or ‘oh’ were occasionally produced. It is therefore, plausible that, unlike other numbers, following exposure to the stimulus ‘0,’ a number of terms may
be activated in memory that compete or interfere with the production of the final verbal response.

The present study employed a priming procedure that enabled the comparison of congruent condition naming times with incongruent condition naming times, thereby producing an overall priming effect for each problem type that was independent of encoding effects. This manipulation revealed similar priming effects for all problem types, except for multiplication zero-problems, the reaction times for which were influenced by a zero-target naming disadvantage. Thus, in the context of the present study, the results showed that, like standard problems, one and addition zero-problems are stored in memory as facts and are accessed directly. Furthermore, they suggest that the advantage in access to tie problems identified in earlier investigations resulted simply due to encoding processes.

References


Appendix A

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<tr>
<th>Problem Type</th>
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*Note. Neutral condition primes were X x Y for multiplication and X + Y for addition. Neutral condition targets were the same as congruent condition targets.*
2.5. The Reversed Split Effect.

The final study investigated whether the split between correct and incorrect solutions to simple arithmetic problems effects the distribution of reaction times produced in the context of arithmetic priming tasks. The same procedure employed in the earlier investigations was again employed. However, four prime-target relationship conditions were utilized, including: a congruent condition (e.g., 3 + 5 and 8), a close incongruent condition (e.g., 3 + 5 and 7), a distant incongruent condition (e.g., 3 + 5 and 13) and a neutral condition (e.g., X + Y and 8). A high skilled sample and SOAs of 300 and 1000 ms were again employed.

Consistent with Campbell’s (1987) findings, the results showed that the time taken to name distant incongruent targets was significantly greater than the time taken to name close incongruent targets in all conditions. Additionally, some interesting priming effects were found. Congruent target naming was facilitated in the multiplication condition at 300 ms and 1000 ms, and in the addition condition at 1000 ms. Incongruent target naming was inhibited at 300 ms only. In the multiplication condition, inhibition occurred in distant incongruent target naming, whilst in the addition condition, inhibition occurred in both close and distant incongruent target naming.

As in the earlier surface form investigation, the facilitation function was again explained in terms of an automatic spreading activation mechanism operating on visual representations of the arithmetic stimuli. However, in contrast to the earlier surface form investigation (which employed numerical word stimuli), it seemed less likely that the inhibitory mechanism operated on phonological representations in this study. Accordingly, the inhibitory effects were explained in terms of the obligatory activation of a numerical magnitude representation that is analogous to a mental
number line and is logarithmic in nature. It was suggested that, at the short SOA, this representation influenced the participant’s confidence in their response. When the presented target was further away from the activated region, the participant was less confident and it took longer to name the target. At the long SOA, the activation of this representation had diminished so that little inhibition was found.
Simple Arithmetic Processing: The Reversed Split Effect

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Abstract

This study aimed to explore the nature of split effects in an arithmetic based variant of the single word semantic priming paradigm, with naming task. Single digit addition and multiplication problems were presented to participants in each of four prime-target relationship conditions, including congruent (e.g., 3 + 5 and 8), close incongruent (e.g., 3 + 5 and 7), distant incongruent (e.g., 3 + 5 and 13), and neutral conditions (e.g., X + Y and 8). The results revealed a reversed split effect in all conditions, with close incongruent targets named significantly faster than distant incongruent targets in every condition of the present study. Additionally, congruent target naming was facilitated in the multiplication condition at 300 and 1000 ms, and in the addition condition at 1000 ms. Target naming in the distant incongruent condition was inhibited at 300 ms in both the multiplication and addition operations, whilst target naming in the close incongruent condition was inhibited at 300 ms in
the addition condition only. Explanations of the facilitation effects in terms of automatic spreading activation, and the inhibitory effects in terms of the activation of a magnitude representation, are offered.

PsycINFO classification: 2343; 2346

Key Words: Arithmetic, Priming, Reversed Split Effect, Verification, Addition, Multiplication,
1.0 Introduction

In the investigation of simple arithmetic processing, verification tasks require that participants decide quickly and accurately whether an equation is true (e.g., $3 + 5 = 8$) or false (e.g., $3 + 5 = 13$) (Campbell, 1987). A robust finding within the literature is that the verification of true problems occurs rapidly, whilst the verification of false problems occurs more slowly and varies as an inverse function of the split between the correct and the incorrect solution to the presented problem (Ashcraft, 1992; Campbell, 1987; Zbrodoff & Logan, 1990). For example, it takes longer to determine that ‘$3 + 5 = 9$’ (split = 1) is false than it does to determine that ‘$3 + 5 = 13$’ (split = 5) is false. This finding is referred to as the split effect.

Associative network approaches are commonly employed to explain how simple arithmetic facts are organised in and accessed from memory (e.g., see Ashcraft, 1992, Campbell & Graham, 1985, and Campbell, 1987). At the basis of associative network approaches is the assumption that simple arithmetic facts are stored in a network of associations in which structural links or pathways exist between related nodes (Campbell, 1987). Retrieval begins with the encoding of a problem, leading to the activation of its corresponding node in memory. Activation then spreads automatically throughout the network via these structural links to a set of nodes that are related to the problem (possibly including the correct solution, products related to each operand, the correct solution to a different operation, and near neighbours via the counting sequence) (Ashcraft, 1992; Campbell, 1987; Galfano, Rusconi and Umilta, 2003). The probability of retrieval of the correct solution and the time taken to retrieve this solution is directly related to the level of activation that it receives (Anderson, 1981, 1983; Campbell, 1987). The incidental
activation of other numbers within the related set directly interferes with access to the correct solution (Anderson, 1981, 1983; Campbell, 1987).

Within the context of associative network approaches, split effects may thus be explained in terms of the varying levels of activation received by different solutions (e.g., Campbell, 1987; Stazyk, Ashcraft & Hamann, 1982). In true trials, the correct solution is automatically activated in memory through spreading activation from encoding the problem and via simultaneous exposure to this same number as part of the equation to be verified. Consequently, verification occurs rapidly and accurately. In contrast, in false trials, priming of the correct solution from memory may be interfered with by the direct activation of the simultaneously presented incorrect solution (Campbell, 1987). When the incorrect solution is distant from the correct solution or is unrelated to the problem (or the correct solution e.g., see Galfano et al., 2003), this interference is minimal. However, when the incorrect solution is close to the correct solution or is related to the problem, its activation level may be increased through spreading activation (Campbell, 1987; De Rammelaere, Stuyven & Vandierendonck, 2001; Galfano et al., 2003; Stazyk et al., 1982; Winkelman & Schmidt, 1974; Zbrodoff & Logan, 1986, 1990). As a result, the interference effect may be exacerbated for these equations and consequently, they take longer to verify.

The notion that simple arithmetic facts are organised in an associative network in memory, with solutions retrieved via automatic spreading activation, was investigated recently by Jackson and Coney (2005; also see Jackson & Coney 2006a, 2006b). In this study, arithmetic stimuli were employed in a priming procedure analogous to the single word semantic priming paradigm. Participants were presented with problems as primes and solutions as targets in one of three prime-target
relationship conditions i.e., a congruent condition ($2 + 3$ and $5$), an incongruent condition ($9 + 7$ and $5$), and a neutral condition ($X + Y$ and $5$). Three stimulus onset asynchronies (SOAs; the time periods between the onset of the prime and exposure to the target) were employed, including time periods of 120, 240 and 1500 ms. The effect of the congruent and incongruent conditions were assessed by subtracting the reaction time taken to name the target in each condition from the reaction time taken to name the same target following exposure to the neutral prime (Neely, 1991). When the difference was positive, the effect was referred to as facilitation and when the difference was negative, it was referred to as inhibition. The results revealed facilitation in congruent target naming, and inhibition in incongruent target naming, that varied as a function of time. The facilitation function emerged at 240 ms, a time period too brief to enable strategic processing of the prime prior to the appearance of the target. Moreover, no significant facilitation was observed at the shortest SOA. If the facilitation had resulted from strategic processing that occurred after presentation of the target, then it should also have been observed at 120 ms. These findings, combined with the use of a target naming task (in which intentional processing of the prime and calculation were not required), led to the conclusion that simple arithmetic facts are indeed automatically retrieved from memory. In contrast, the inhibition function emerged at an SOA of 120 ms and remained relatively constant over time, a pattern of reaction times more consistent with the use of a strategic process following exposure to the target. Accordingly, it was explained in terms of the operation of a response validity checking mechanism in which the target was compared to the correct solution evoked from memory. In the incongruent condition, when the target and correct solution did not match, this led to hesitation in target naming.
The methodology employed in the Jackson and Coney (2005) investigation was thus useful for accessing and distinguishing between the automatic and strategic processes in operation in simple arithmetic processing. However, to date, this methodology has not been employed in an investigation of just how the split between a correct and an incorrect solution influences processing in arithmetic priming tasks. Arithmetic priming tasks differ from verification tasks because they involve the successive presentation of problems and solutions, rather than the simultaneous presentation of these stimuli. Consequently, the processes leading to split effects in arithmetic priming tasks may be different to those employed in verification tasks.

There is some evidence to suggest that the split effects produced in the context of arithmetic priming tasks do in fact differ to the standard split effect. In a study by Campbell (1987: Experiment 2) examining the confusion-product effect, solutions were presented as primes (e.g., 8), and problems were presented as targets (e.g., 2 x 3). The participants were then required to produce the correct solutions to the given problems. Consistent with the expectations of this study, the results revealed slower reaction times in a table related condition than in an unrelated condition. However, surprisingly, the results also indicated a reversed split effect to that commonly found in verification tasks, with reaction times increasing with larger differences between incorrect primes and correct solutions. Campbell offered two main explanations for this finding. Firstly, he suggested that it was possibly attributable to predictive information provided by the prime about the approximate magnitude of the correct answer, with splits greater than 10 accounting for less than 15% of trials in his study. Such an account of the reversed split effect suggests that strategic processing played an important role in shaping the distribution of reaction times, and hence, the split effect, that Campbell found. Secondly, Campbell
suggested that magnitude may be a primeable arithmetic dimension, with retrieval performance facilitated when evidence about the magnitude of a correct answer was activated in memory. Unfortunately, however, Campbell did not elaborate on this explanation. The finding of a reversed split effect was secondary to the main findings produced in Campbell’s research and consequently further investigation into its origins was not undertaken.

The main aim of the present study was thus to explore what effect, the split between correct and incorrect solutions to simple arithmetic problems has in shaping the reaction times that are produced in arithmetic priming tasks. The arithmetic priming procedure utilised by Jackson and Coney (2005) was employed, with four prime-target relationship conditions i.e., congruent (‘2 + 4’ and ‘6’), close incongruent (‘2 + 4’ and ‘5’), distant incongruent (‘2 + 4’ and ‘11’), and neutral conditions (‘X + Y’ and ‘6’). The benefit of using this procedure was that it enabled the measurement of both automatic and strategic processes in performance. The short SOA condition used in Campbell’s (1987) study was employed (i.e., 300 ms), as was an SOA of 1000 ms. These SOAs are consistent with those employed in the single word semantic priming paradigm in which automatic effects are measured at SOAs in the order of 250 ms and strategic effects are measured at SOAs of greater than 400 ms (Perea & Rosa, 2002; Velmans, 1999). The naming task (cf. Campbell’s, 1987, priming task, with production) was employed to minimise any effects of calculation-induced attentional processing that might arise in the measurement of automatic processes (Neely, 1991).

Additionally, the present study aimed to address a possible confound inherent in many earlier investigations resulting from the use of stimulus sets that have either varied between conditions or that were not generalizable. For example, in an
investigation into split effects by Stazyk et al. (1982) the stimulus set was divided into 81 true non-zero multiplication equations and 81 false non-zero equations. The 81 false equations were then divided into table-related and unrelated conditions, such that the equations and the effects of problem size in each of the false conditions varied and were different again, to those in the true condition. Similarly, in a study undertaken by Zbrodoff and Logan (1990), 36 of the possible 81 non-zero, addition and multiplication problems, excluding ties and involving only those equations with the smallest number on the left of the equation, were used to create true and false conditions. Furthermore, the false equations in this study only included positive splits (+ 2 and + 12), and no negative splits were employed. To address this, each condition in the present study comprised the same set of problems and solutions. In the false condition, the equations were developed through the pairing of solutions with alternative problems, such that they were mathematically erroneous (i.e., incongruent; Jackson & Coney, 2005). In the distant incongruent condition, the incorrect solution was further in magnitude from the correct solution than in the close incongruent condition. The splits in the present study were therefore created on the basis of proximity along the number line, and both positive and negative splits were utilised.

Given the paucity of information available on split effects in arithmetic priming tasks, no specific predictions were made regarding the effect of the split between correct and incorrect solutions to simple arithmetic problems on target naming times. However, in line with the findings of the earlier Jackson and Coney (2005) investigation, in each operation, significant facilitation in congruent target naming was expected at both SOAs. Moreover, consistent with the use of strategic processing at the long SOA, the level of facilitation was expected to increase
significantly over time in each operation. Finally, a pattern of significant inhibition that remained generally consistent over time was expected in each operation.

2.0 Method

2.1 Participants

Twenty eight students enrolled in Psychology at Murdoch University, including 8 males and 20 females, participated in the present study. The participants’ ages ranged from 18 to 53, with a mean age of 28 years. The participants’ mean correct score on the arithmetic section of the Australian Council for Educational Research Short Clerical Test (ACER SCT) was 23, with a standard deviation of 4.8. This score was equivalent to a percentile rank of 58% in normed samples of 124 tertiary students and 973 administrative officer/assistant applicants. All students participated in exchange for course credit.

2.2 Design and stimulus materials

For each of the addition and multiplication operations, two repeated measures variables were investigated. These included SOA (300 ms and 1000 ms) and prime-target relationship (congruent e.g., 2 x 4 = 8, close-incongruent e.g., 2 x 4 = 10, distant-incongruent e.g., 2 x 4 = 27 and neutral e.g., X x Y = 8 conditions).

Two sets of primes containing 20 standard simple arithmetic problems were constructed for each operation. The first set for each operation is presented in Appendix A. The second set comprised the reverse operand placement equivalent problems of the first set. Each set was balanced so that the number that was smallest in magnitude was the left operand in half of the problems and it was the right operand in the remaining half. Additionally, to balance for overall problem size, each set contained six problems with operands less than or equal to five, six problems with
operands greater than or equal to five, and eight problems with operands of mixed magnitude. Arithmetic ties were excluded from use as primes as these problems have been shown to be solved more quickly than others (LeFevre, Bisanz, & MrKonjic, 1988).

As noted earlier, the correct solutions corresponding to the 20 primes in each operation were employed as targets in all prime target relationship conditions. This balanced for any effects due to parity and size between conditions and ensured that the splits were proportional to the solution range characterising each operation. In the incongruent conditions, the correct solutions were paired with an alternative problem so that they were mathematically erroneous. In the addition condition all close-incongruent targets were paired with problems in such a way that they were split from congruent targets by a maximum distance of two. In contrast, distant-incongruent targets were paired so that they were split from congruent targets by a minimum distance of four. In the multiplication condition, close incongruent targets were split from congruent targets by a maximum distance of four, whilst distant-incongruent targets were split from congruent targets by a minimum distance of nine. Both positive and negative splits were employed in the present study. As far as possible, the number of positive and negative splits in each condition was generally kept equivalent. In the multiplication close incongruent condition and the addition distant incongruent condition, the ratio was 9 negative to 11 positive splits, whilst in the remaining two incongruent conditions the ratio was 10 positive to 10 negative splits.

The neutral primes employed in the present study for the addition and multiplication operations consisted of ‘X + Y’ and ‘X x Y’, respectively. These primes were previously employed in studies by Jackson and Coney (2005, 2006a,
and have provided a useful baseline to which to compare congruent and incongruent reaction times in the demonstration of facilitatory and inhibitory effects in simple arithmetic processing.

2.4 Procedure

Participants completed the Arithmetic section of the ACER SCT and then were individually tested in the computer task. Testing was undertaken in a well-lit cubicle room containing an Amiga 1200 microcomputer, with 1084S monitor. This system controlled stimulus presentation, trial sequencing, timing and data collection. All stimuli were presented centrally on the computer screen, and were white against an amber background. Individual operands had dimensions of no more than 5 x 15 mm and were situated 5 mm on either side of the arithmetic operator (i.e., the x or + sign), which had dimensions of no more than 5 x 10 mm. A chin rest was placed 60 cm in front of the screen and was used to stabilise the participant’s head during testing.

Addition and multiplication trials were blocked separately so as not to induce cross operation errors. Participants completed two blocks of 80 trials (with 20 trials for each of the four prime-target relationship conditions) in each operation, i.e., one block at each of the 300 and 1000 ms SOAs. Exposure to all stimuli was counterbalanced across participants, with half of the participants completing the addition condition first and half completing the multiplication condition first. At the short SOA, for each operation, half of the participants were exposed to the first set and half were exposed to their reverse operand placement equivalents. Each participant was then exposed to the exact same set at the long SOA. This was done to enable a level of familiarity with the stimuli and to draw attention to the prime-target relationship so that strategic processing could be measured. This process was then
repeated in the operation that was not tested in the first two blocks. The computer randomly generated the order of presentation of all prime target relationships within each block.

Equal emphasis was placed on instructing the participants to respond both quickly and accurately. Each trial began with the participant fixing their gaze on a 1 x 1 mm blue central fixation dot that was exposed for 600 ms. Following this, the screen went blank for 150 ms and then the prime was presented for 100 ms. SOAs of 300 and 1000 ms were employed. The target number remained exposed until the participant verbally responded with the target number. The time between the onset of the target and the participant’s verbal response was recorded via a microphone connected to a headset. The microphone amplifier was used to trigger an electronic relay that was interfaced to the computer and determined the time of relay closure using a hardware timer that was accurate to 1 millisecond. Interference from external noises was guarded against using ear defenders. The experimental session took approximately 35 minutes to complete.

3.0 Results

The mean correct target naming latency for each prime-target relationship condition, at each SOA, was calculated. Due to the negligible error rates produced in target naming, they were not considered in the present analysis. The resulting reaction time data are presented in Table 1.
Table 1. Showing Mean Reaction Times and Standard Deviations (in brackets) for all Prime-Target Relationship Conditions as a Function of Operation and SOA.

<table>
<thead>
<tr>
<th>SOA</th>
<th>Multiplication</th>
<th>Addition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>300 ms</td>
<td>1000 ms</td>
</tr>
<tr>
<td></td>
<td>459 (47)</td>
<td>453 (46)</td>
</tr>
<tr>
<td>Congruent</td>
<td>472 (46)</td>
<td>477 (44)</td>
</tr>
<tr>
<td>Close-incongruent</td>
<td>481 (45)</td>
<td>485 (42)</td>
</tr>
<tr>
<td>Distant-incongruent</td>
<td>469 (44)</td>
<td>479 (48)</td>
</tr>
<tr>
<td>Neutral</td>
<td>438 (43)</td>
<td>435 (45)</td>
</tr>
<tr>
<td>Congruent</td>
<td>452 (44)</td>
<td>451 (47)</td>
</tr>
<tr>
<td>Close-incongruent</td>
<td>459 (48)</td>
<td>458 (51)</td>
</tr>
<tr>
<td>Distant-incongruent</td>
<td>440 (41)</td>
<td>451 (47)</td>
</tr>
</tbody>
</table>

A repeated measures t-test comparison showed that the mean target naming time in the addition condition (i.e., 448 ms) was significantly faster than the mean target naming time in the multiplication condition (i.e., 472 ms) ($t(27) = 5.68, p < .001$). This finding is consistent with operation differences found in earlier investigations and possibly reflects differences in solution magnitudes between the two operations (i.e., addition solutions only ranged between 5 and 17, whilst multiplication solutions ranged between 6 and 72) (Jackson & Coney, 2005, 2006a, 2006b; Zbrodoff & Logan, 1986). Additionally, earlier research by Brysbaert (1995) and Jackson and Coney (2005, 2006a, 2006b) shows that it takes longer to perform number naming tasks when the numbers are large than when they are small. Consequently, in the present analyses, the addition and multiplication operations were considered separately.

3.1 Multiplication
The multiplication data were entered into a two-way repeated measures analysis of variance, with SOA and prime-target relationship as the within group variables. This analysis revealed a significant main effect of prime-target relationship \((F(2.44, 65.74) = 22.94, MSe = 401.00, p < .001;\) Violations of the assumption of compound symmetry were corrected throughout these analyses by adjusting the degrees of freedom using Huynh Feldt Epsilons). When tested at an adjusted alpha level of .038 (using a modified Bonferroni adjustment i.e., \(\{(4 – 1) \times .05/ 4\}\)) target naming in the congruent condition was facilitated by 19 ms in comparison to the neutral condition \((t(27) = 5.86, p < .001).\) Target naming in the distant incongruent condition was inhibited by 9 ms in comparison to the neutral condition \((t(27) = 2.26, p = .032),\) and was 9 ms slower than in the close incongruent condition \((t(27) = 3.61, p = .001).\)

Additionally, a significant interaction between SOA and prime-target relationship was found \((F(3, 81) = 3.53, MSe = 174.55, p = .018).\) The facilitatory (i.e., neutral – congruent naming times) and inhibitory effects (i.e., neutral - close incongruent naming times = close inhibition; and neutral - distant incongruent naming times = distant inhibition) from this interaction are illustrated in Figure 1.
Fig. 1 Showing facilitation (neutral – congruent) in naming congruent targets and inhibition (incongruent – neutral) in naming close-incongruent and distant-incongruent targets as a function of SOA, in each operation. Values below the baseline of 0 represent inhibition and those above the baseline represent facilitation. The 95% confidence intervals were calculated using the $MSe$ term for individual one factor repeated measures ANOVAs of the difference scores representing each of the facilitatory and inhibitory effects.

At the short SOA, target naming in the congruent condition was significantly facilitated in comparison to the neutral condition ($t(27) = 3.71, p = .001$) and target
naming in the distant incongruent condition was significantly inhibited in comparison to the neutral condition ($t(27) = 2.64$, $p = .014$). Additionally, target naming in the distant incongruent condition was significantly slower by 9 ms in comparison to the close incongruent condition ($t(27) = 2.84$, $p = .008$). At the long SOA, target naming in the congruent condition was significantly facilitated in comparison to the neutral condition ($t(27) = 5.37$, $p < .001$). Target naming in the distant incongruent condition was significantly slower (by 8 ms) than in the close incongruent condition ($t(27) = 2.56$, $p = .016$). The level of facilitation observed at the long SOA was significantly greater than that observed at the short SOA ($t(27) = 3.12$, $p = .004$). No significant difference in the level of inhibition was found between the two SOAs in either the close ($t(27) = 1.07$, $p = .295$) or distant ($t(27) = .98$, $p = .336$) incongruent conditions.

### 3.2 Addition

A two-way repeated measures analysis of variance was then performed on the addition data, with SOA and prime-target relationship as the within group variables. A significant main effect of prime-target relationship was again found ($F(1.34, 36.18) = 13.15$, $MSe = 801.92$, $p < .001$). Target naming in the congruent condition was facilitated by 9 ms in comparison to the neutral condition ($t(27) = 3.85$, $p = .001$). Target naming in the distant incongruent condition was inhibited by 12 ms in comparison to the neutral condition ($t(27) = 3.18$, $p = .004$), and was 7 ms slower than in the close incongruent condition ($t(27) = 4.04$, $p < .001$).

Furthermore, a significant interaction between SOA and prime-target relationship was again found ($F(2.41, 65.15) = 4.71$, $MSe = 151.10$, $p = .008$; See Fig. 1 for facilitatory and inhibitory effects). At the short SOA, no significant facilitation in congruent target naming was found ($t(27) = 1.24$, $p = .228$). Target
naming in both the close incongruent condition ($t(27) = 3.62, p = .001$) and in the distant incongruent condition ($t(27) = 4.20, p < .001$) was significantly inhibited in comparison to naming in the neutral condition. Target naming in the distant incongruent condition was significantly slower (by 7 ms) than in the close incongruent condition ($t(27) = 3.00, p = .006$). At the long SOA, target naming in the congruent condition was significantly facilitated in comparison to the neutral condition ($t(27) = 4.48, p < .001$) and target naming in the distant incongruent condition was significantly slower by 7 ms than in the close incongruent condition ($t(27) = 3.11, p = .004$).

As in the multiplication condition, the level of facilitation observed at the long SOA was significantly greater than that observed at the short SOA ($t(27) = 4.09, p < .001$). However, in contrast to the multiplication condition, the level of inhibition observed in target naming in the close incongruent condition at the long SOA was significantly less than that observed at the short SOA ($t(27) = 2.65, p = .013$). Similarly, the level of inhibition observed in target naming in the distant incongruent condition at the long SOA was significantly less than that observed at the short SOA ($t(27) = 2.84, p = .008$).

### 4.0 Discussion

The main aim of the present study was to determine whether the split between correct and incorrect solutions to simple arithmetic problems influences the reaction times that are produced in the context of arithmetic priming tasks. The results of the present study showed that this is the case. Close incongruent targets were named significantly faster than distant incongruent targets in all conditions of the present study. Additionally, the present study revealed some interesting priming effects. In the multiplication condition, significant facilitation in congruent target naming was
found at 300 and 1000 ms, and the level of facilitation increased significantly over time. Significant inhibition was observed in naming distant incongruent targets only, and this occurred only at the short SOA. In the addition condition, the facilitation obtained at the short SOA failed to reach significance. However, the level of facilitation increased significantly over time and was significant at 1000 ms. Significant inhibition in naming both close and distant incongruent targets was observed at 300 ms only. The level of inhibition observed in distant incongruent target naming decreased significantly over time. These findings have some interesting implications for our understanding of simple arithmetic processing.

The finding that close incongruent targets were named significantly faster than distant incongruent targets in all conditions of the present study is consistent with the results of the Campbell (1987) study, in which reaction times were found to increase with larger differences between incorrect primes and correct solutions. Notably, this effect resulted in multiplication and addition (the latter was not tested by Campbell), and it occurred even though splits of greater than or equal to 10 accounted for approximately only 50% of trials in the multiplication condition (cf. 85% in Campbell’s study). This finding suggests that Campbell’s (1987) explanation of his findings in terms of the strategic use of predictive information provided by the prime about the approximate magnitude of the correct answer may have been incorrect. Moreover, it indicates that the effects of split on arithmetic processing are task specific, and that in the context of arithmetic priming tasks, a reversed split effect to that observed in verification tasks is found.

What mechanisms are responsible for the priming effects observed in the present study? As in the earlier Jackson and Coney (2005) investigation, the facilitation effect observed at the short SOA in the multiplication condition can
easily be explained in terms of the operation of an automatic spreading activation mechanism. This is supported in the present study by the fact that this effect occurred even though processing of the prime and calculation of the solution were not necessary to performance of this task. Furthermore, this effect occurred in the multiplication condition only. In contrast to the addition operation, the multiplication operation is traditionally over-learned through rote learning, thereby possibly producing stronger associations between multiplication problems and their correct solutions, and thus, greater levels of automaticity in multiplication fact retrieval. Finally, the large and significant increases in facilitation at the long SOA in both operations (i.e., when the participants had ample time to process the prime before presentation of the target) indicate that strategic processing of the prime was used in order to speed responding to the target in this condition.

As noted by Campbell (1987) it is possible that the pattern of inhibitory effects, and indeed, the difference in the time taken to name close and distant incongruent targets, resulted from the activation of some numerical magnitude representation in memory. The assumption that representations of this type exist in memory is central to Dehaene’s (1997) proposal that humans and animals share an evolutionary number sense that is specialised for detecting approximate quantity. This proposition is supported in the literature by research that shows that animals (such as primates, rats and pigeons) and human infants who have not yet developed language skills have the capacity to attend to numerosity and to perform elementary computations (Feigenson, Dehaene & Spelke, 2004; Dehaene, 1997). Additionally, studies involving Amazonian indigenous groups, whose language systems do not contain fully elaborated counting systems, indicate the use of a non-verbal, analogue, number estimation process in simple enumeration and arithmetic computation tasks.
Furthermore, the findings of a recent neuroimaging investigation by Dehaene, Piazza, Pinel and Cohen (2003) indicate that the detection of quantity relies on a distinct neural circuit in the brain. In this study, fMRI was used in conjunction with various numerical tasks to reveal a system in the brain that was activated whenever numbers were manipulated, and that was increasingly activated in tasks involving quantity processing (i.e., the horizontal segment of the intraparietal sulcus; HIPS). This system, described as being analogous to a ‘mental number line,’ was supplemented by two other systems. These included a system responsible for the manipulation of numbers in verbal form (i.e., the left angular gyrus), and a system supporting attentional orientation along the number line (i.e., a bilateral parietal system).

Behavioural research also supports the notion that semantic quantities are specified along an oriented, internal number line, and provides added insight into the nature of this representation. For example, a study by Dehaene, Bossini and Giraux (1993) involving parity judgements, found that participants responded faster to numbers that were small in magnitude with their left hand, and responded faster to numbers that were large in magnitude with their right hand (referred to as a spatial numerical association of response codes or SNARC effect). Furthermore, this effect was found even though access to semantic quantity was not necessary for performance of Dehaene et al.’s (1993) task. These findings indicate that the mental number line is oriented left to right, and that its activation occurs as an obligatory process. Moreover, a wide range of research using various response formats, measures, and samples (including children, adults, and non human species), has produced evidence to suggest that the proposed mental number line is logarithmically compressed (Dehaene, 1993, 1997; Nuerk, Zoppoth, Kaufmann & Willmes, 2004;
Roberts, 2005; Siegler & Booth, 2004). That is, the distance between the numbers 10 and 20 would be the same as the distance between the numbers 100 and 200 on this number line (Nuerk, Weger & Willmes, 2005).

Given this growing body of converging evidence, it seems plausible that a similar ‘number line’ representation was activated in the present context. Presumably, exposure to the two digits in each arithmetic problem led to the activation of a region of the number line corresponding to a rough estimate of the location of its correct solution. This region of activation, possibly represented by a distribution extending out on either side from a peak around the correct solution, influenced the participant’s confidence in their response. Theoretically, if the presented target was consistent with the activated region of the number line, no hesitation in vocal responding resulted. However, if there was a discrepancy between the number that the participant was trying to name and the region of the number line that was activated, then some degree of inhibition in responding occurred. When the target was further away from the activated region, the participant was less confident in their response and a larger inhibitory effect was produced than when the target was close to the activated region.

Thus, an explanation of the present findings in terms of the activation of the mental number line accounts nicely for the finding of a reversed split effect. What is more, the notion that this representation is logarithmic in nature allows for the prediction of stronger and clearer effects in the context of a smaller numerical size and range of targets. This is notably consistent with the present finding of close incongruent inhibition in the addition condition only. Furthermore, given that the participants were simply required to name target numbers and were not required to perform controlled number magnitude comparisons or calculations, it appears that
the activation of this representation was obligatory. Like the activation of automatic processes that are not subsumed or reinforced by strategic processes, the activation of this representation would be expected to diminish over time (Stolz & Neely, 1995). That is, the inhibitory effects would be expected to be smaller at the long SOA than at the short SOA (Stolz & Neely, 1995). The mental number line explanation is therefore consistent with the observed pattern of diminishing inhibitory effects in the present study.

Finally, it is worth mentioning that the present interpretation has implications for the findings of previous investigations employing a similar arithmetic priming methodology (e.g., Jackson and Coney, 2005, 2006a, 2006b). For example, it may be the case that the inhibition observed at the short SOA in these earlier investigations resulted due to the activation of the mental number line representation described above, with inhibition only occurring at the long SOA when strategic processing was employed. Whilst this possibility seems less likely in the context of the surface form investigation in which number word stimuli were presented, it is worth considering given that Dehaene et al. (2003) found that the HIPS area was activated independently of number notation. Accordingly, future research should investigate what effect, if any, the split between correct and incorrect solutions to simple arithmetic problems represented in number word form has in shaping the reaction times that are produced in arithmetic priming tasks.

In summary, the results of the present study demonstrated the fact that split effects are task specific. A priming procedure, with a naming task, revealed a reversed split effect i.e., the time taken to name distant incongruent targets was significantly greater than the time taken to name close incongruent targets. A proposal was put forward to suggest that, in the context of arithmetic priming tasks,
split effects result from the obligatory activation of a numerical magnitude representation that is logarithmic in nature, and that influences the participant’s confidence in their response. Finally, the results of the present study confirm the need to develop stimulus sets that are balanced in terms of split in future priming investigations of simple arithmetic processing.

References


## Appendix A

<table>
<thead>
<tr>
<th>Operation</th>
<th>Prime</th>
<th>Congruent Target</th>
<th>Close-incongruent Target</th>
<th>Split</th>
<th>Distant Incongruent Target</th>
<th>Split</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Multiplication</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 x 4</td>
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<td>10</td>
<td>+2</td>
<td>27</td>
<td>+19</td>
<td></td>
</tr>
<tr>
<td>2 x 7</td>
<td>14</td>
<td>18</td>
<td>+4</td>
<td>54</td>
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<td></td>
</tr>
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<td>+2</td>
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<tr>
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<tr>
<td>3 x 7</td>
<td>21</td>
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3. DISCUSSION

3.1 Arithmetic Processing in the Priming Paradigm.

The first investigation in the present series aimed to determine whether the numerical variant of the single word semantic priming paradigm would uncover facilitatory and inhibitory effects as a result of number priming using simple arithmetic stimuli. Significant priming effects were identified in both the addition and multiplication operations. These effects were somewhat smaller than those identified in earlier arithmetic research employing a priming procedure with a verification task (e.g., Ashcraft, Donley, Halas & Vakali, 1992) but they were, nonetheless, comparable to the effects identified in the investigation of associative word primes and targets (e.g., see Neely, 1991). This finding supports the idea that number and word knowledge may be stored in memory and accessed in the same way.

The facilitation function identified in the first study appeared to result from the operation of an automatic spreading activation mechanism. Three main factors supported this conclusion. Firstly, the facilitation function arose at 240 ms, a time period too short to allow for strategic processing of the prime in order to speed responding to the target. Secondly, no facilitation was found at 120 ms. If the facilitation function had resulted from strategic processing that occurred after presentation of the target, then facilitation should also have been found at the short SOA. Thirdly, the use of a target naming task ensured that attentional processing of the prime and calculations were not necessary for performance of this procedure. At the long SOA, the increased facilitation effect appeared consistent with the use of an expectancy strategy i.e., the use of the correct solution activated in memory to speed target naming in the congruent condition.
If the facilitation and inhibition functions were produced by a common mechanism, then greater levels of facilitation should have been accompanied by greater levels of inhibition. In contrast, the inhibition function emerged at the shortest SOA and remained relatively constant or decreased over time. Given that SOAs of 120 and 240 ms are too brief to allow for processing of the prime to influence responding to the target, the inhibition must have resulted from processing that occurred after presentation of the target. Accordingly, this effect was explained in terms of the operation of a self regulatory response validity checking mechanism. This mechanism was thought to operate after exposure to the target and before vocalisation, and involved a comparison of the target to the correct solution evoked from memory. In the incongruent condition, when the target and correct solution did not match, hesitation in vocalisation resulted. Furthermore, the operation of this mechanism appeared to occur at an obligatory level because participants were not required to produce the correct solutions to problems or to verify the relationships between the primes and the targets. They were simply required to vocalise the target numbers as they appeared on the computer screen.

Therefore, the first investigation in this series of studies provided valuable insight into the cognitive mechanisms underlying simple arithmetic processing. Not only did it allow for a more valid and reliable investigation into automatic fact retrieval but it was the first study in the literature to identify the operation of an inhibitory mechanism in simple arithmetic processing.

A second significant finding in the first study was the finding that automaticity in fact retrieval was modulated both by the person’s arithmetic fluency and by the operation under investigation. The correct solutions to simple addition problems appeared to be automatically activated from memory in people of all
abilities, whilst only high ability arithmeticians were able to apply their multiplication knowledge strategically. This finding possibly resulted from a greater sensitivity of the word priming methodology to individual differences in multiplication ability, which is traditionally rote learnt and relies on the development of associations between words. Furthermore, it may be the case that earlier and greater exposure to simple addition facts, which tend to be more concrete than simple multiplication facts, leads to fewer observable differences in addition performance. Nevertheless, it is noteworthy that the investigation of individual differences in the first study was secondary to its main aim. Consequently, a median split was employed to create the two fluency groups, making them somewhat less distinctive in terms of ability than was desired. Further investigation employing more extreme groups was therefore required to determine the reliability of this finding.

Thirdly, the results of the first study revealed an influence of problem size on priming effects. Priming by small addition and multiplication problems produced significant inhibition in target naming in both operations. However, the facilitation effects were mixed. Significant facilitation was found following priming by small multiplication problems, whilst in the addition condition, the facilitation approached significance in the small problem condition and reached significance in the large problem condition. These results were therefore, generally consistent with the notion that problems of differing size are processed differently, with advantages in the processing of small problems that are more frequently encountered in natural and formal educational settings (Ashcraft et al., 1992; Campbell, 1987, 1991; Koshmider & Ashcraft, 1991; LeFevre et al., 1996a, 1996b). Unfortunately, the results of further problem size analyses undertaken at each level of SOA appeared somewhat inconsistent, suggesting again, the need for additional testing.
In summary, the first study in this series of investigations showed that the numerical variant of the single word semantic priming paradigm was an effective method for the investigation of the cognitive processes underlying simple arithmetic performance. When combined with a naming task, this method enabled a valid and reliable investigation into automaticity in simple arithmetic processing. The results of this study led to the proposal that two independent mechanisms operate in simple arithmetic performance. The first was an automatic spreading activation mechanism, with activation spreading from the simple arithmetic problem to the correct solution in memory, leading to facilitation in naming congruent targets. The second mechanism was a response validity checking mechanism that operated after exposure to the target and before vocalisation, producing inhibition in naming incongruent targets. Finally, the results of the first investigation indicated that both arithmetic fluency and problem size influence priming effects, although further research involving a larger sample size and more extreme groups was required to confirm the reliability of these findings. These topics formed the basis of the second investigation.

3.2 Individual Differences in Processing.

Following on from the findings of the first investigation, the second study aimed to more thoroughly examine individual differences in simple arithmetic processing by employing a larger sample size and more distinguishable skill groups. The results of this study were consistent with earlier research (e.g., LeFevre et al., 1991, and LeFevre & Kulak, 1994) in showing that high skilled individuals access simple arithmetic facts earlier in the processing sequence than low skilled individuals do. Significant facilitatory effects were identified at the 300 ms SOA in both the addition and multiplication operations for the high skilled participants alone.
Therefore, only high skilled participants demonstrated automaticity in access to simple arithmetic facts. In contrast, significant facilitation was observed for both groups in both operations at the long SOA. However, as in the first investigation in this series, only the level of facilitation produced by the high skilled group in the multiplication condition increased markedly over time. Thus, individual differences in arithmetic skill also appeared to result from strategic access to multiplication solutions.

Importantly, significant inhibition in naming incongruent targets was found at the 300 ms SOA in both operations, and at 1000 ms in the addition operation, for the high skilled group only. Therefore, the findings of the second study supported and extended those of the first investigation in this series by demonstrating the operation of an inhibitory mechanism in multiplication and addition processing. What is more, the finding of inhibitory effects only for the high skilled group supported an explanation of these effects in terms of the operation of an obligatory, response validity checking mechanism. As noted earlier, this mechanism was proposed to operate via the comparison of a given target to the correct solution evoked from memory. Consequently, only those participants possessing stable semantic representations of associations between problems and their correct solutions (i.e., skilled participants) would be likely to, and indeed did, produce such effects.

The results of the multiplication analysis in the second study were again consistent with the notion that simple arithmetic knowledge may be represented in memory in a similar form to word knowledge. Notably, equivalent levels of facilitation and inhibition were found at the short SOA, and facilitation dominance was found at the long SOA. This pattern of priming effects is similar to that observed in the investigation of associatively related word primes and targets (e.g., see Neely,
1991), and may reflect a reliance on the verbal rote learning of associations between words in the acquisition of multiplication knowledge.

A second aim of the second study was to re-examine the influence of problem size on arithmetic processing. Individual differences in access to solutions of small and large problems that varied by operation were found. In the multiplication condition at the long SOA, the facilitation observed following priming by small problems was significantly greater for high skilled participants than for low skilled participants. Furthermore, significant facilitation was observed following priming by large problems for the high skilled participants alone. In the addition condition, significant facilitation and inhibition was observed following priming by small problems at both SOAs for the high skilled group, whilst facilitation only was observed at the long SOA for the low skilled group. Thus, high skilled participants appeared to have greater access to solutions of small and large multiplication problems and automatic access to solutions of small addition problems. Notably, however, no significant priming effects were observed for either group in the large problem, addition condition. This latter finding may have reflected differences in the way that large problems are learnt in each operation. For example, rote learning is commonly employed in the development of multiplication fact knowledge. Therefore, large multiplication problems may be verbally practiced to a greater extent than large addition problems. This, in turn, may lead to an advantage in access to the solutions of large multiplication problems that is reflected in the priming effects observed in the present study. In contrast, access to solutions of large addition problems may be at a permanent disadvantage, occurring only via strategic operations.
A further aim of the second study was to examine the utility and validity of a set of different neutral condition stimuli (i.e., $X + Y$ and $X \times Y$) to that employed in the first investigation (i.e., $0 + 0$ and $0 \times 0$). The results revealed longer target naming times following exposure to the letter based neutral primes than was observed in response to the zero based neutral primes at the 1000 ms SOA. Furthermore, a large inhibitory effect that was identified in the first study at the long SOA was absent in the second study. However, this latter finding may have resulted from other factors, such as the differences in skill levels between samples (i.e., with a more skilled sample employed in the second study), a possibility that requires future investigation. Importantly, the new neutral stimuli guarded against inducing the expectation that the solution ‘0’ (i.e., the correct solution to the primes $0 + 0$ and $0 \times 0$) would be presented. Moreover, significant priming effects were identified using the new letter based stimuli that essentially, were of a similar magnitude to the priming effects produced in the first study. The letter based neutral primes therefore, appeared to provide a useful baseline by which to compare congruent and incongruent condition reaction times in the demonstration of facilitatory and inhibitory effects in simple arithmetic processing.

A final aim of the second study was to examine the utility of a 300 ms, short SOA, condition. This SOA was effective in demonstrating priming effects using Arabic digit stimuli, with significant facilitation and inhibition identified for the high skilled group, in both operations. The 300 ms SOA was somewhat lengthier than the SOAs employed in the first investigation in this series of studies (i.e., 120 and 240 ms SOA conditions) and therefore, appeared well suited for the investigation into the priming effects produced by exposure to lengthier, word problem, stimuli planned for the third study in this series.
In summary, the results of the second study in this series of investigations were consistent with the results of the first investigation in again, suggesting that high skilled individuals automatically access correct arithmetic solutions from memory. Additionally, the results revealed a skilled advantage in strategic access to multiplication solutions. However, the problem size analysis revealed no significant priming effects following exposure to large addition problems, for either skill group. This finding suggested an enduring disadvantage in access to solutions to large addition problems that may result from differences in learning practices between the two operations. Finally, the new letter based neutral condition, and the 300 ms short SOA condition both appeared to be useful methodological manipulations in the context of the arithmetic variant of the single word semantic priming paradigm.

3.3 Surface Form Effects.

The third investigation in the present series of studies centred on the question of whether problems represented in different surface forms i.e., as words or Arabic digits, are processed along the same or different pathways. In the Arabic digit condition, significant facilitation in naming congruent digit targets was found in both operations, at both SOAs. In contrast, in the word problem condition, only significant inhibition was identified. This occurred in both operations at 300 ms and in the addition operation at 1000 ms. The results of this study were therefore, consistent with the notion that problems represented as digits are processed along a different pathway to problems represented as words.

Importantly, in the third investigation, the same priming technique, involving the same proportions of congruent, incongruent and neutral trials, and almost exactly the same stimulus set as that employed in the first two investigations was used. Additionally, this study employed a more skilled sample than was employed in the
first investigation. Thus, it was considered likely that the facilitation effect observed in the digit condition reflected the operation of the same automatic spreading activation mechanism as the one that was suggested to operate in simple arithmetic fact retrieval in the first two investigations.

However, unlike the first two investigations, the third investigation did not reveal significant inhibition in Arabic digit processing. Nevertheless, the level of inhibition produced in both operations at the short SOA of the third investigation was comparable to that observed previously for the digit stimuli. Accordingly, the failure to reach significance possibly resulted because of differences between the samples employed in each study. Notably, whilst the mean correct score for the ACER SCT in the third investigation was only 23, the mean correct score on the ACER SCT in the second investigation was 31. Thus, the sample employed in the third investigation was less skilled than the sample employed in the study that examined individual differences in processing.

Given the constant pattern of inhibition identified over time in the word condition, it was again considered likely that the proposed response validity checking mechanism was employed. The finding of significant inhibition only in response to the novel, number word stimuli, which would be more likely to benefit from such a process, was consistent with this assumption.

Following on from the suggestion that similar processes were in operation in the first three studies, it was pertinent to consider just what type of mental representations these processes acted upon. In the case of the facilitation mechanism, two possibilities were suggested. Firstly, consistent with the high frequency of exposure to arithmetic problems represented in Arabic digit form in formal schooling, it was suggested that the activation of the correct solutions from memory
occurred directly via visual representations of this stimuli. Secondly, in line with the encoding complex hypothesis and the triple code model, it was suggested that the visual representations may have first been converted to phonological representations that then elicited the automatic activation of the correct solutions from memory. However, in the latter case, the conversion of a visual representation to a sequential phonological representation appeared an inefficient process, which was possibly more likely to be used in the context of the novel word stimuli.

Support for the notion that phonological representations are employed in the context of word stimuli stemmed firstly, from the finding that the digit and word stimuli were in fact processed differently. Secondly, it seemed improbable that correct solutions would be activated from a stable semantic memory representation of arithmetic problems represented as number words in memory, given that we are rarely exposed to arithmetic problems represented in this form. Moreover, had this information been represented in memory, then facilitation should have been observed in congruent target naming. With number words more commonly found in reading contexts, it seemed more feasible that the priming effects observed for the word stimuli were activated through a strong, verbal, reading based mechanism (e.g., via subvocalisation). In turn, this may have activated the phonological representations of the problems, leading to inhibition in target naming when an incongruent target was presented.

The interpretation of the facilitation observed in the congruent digit condition in terms of visual codes and the inhibition observed in the incongruent word condition in terms of phonological codes was consistent with previous theorising and research in this area. For example, Campbell and Clark (1992) put forward a similar proposition over ten years ago. More recently, a study by Trbovich and LeFevre
(2003) showed that fact retrieval is more difficult under phonological load when problem stimuli are presented in an atypical horizontal format, and is more difficult under visual load when presented in a vertical format.

Together the results of the third study in this series and those of Trbovich and LeFevre (2003) implied that, when confronted with problems represented in an unusual visual form, fact retrieval relies on the obligatory processing of more familiar, phonological representations. Such a process can be likened to a ‘backup’ procedure, which allows for the faster and more accurate retrieval of facts from memory (Siegler, 1988; Siegler & Jenkins, 1989; Siegler & Shipley, 1995). Moreover, this explanation closely paralleled the dual-route cascaded model of visual word recognition and pronunciation (Coltheart, Curtis, Atkins & Haller, 1993; Coltheart, Rastle, Perry, Langdon & Ziegler, 2001; Coltheart & Rastle, 1994). According to this model, two procedures are employed to covert printed words to speech. The first is a lexical procedure that operates via the activation of learned word representations in an orthographic lexicon that is directly linked to the semantic system (Coltheart et al., 2001). The second is a non-lexical, grapheme to phoneme conversion procedure that operates on pronounceable non-words and novel words that obey the spelling-sound rules of English (Coltheart et al., 2001). Research indicates that whilst initial reading is phonologically based, highly skilled reading is able to bypass the phonological stage and directly access semantic representations from the visual form of words alone (Coltheart et al., 1993; Frost, 1998; Visser & Besner, 2001). This can be likened to a situation where the highly over learned Arabic digit stimuli directly accessed semantic representations via visual input, whilst the poorly learned number word stimuli relied upon phonological processing.
Given this explanation, it was suggested that the differing patterns of inhibition observed between the addition and multiplication operations in the word condition resulted from differences in learning practices. Presumably, the verbal rote learning of simple multiplication facts leads to the development of quite strong associations between multiplication problems and their correct solutions. Consequently, at the long SOA, a pattern of priming effects that approached that of the well practiced Arabic digit stimuli was found for the word stimuli. In contrast, addition facts are not generally verbally rote learnt. As a result, only weak associations may develop between addition problems and their correct solutions, leading to a greater reliance on a backup procedure in performance.

In addition to surface form effects, the third study again revealed problem size effects in the processing of problems represented as Arabic digits. Facilitation was observed in naming congruent digit targets following exposure to small addition and multiplication problems, and large multiplication problems, at 1000 ms. Furthermore, inhibition was found in naming incongruent targets following exposure to small addition problems at 300 ms only. Consistent with the second study in this series, no priming effects were observed in performance following exposure to large addition problems. This finding suggests that no stable semantic representation of large addition problems exists in memory. As noted earlier, this may reflect differences in the way that large problems are learnt in each operation, with large multiplication problems generally rote learnt and practiced to a greater extent than large addition problems.

Notably, unlike previous verification and production research in the cognitive arithmetic area, no significant main effect of surface form was identified using the present priming procedure (Campbell & Clark, 1992; Campbell, 1994; Campbell,
1999; Campbell & Fugelsang, 2001; Noel et al., 1997). In the case of the verification research, the difference between the third investigation and the findings of the Campbell and Fugelsang (2001) study can easily be explained in terms of solution encoding time differences. For example, in the third investigation, target solutions were maintained as digits, whilst in Campbell and Fugelsang’s study, the solution form varied with the problem form (e.g., 2 + 5 = 8 and two + five = eight), thereby leading to an advantage in digit processing times.

The difference between the results of the third investigation and the finding of a main effect of surface form in production tasks is more difficult to explain. One possibility is that the absence of a digit processing advantage in the third investigation resulted from the processing of the word primes at a more superficial level than the digit primes. Such an explanation is supported in the third investigation by the fact that it was not necessary to encode and process the prime in order to successfully perform the naming task. Furthermore, shallow encoding of the word stimuli may have been encouraged due to its greater physical length and its exposure within the short SOA time periods employed in the third investigation (McCloskey et al., 1992). However, had the word stimuli been under greater speed pressure for encoding, slower responses due to interference in target naming would have been expected at the shorter SOA when compared to the long SOA. Indeed, such a pattern of performance was actually evident in the long SOA, multiplication data, yet it was noticeably absent in the long SOA addition condition in which the same stimuli had to be processed (i.e., except for the arithmetic operator). Moreover, as noted earlier, the pattern of performance observed at the long SOA in the word multiplication condition began to resemble that observed in the processing of digit stimuli. This suggests that the shorter reaction times at this SOA were not the result of greater
speed pressure in encoding but rather that they reflected a greater proficiency in the processing of these stimuli.

Thus, assuming that the absence of a main effect of surface form in the third investigation did not result from superficial encoding of the word stimuli, an alternative explanation is that, in the previous research, the digit advantage resulted from processing that occurred further downstream than encoding. That is, given that the encoding stage was held constant between studies, with all involving the visual presentation of Arabic digit and English word problems, and that only the tasks differed (i.e., naming cf. fact retrieval), the digit advantage must have resulted from fact retrieval mechanisms. Such an explanation is supported in the findings of the third investigation by the fact that facilitation effects were observed only following exposure to congruent digit stimuli.

It is noteworthy that the results of the third investigation in this series were similar to those of earlier simple arithmetic investigations in revealing differences in processing between the two surface forms (Campbell & Clark, 1992; Campbell, 1994; Campbell, 1999; Campbell & Fugelsang, 2001; Noel et al., 1997). However, an interesting observation that stems from the discussion of number manipulation tasks (e.g., number comparison) in Section 1.2.3 above is that they generally show that digit and word problems are processed in the same way (Dehaene et al., 1998; Dehaene & Akhavein, 1995; LeFevre et al., 1988; Noel & Seron, 1997). One possible explanation for this inconsistency in findings between the arithmetic and number processing literature is that it reflects the operation of two separate quantity and verbal based knowledge systems (respectively) that are organized in memory and accessed differently to one another. Such an explanation is inspired by and adds
support to the basic principles underlying Dehaene and colleagues’ triple code model.

Nevertheless, a second (although not necessarily mutually exclusive) interpretation of the differences in outcomes identified between number manipulation and arithmetic tasks is possible. This explanation is motivated by the results of the third investigation, which suggest that no stable semantic network of arithmetic problems represented visually in number word form exists in memory. Thus, whilst it is reasonable to assume that stable, semantic representations of ‘2 + 3’ may be accessed from memory in the digit condition, the only way to access any meaning from visual exposure to ‘two + three’ is via an alternative phonological representation. Hence, the finding of differences in processing between the two surface forms in arithmetic operations. In contrast to this, given that both representations are commonly encountered in mathematical and reading based contexts, it would seem plausible that stable visual semantic representations of numbers represented as digits (e.g., ‘3’) and number words (e.g., ‘three’) both exist. Moreover, given that both representations are used to indicate the same semantic quantity, it is possible that the meaning derived from each is accessed in the same way, from the same or a similar underlying representation, in number manipulation tasks.

This latter explanation of the differences in outcomes between tasks in terms of the absence of a stable semantic memory representation for word problems may also help to explain why differences exist between the findings of the word priming literature and the present results. For example, a plethora of psycholinguistic research reveals priming effects that occur between visually presented single words (e.g., ‘doctor’ and ‘nurse’), short sentences and words (e.g., ‘I saw a duck’ and ‘quack’),
and pictures and words (the word ‘nest’ primes a picture of a ‘bird’ or vice versa) (Alario, Segui & Ferrand, 2000; Dell’Acqua & Grainger, 1999; Neely, 1991; Paul & Kellas, 2004). It therefore, seems odd that, in the third investigation, ‘two + three’ did not prime ‘5’. However, given that the stimuli employed in much of the word priming research are encountered at an early age and regularly, throughout a persons reading and viewing history, such a finding does not appear too unusual.

To sum up, the findings of the third investigation indicated that solutions to simple arithmetic problems represented in digit form are accessed differently to solutions of problems represented in number word form. Accordingly, it was suggested that solutions to digit problems are accessed automatically from stable visual semantic representations in memory, whilst solutions to number word problems are accessed via phonological representations.

3.4 Direct Access to Different Problem Types.

The fourth investigation in this series of studies addressed the question of whether problem type (including: standard, tie, one and zero-problems) influences fact retrieval mechanisms. Two main hypotheses were presented in this study. Firstly, given that tie problems can not be solved via a general rule, it was hypothesised that tie problems are accessed in the same way as standard problems and consequently, that they would produce a similar pattern of priming effects to that observed for standard problems. The results supported this hypothesis, revealing similar patterns of facilitation and inhibition for both problem types. This finding supports the notion that the tie-advantage identified in earlier investigations resulted from encoding processes.

Secondly, given that zero and one-problems are solved via conscious processing strategies (i.e., via the verbal counting sequence or rules such as \( n \times 1 = \))
(n), it was hypothesised that a pattern of priming effects that includes increased facilitation and inhibition at the long SOA would be found in these conditions. However, the patterns of priming effects observed for addition zero and one-problems, and multiplication one-problems, did not differ markedly to that found for the tie and standard problems. Moreover, performance in the multiplication one-problem condition was similar to that observed in the addition one-problem condition. However, prior exposure to addition one problems led to significant facilitation in congruent target naming at the short SOA, a time period too brief to allow for strategic processing of the prime in order to speed naming of the target. This finding indicated that the solutions to addition one problems were actually retrieved automatically. Thus, the results of the fourth investigation suggested a direct access route to solutions of addition and multiplication one-problems, and addition zero-problems.

The findings for the multiplication zero-problem condition were also somewhat different to what was expected. In the multiplication congruent and neutral zero-problem conditions, given that the target ‘0’ was presented far more frequently than any other target number in this study, faster responses than in any of the other conditions would have been predicted. Nevertheless, surprisingly, a zero-target naming disadvantage was identified in both of these conditions. Accordingly, this effect was explained in terms of response competition that occurs after exposure to the stimulus ‘0’ and that is produced by the activation of the various other terms for zero (e.g., nil, nought, and oh) in memory. Importantly, given such an explanation, it would seem that an alternative methodology to one that relies on naming the digit ‘0’ is required in the investigation of fact retrieval for this problem type. Moreover, it
suggests that generally, access to concepts that have only one interpretation occurs more rapidly than access to concepts that have many interpretations.

In brief, the results of the fourth investigation in this series of studies indicated that solutions to tie, standard, zero and one-problem types are accessed in much the same way i.e., directly from memory. However, further investigation using an alternative methodology is required in determining how access to solutions of multiplication zero-problems occurs.

3.5 The Split Effect.

The final study in this series of investigations aimed to determine whether split effects are produced in the present priming procedure, and if so, what the nature of these effects are. Consistent with the results of an earlier investigation by Campbell (1987), the results of the final investigation revealed a reversed split effect in simple multiplication and addition processing. That is, the results showed that the time taken to name distant incongruent targets was significantly greater than the time taken to name close incongruent targets, in all conditions of this study. This finding indicates that split effects are task specific and that, in the context of the present arithmetic priming task, they produce a reversed split effect to the standard effect found in verification tasks.

In addition to the reversed split effect, the final study produced some interesting priming effects. Facilitation in congruent target naming was found at 300 and 1000 ms in the multiplication condition, and at 1000 ms in the addition condition. In contrast, inhibition in incongruent target naming was only found at 300 ms. In the multiplication condition, inhibition in distant incongruent target naming resulted, whilst in the addition condition, inhibition in both close and distant incongruent target naming was found.
The facilitation effects observed in the final study were consistent with the pattern of effects observed in earlier investigations employing this methodology. Accordingly, the facilitation in the multiplication condition at 300 ms was attributed to the operation of an automatic spreading activation mechanism, whilst the significant increase in facilitation observed in both operations over time was attributed to the use of a strategic process to speed target naming at the long SOA.

Whilst not addressed in the final manuscript, in the context of the present series of investigations, the assumption that the increase in facilitation at the long SOA was produced by strategic processing leads naturally to the question of just what type of process was employed to produce facilitation and not inhibition. This is a relevant question to ask because word priming theories predict increased levels of facilitation and inhibition at long SOAs. As noted in section 1.3.1 above, this is because, in word priming theories, it is assumed that participants can use the prime strategically to generate an expectancy set of related targets (Neely, 1991). This then leads to speeded processing of related targets, whilst a time consuming search through the expectancy set inhibits processing of unrelated (i.e., incongruent) targets (Neely, 1991). Nevertheless, it is noteworthy that even in Neely’s (1991) review of the word priming literature, the inhibitory effects that are presented are often not as large, reliable, or robust as the facilitation effects that he presents. Moreover, the relationships between the primes and targets in the word priming and arithmetic priming paradigms are quite different. As noted in the discussion section of the first investigation, unlike words, arithmetic problems have only one correct solution, and there are only 10 basic symbols, leading to a finite set of varying relationships between these numbers (Anderson, 1983). Accordingly, on this basis, a good deal of
variation in the level of inhibition, both within the present series of investigations and between the number and word priming paradigms, was probably to be expected.

However, one tentative explanation for why facilitation is found in the absence of inhibition at the long SOA is that the types of representations that are activated in the arithmetic priming paradigm may influence responding. For example, given the similarities between the methodology employed in the surface form investigation and the final study, it is reasonable to assume that the automatic spreading activation mechanism again operated on visual representations of the digit stimuli. Thus, in the congruent condition, the early activation of a visual representation of the correct solution (through exposure to the prime), coupled with its subsequent presentation as the target, may have combined to produce the observed facilitation effects. In contrast, in the incongruent condition, the early activation of a visual representation of the correct solution (cf. a verbal representation of the correct solution) would presumably have interfered little with the development and execution of the vocal response required in naming the incongruent target. Such an account of the operation of this mechanism therefore allows for a plausible explanation as to why it produces facilitation and no inhibition. Nonetheless, at present, there is little in the way of empirical evidence to support this hypothesis, and a good deal of research into the types of representations activated in memory in arithmetic processing remains to be done (Fayol & Seron, 2005).

In the context of the surface form investigation, it was suggested that the inhibition that was observed resulted from the processing of phonological representations of the novel word problems that were activated via a reading based mechanism. However, in the context of the final investigation, it seemed implausible that the primes, which were represented in the usual Arabic digit format, would incur
a similar process. Accordingly, as with the findings of the Campbell (1987) investigation, it was suggested that the inhibition (and the difference in the time taken to name close and distant incongruent targets) resulted because of the activation of a magnitude based representation in memory.

According to Dehaene and colleagues (e.g., Dehaene, 1997; Dehaene et al., 2003), magnitude representations underlie an evolutionary number sense that is found in both humans and animals and that is specialised for detecting approximate quantity. This is supported in the literature by the finding that animals, human infants, and human adults that do not have extended counting systems, are capable of attending to numerosity and performing elementary computations (Feigenson, Dehaene & Spelke, 2004; Dehaene, 1997; Gordon, 2004; Pica, Lemer, Izard & Dehaene, 2004). Furthermore, neuroimaging research by Dehaene et al. (2003) has identified a neural system in the brain that is activated whenever numbers are manipulated, and that is increasingly activated in tasks involving quantity processing i.e., the horizontal segment of the intraparietal sulcus. This system is described as being analogous to an internal mental number line and is thought to be supported by a second, bilateral parietal, system responsible for attentional orientation along this line. Behavioural research maintains this description and also shows that the mental number line is oriented from left to right, is logarithmically compressed, and is activated in memory as an obligatory process (e.g., see Dehaene et al., 1993; Dehaene, 1993, 1997; Dehaene, Dupoux & Mehler, 1990; Nuerk, Weger & Willmes, 2005; Nuerk, Kaufmann, Zoppoth & Willmes, 2004; Roberts, 2005; Siegler & Booth, 2004).

Thus, in the context of the final investigation, it is possible that a similar ‘number line’ representation was activated in memory following exposure to the
numerical stimuli in the arithmetic priming task and that this representation influenced the pattern of findings. As outlined in the final study, exposure to the two digits in each problem possibly led to the activation of a region of the number line corresponding to a rough estimate of the location of its correct solution. This region of activation was probably represented by a distribution that was peaked around the correct solution and that extended out on either side. If the target was consistent with the activated region on the number line, no hesitation in target naming resulted but if the number that the participant was trying to name differed to the activated region, then some degree of inhibition in responding occurred. Moreover, the participant was presumably less confident in their response when the target was further away from the activated region, leading to a larger inhibitory effect than when the target was close to the activated region. Thus, the number line explanation accounted nicely for the finding of a reversed split effect. What is more, given its logarithmic nature, it allowed for the prediction of stronger and clearer effects in the context of the smaller numerical size and range of targets in the addition condition. Finally, with the participants simply required to name target numbers and not to perform magnitude comparisons or calculations, it appeared that the activation of this representation was obligatory. Like automatic processes that are not acted upon by strategic processes, the activation of the mental number line would therefore, be expected to diminish over time, such that smaller inhibitory effects would be found at the long SOA than at the short SOA (Stolz & Neely, 1995). This was notably consistent with the findings of the final investigation.

Finally, whilst the findings of the earlier investigation in this series indicated that the facilitation and inhibition produced in the context of the priming task were produced by different mechanisms, there did appear to be some degree of
interrelatedness between these two mechanisms in the final study. It is noteworthy, for example, that the directions of change in the levels of facilitation and inhibition in each of the conditions in this study were remarkably consistent. An important case in point occurred in the addition, short SOA, condition. Notably, no significant facilitation was found in naming congruent targets in this condition, which indicates that the present sample did not automatically retrieve addition facts from memory. Indeed, the pattern of facilitation produced in the present study was the same as that produced by the low skilled sample in Jackson and Coney (2006a), i.e., regardless of the fact that the present sample ($M = 23, SD = 4.85$) scored significantly higher overall on the ACER SCT than did the previous sample ($M = 11, SD = 1.73$) ($t(33.78) = 12.33, p < 0.001$). Nevertheless, as noted above, this same sample produced significant inhibitory effects in naming both close and distant incongruent targets at this SOA. Thus, it may be the case, that when arithmetic facts are not automatized, arithmetic processing relies on the activation of a more fundamental representation specifying approximate quantity (i.e., the mental number line). This proposition is consistent with current theorizing suggesting that magnitude representations serve as the default option for quantitative computations (Booth & Siegler, 2006; Dehaene, 1997).

In summary, the results of the final study revealed a reversed split effect in target naming times following exposure to simple addition and multiplication problems. This finding indicated that split effects are in fact task specific. Furthermore, this investigation revealed some interesting facilitatory and inhibitory effects. Consistent with the conclusions drawn in the earlier investigations, the facilitation function was explained in terms of the operation of an automatic spreading activation mechanism. However, in contrast to this, a new explanation of
the inhibitory effects observed in the arithmetic priming task was offered. Specifically, it was suggested that the inhibition resulted from the activation of regions along an internal magnitude representation (analogous to Dehaene and colleagues’ mental number line) corresponding to the approximate solutions to the primes in memory. The distance between the activated region and the presented target was suggested to directly influence the participants’ confidence in their response. When the target was further from the activated region, the participant was less confident and more inhibition in responding was observed. This explanation was notably consistent with the conclusions drawn from current behavioural and neuroimaging research.

3.6 The Cognitive Mechanisms Underlying Simple Arithmetic Processing

One of the most important outcomes of the present research is that it has introduced an alternative priming methodology to the investigation of the cognitive processes underlying simple arithmetic processing. As a direct result of this, the present investigations were able to identify some interesting facilitatory and inhibitory effects in target naming. On the basis of these findings, four main mechanisms were suggested to operate in simple arithmetic fact retrieval. One of these mechanisms accounted for the facilitation functions, which were generally consistent across investigations. In contrast, the inhibitory functions varied between studies, across SOA, skill level, operation, and surface form. This led to the proposal of three new inhibitory mechanisms operating in arithmetic performance. The following subsections describe each of the four mechanisms in turn.

3.6.1 Automatic Spreading Activation
Given that the mechanism that operated to produce the facilitation in the present series of investigations appeared to be automatic, it was suggested that this mechanism was spreading activation. Support for the notion that this mechanism operated automatically was provided by the fact that the facilitation was found in the priming paradigm, regardless of the requirement to simply name target numbers. Moreover, the time course of the facilitation function supported this conclusion. No facilitation was observed at very brief SOAs (i.e., at 120 ms). If the facilitation effect resulted from strategic processing that occurred after presentation of a target, then it should have been found no matter how brief the SOA. Instead, the facilitation function emerged at medium SOAs (i.e., 240 and 300 ms) i.e., time periods too short for the use of strategic processing to speed target naming. At long SOAs (i.e., 1500 and 1000 ms), the facilitation function increased markedly. This was consistent with the use of the correct solution activated in memory to speed congruent target naming.

The operation of the proposed automatic spreading activation mechanism was only evident in high skilled performance in the naming task. In the present studies, significant facilitatory effects were identified at 300 ms in both operations, for the high skilled group alone. In contrast, at the long SOA, significant facilitation was observed in multiplication and addition performance for both the high and the low skilled group. Nevertheless, only the level of facilitation produced in the multiplication condition by the high skilled group increased markedly over time. Therefore, only high skilled individuals appeared to use their multiplication fact knowledge strategically to speed processing in the naming task.

The operation of the proposed automatic spreading activation mechanism in high skilled performance appeared to be modulated both by problem size and operation. High skilled individuals appeared to have automatic access to small
addition problems and greater access to solutions of small and large multiplication problems at long SOAs. However, access to large addition problems did not appear to occur, with no significant priming effects resulting following exposure to these problems at all. Thus, it appeared that no stable semantic representation of large addition problems exists in memory. As noted earlier, this is probably the result of less practice with this problem type. In contrast to large multiplication problems, large addition problems are not generally rote learnt. Furthermore, unlike small addition problems, large addition problems are possibly encountered less often in early formal schooling and naturally occurring settings.

The proposed automatic spreading activation mechanism appeared to operate on mental representations of Arabic digit addition and multiplication problems that were visual. The surface form investigation showed that the problems represented in Arabic digit and word form were processed differently. Significant facilitation was found in congruent target naming in the digit condition only, whilst significant inhibition was identified in the number word condition only. Thus, given the novelty of the visual representation of the number word problem stimuli, the inhibitory effects were explained in terms of the processing of phonological representations of the word stimuli. In contrast, consistent with the high frequency of exposure to Arabic digit stimuli in formal schooling, the facilitatory effects were explained in terms of the processing of visual representations of these stimuli.

In summary, together, the nature of the present naming task and the time course of the facilitation function implied that an automatic spreading activation mechanism operated in the arithmetic priming task, leading to earlier and greater access to correct solutions. Spreading activation appeared to occur directly, through
visual exposure to Arabic digit stimuli. It only emerged in skilled arithmetic performance, and in the context of more frequently encountered problems.

3.6.2 Inhibitory Mechanisms

Inhibitory effects were observed in the incongruent conditions of all investigations in the present series of studies. Moreover, the inhibition was found even though the target naming task did not require production or verification operations. This suggests that the inhibitory effects resulted from the operation of obligatory mechanisms in processing.

The pattern of inhibitory effects appeared to differ with the surface form of the arithmetic stimuli (i.e., either Arabic digit or number word stimuli), the skill level of the participants, and the split between correct solutions and incongruent targets. Consequently, the explanations for what types of mechanisms caused these effects evolved over the course of the investigations. For instance, the findings of the first two investigations in this series led to the proposal that a ‘response validity checking mechanism’ operated in skilled arithmetic performance. The findings of the third investigation led to the proposal that the inhibition observed in processing incongruent ‘number word’ stimuli resulted from the activation of phonological representations of these stimuli via a reading based mechanism. The final investigation led to the proposal that the inhibitory effects observed in Arabic digit processing resulted from the activation of magnitude based representations of the presented problems and their approximate solutions in memory. This mechanism, which can be likened to Dehaene’s (1997) ‘number sense,’ influenced the participant’s confidence in their response. The following subsections provide a chronological description of the conclusions surrounding each these mechanisms.
3.6.2.1 The Response Validity Checking Mechanism

Unlike the facilitation functions in the first two investigations of the present series, the inhibition functions emerged at short SOAs (i.e., at 120, 240 and 300 ms), i.e., time periods that were too brief to allow for processing of the prime to influence responding before presentation of the target. Thus, the inhibition in the incongruent condition must have resulted from processing that occurred after presentation of the target i.e., a stage that is not strictly automatic. Furthermore, significant inhibition was found in high skilled performance only i.e., in individuals possessing stable semantic representations of associations between simple addition and multiplication problems and their correct solutions. Accordingly, the inhibitory effects were explained in terms of the operation of a ‘response validity checking mechanism’ that operated by a comparison of the presented target to the correct solution evoked in memory. Theoretically, inhibition in verbal target naming resulted when a mismatch between these two numbers occurred. This explanation was consistent with the emphasis placed on accuracy in responding in simple arithmetic operations in formal learning procedures.

3.6.2.2 Number Word Problems and Reading Mechanisms

The third investigation in the present series showed that arithmetic problems represented in number word form are processed differently to problems represented in Arabic digit form. Importantly, in the context of this third investigation, significant inhibition in incongruent target naming was only found in the number word problem condition. Given the novelty of the number word stimuli (e.g., ‘two + three’ cf. ‘2 + 3’), it seemed unlikely that correct solutions would be activated from a stable, semantic, visual representation of these problems in memory. Had this information
actually been represented in memory in this form then significant facilitatory effects should have been found in the congruent word condition but they were not. Accordingly, with number words more commonly found in reading contexts, it was suggested that the priming effects observed in the word condition were activated via a strong reading based mechanism, such as subvocalisation. This, in turn, led to the activation of phonological representations and subsequently, inhibition when an incongruent target was presented.

Notably, this explanation suggests that when faced with problems represented in an unusual visual form (e.g., number word problems) or with problems that are not practiced to mastery (i.e., multiplication facts are usually rote learnt, whilst addition facts are not), phonological processing can be relied upon as a back up procedure for arithmetic fact retrieval. This explanation can be likened to the dual-route cascaded model of visual word recognition and pronunciation (Coltheart et al., 1993; Coltheart et al., 2001; Coltheart & Rastle, 1994). According to this model, there are two procedures that are responsible for converting printed words to speech. The first is phonologically based, involves a grapheme to phoneme conversion procedure, and is employed in early reading. The second is an orthographic, visual word recognition system that is used in skilled reading and accesses semantic representations directly via the visual word form (Coltheart et al., 2001; Coltheart et al., 1993; Frost, 1998; Visser & Besner, 2001). This is similar to the proposal that the novel number word stimuli rely upon phonological processing, whilst the highly over learned Arabic digit stimuli directly access semantic representations via their visual form.

3.6.2.3 Arabic Digit Problems and ‘Number Sense’

In the final investigation of the present series, Arabic digit stimuli were again presented as primes, however, this time, the distance between the correct solutions to
the problems and the incongruent targets was varied. The results revealed increased inhibition in naming more distant incongruent targets, inhibition in naming close incongruent targets in the addition condition, and a pattern of diminishing inhibitory effects over time. These findings were difficult to explain in terms of reading based mechanisms and phonological processing. Moreover, in the context of the final investigation, it seemed implausible that the Arabic digit primes would activate visual representations in the congruent condition, and phonological representations in the incongruent condition. In view of this, it was suggested that the inhibition in the final investigation (and the difference in the time taken to name close and distant incongruent targets) resulted because of the activation of a magnitude representation of number in memory. That is, it was suggested that exposure to the two digits in each problem led to the activation of a region of a mental number line representing its approximate solution. The region of activation, itself, was described as being a ‘distribution’ that was peaked around the correct solution. The more the target differed to the activated region, the less confident the participant was in their response and the greater the level of inhibition produced in target naming.

Support for the above conclusion was provided by a body of converging evidence from behavioural and neuroimaging research, indicating that humans and animals possess an evolutionary number sense that is specialised for detecting approximate quantity (Dehaene, 1997; Dehaene et al., 2003; Feigenson et al., 2004; Gordon, 2004; Pica et al., 2004). Moreover, according to Dehaene and colleagues (Dehaene, 1997; Dehaene et al., 1993), this number sense can be likened to an internal number line that is logarithmically compressed and obligatory in nature. It thus provided a particularly useful account of the findings in the final investigation.
3.6.2.4 Summary and Conclusions

In the first two investigations in which Arabic digit processing was examined, the patterns of inhibitory effects were generally consistent over time and were suggested to occur because of the operation of a response validity checking mechanism. In contrast, in the final investigation that examined Arabic digit processing, the inhibitory effects were found to diminish over time and were suggested to result from the processing of magnitude representations. Thus, two potential explanations for the inhibition in Arabic digit processing were offered in the present series of investigations. Whilst this goes against considerations of parsimony, given that the trials were blocked across SOA and that the short SOA condition was always presented first, it is possible that the participants employed different and finely-tuned approaches to performing the naming task at each SOA. For example, because of the strong resemblance in methodology between the aforementioned studies, it is likely that the obligatory ‘number sense’ mechanism was in operation at the short SOA in all three investigations. At the long SOA, if this magnitude representation was then acted upon by a strategic mechanism, e.g., a response validity checking mechanism, significant inhibition would be observed. However, if the magnitude representation was not acted upon by a strategic mechanism, the level of inhibition would diminish over time, as occurred in the final investigation. In support of this proposition, it is noteworthy that the final investigation employed a lower relatedness proportion than was employed in the earlier investigations (0.25 cf. 0.30, respectively), thereby possibly producing conditions that were less conducive to the use of strategic processing in performance. Future research should thus examine what influence the manipulation of relatedness
portions has on inhibitory effects over time, using the same methodology as that used in the final investigation.

In the context of the third investigation, which involved number word stimuli, it was suggested that the inhibition resulted from the processing of phonological representations of these stimuli that were activated via reading mechanisms. Whilst this conclusion was consistent with reading and word processing theories, it is worth noting that Dehaene et al. (2003) found that the HIPS area (i.e., the area most strongly activated in memory when quantity based processing is required) is activated independently of number notation. Thus, it may be the case that magnitude representations were also activated following exposure to the number word primes. Thus, future research should examine whether the split between correct and incorrect solutions to number word problems influences the inhibition observed in the arithmetic priming task.

To sum up, the present series of investigations led to the proposal that three different inhibitory mechanisms may be in operation in the arithmetic priming paradigm. Whilst other factors were involved (e.g., split, operation and skill level), the surface form of the stimuli and the SOA between the prime and the target appeared influential in determining which mechanism was employed and was thus, responsible for the pattern of inhibitory effects observed in each investigation. However, the assumptions based on the present findings are, at this stage, tentative and further research is needed to examine their validity.

3.7 Future Research

The findings of the present series of investigations demonstrate the utility of the numerical variant of the single-word semantic priming paradigm for accessing the cognitive processes underlying simple arithmetic processing. Given this capacity,
there are a number of important directions for future research. The following sub-sections provide examples of directions for further theoretical enquiry and applied research.

3.7.1 Directions for Further Theoretical Enquiry

A number of directions for further theoretical enquiry exist. These include studies that aim to further explore the main factors influencing simple arithmetic processing, studies aimed at exploring operation differences, and studies that aim to strengthen the priming methodology.

3.7.1.1 Investigations Stemming from the Present Research

A first study following on from the present series of investigations is reminiscent of the investigations undertaken by LeFevre and her colleagues, and involves the use of primes that do not contain an arithmetic operator in the present methodology (i.e., ‘2 + 3’ cf. ‘2   3’). The findings from such a study would potentially solidify the present conclusions relating to automaticity in processing, and may possibly provide greater insight into just how the response validity checking mechanism operates. Theoretically, the exclusion of the arithmetic operator negates the need for validity checking but it may be the case that the response validity checking mechanism is operand driven.

A second study relates to the findings of the problem type investigation. The results of this study revealed some interesting priming effects. For example, in the short (i.e., 300 ms) multiplication condition, only the facilitation in target naming following exposure to tie problems reached significance. Nonetheless, no significant differences in the levels of facilitation were observed between problem types at this SOA, leading to the conclusion that there are indeed no differences in processing.
However, it may be the case that larger differences in access do actually occur between the different problem types at shorter SOAs, and that these differences are just not evident at 300 ms. In view of this possibility, future research aimed at mapping the time course of semantic activation for each of the different problem types at shorter SOAs (e.g., SOAs of 120 and 240 ms) would be beneficial.

A third possible study stems from the split effect literature and, in particular, from the findings of an investigation by De Rammelaere et al. (2001). In their study, De Rammelaere et al. (2001) found a problem size effect for true and close false equations but not for distant false equations. According to the authors, this indicated that the solutions to the distant false equations had not been activated in memory and thus, that they must have been verified using a plausibility judgement. Future research could thus employ the present methodology in an investigation of problem size and split effects in an attempt to replicate and further explore this finding.

Yet another study also involves the investigation of split effects but has a greater focus on the multiplication operation. In the present split effect investigation, the splits between the correct and incorrect solutions were created on the basis of the distance between these two numbers along the number line. An alternative method, introduced by Galfano et al. (2003), is to create the splits on the basis of the multiplicative relatedness between the correct and incorrect solution. Here, close incongruent targets would be the numbers adjacent to the product in the table related to one of the operands, whilst distant incongruent targets would include numbers further than the numbers adjacent to the product in the table. For example, given the problem ‘4 x 7’ (with solution 28) the close incongruent target set would consist of 21, 35, 24 and 32 (corresponding to 3 x 7, 5 x 7, 4 x 6 and 4 x 8). In contrast, the distant incongruent target set would consist of multiples of the operands 4 and 7.
greater than 35 and less than 21. In this way, rather than measuring split effects that are due to a similarity in magnitude, split effects that are due to the activation of concepts that are closely linked within an associative multiplication network can be measured.

Finally, two important investigations were highlighted in the discussion of the inhibitory mechanisms above. Firstly, it was suggested that the lack of inhibition at the long SOA in the final study may have resulted due to the use of a lower relatedness proportion than was employed in the earlier investigations (0.25 cf. 0.30, respectively). Thus, it was suggested that future research should include a relatedness portion manipulation to determine its influence on inhibitory effects over time. Secondly, Research by Dehaene et al. (2003) indicates that the area most strongly activated in memory when quantity based processing is required is activated independently of number notation. Future research should thus examine whether split influences the inhibition observed in the context of arithmetic priming tasks involving number word stimuli.

3.7.1.2 Operation Differences in Cognitive Processing

Future research into arithmetic processing should aim to provide a more thorough exploration of processing differences between the addition and multiplication operations. A common finding in most of the present investigations was the finding of a main effect of operation, with target naming times in the addition operation found to occur more rapidly than in the multiplication operation. With previous research having shown that it takes longer to perform number naming tasks when numbers are large than when they are small (e.g., see Brysbaert, 1995), this difference was explained in terms of differences in target magnitude between the operations. In the present series of investigations, addition-related targets ranged
between 5 and 17, whilst multiplication-related targets ranged between 6 and 72. Nevertheless, the differences in target magnitude may not be the only explanation for the operation differences observed in the present investigations. It may be the case that differences in access to solutions arise due to problem size and frequency differences. In the addition operation, the magnitude of the problems that are dealt with are much smaller and are introduced earlier in formal schooling (and with possibly far greater frequency in natural settings) than the problems in the multiplication operation. Consequently, in the addition operation, the time course of semantic activation may be shorter than occurs in the multiplication operation. As a result, when the present procedure is employed, the level of addition facilitation may peak and fade earlier than 1000 ms, such that there appears to be less facilitation than occurs for the multiplication operation at this SOA. Accordingly, to address this possibility, further research that aims to map the time course of semantic activation at shorter long SOAs than 1000 ms (e.g., 500 ms and 750 ms) is required in testing.

Additionally, it is noteworthy that the levels of inhibition that were found for each operation at the long SOAs in each study were not consistent. For example, in the first study, significant inhibition was found for the high skilled group in the multiplication condition at 1500 ms, whilst no inhibition was evident for this group in the addition condition. In contrast, in the second study, significant inhibition was found at 1000 ms in the addition condition, whilst no significant inhibition was found in the multiplication condition. One possible explanation for this finding is that although the groups scored well on the ASCT, they differed in their simple addition and multiplication skill levels. Accordingly, each group may have relied upon different strategies in target naming at these SOAs. In view of this, future research should examine whether differences in the levels of inhibition that are produced in
the performance of the arithmetic priming task result between high and low addition skill groups, and between high and low multiplication skill groups.

The present series of investigations has focused primarily on the cognitive processes underlying simple addition and multiplication performance, and little consideration has been given to the cognitive processes underlying subtraction and division performance. Unlike multiplication facts, subtraction facts are not learned by rote in formal schooling, and instead may be solved through the mental manipulation of quantities (Lemer et al., 2003). Support for this proposition stems from the finding that, whilst multiplication impairments are associated with aphasia due to left subcortical damage, subtraction impairments are associated with dysfunction of the right inferior parietal lobule, which is thought to be involved in the processing of quantity (Dehaene & Cohen, 1997; Lemer et al., 2003). Moreover, research by Lee and Kang (2002) shows that simultaneous phonological rehearsal interferes with multiplication but not subtraction, whilst the simultaneous holding of an image in mind interferes with subtraction but not multiplication (Dehaene et al., 2003). It should therefore, be possible to demonstrate differences in multiplication and subtraction processing using the present procedure.

However, the situation with respect to the division operation is a little more ambiguous (Cohen et al., 2000). On the one hand, like subtraction, the division operation is not rote learnt and therefore, may rely on strategic processing. On the other hand, the solution of division problems could theoretically, rely on direct access to practiced verbal associations that are learnt in the acquisition of multiplication fact knowledge. Certainly, research into the cognitive processes underlying the division operation is underrepresented in the literature, with little
empirical support available for either position. The present procedure can therefore, usefully be applied to address this limitation.

3.7.1.3 Improving the Arithmetic Priming Methodology

A final study stemming directly from the present research involves the exploration of alternative neutral condition stimuli. In the present investigations, the letter based neutral condition stimuli (i.e., $X + Y$ and $X \times Y$) were introduced after the first study. This was done for a number of reasons. Firstly, earlier research had indicated that the processing of zero-based stimuli occurs more slowly than other numerical stimuli and therefore, any facilitatory effects that were identified in the first study were possibly exaggerated (Stazyk et al., 1982). Secondly, the incongruence between the prime and the target in the zero-based neutral condition (e.g., $0 + 0$ presented with 14) may have lead to slowed target naming in this condition, again possibly exaggerating any facilitatory effects. In contrast, the letter based stimuli were considered particularly well suited to the purposes of the present investigations as they were perceptually similar to the numerical primes, and the symbols $X$ and $Y$ are often employed to denote separate unknown quantities in formal schooling.

However, having introduced the letter based neutral condition stimuli after the first investigation, an examination of the cognitive processes in operation at the SOAs of 120 and 240 ms using this baseline, was not undertaken. Moreover, the repetition of the symbol $X$ in the multiplication neutral prime through its use as an arithmetic operator (i.e., $X \times Y$), and the repetition of stimuli in the word condition of the surface form study (i.e., ‘blank x blank’ and ‘blank + blank’) is arguably, problematic. Theoretically, speeded processing of a neutral prime through stimulus repetition should have little impact on the processing of a target. However, it may be
the case that at shorter SOAs, prime encoding actually interferes with target encoding, thereby delaying naming times. Given such a scenario, less interference in target encoding would be expected in neutral conditions employing stimulus repetition. This would lead to faster target naming times in these conditions and thereby, reduced overall facilitation effects. Therefore, future testing at shorter SOAs (i.e., 120 and 240 ms) that does not involve neutral condition stimulus repetition may be a worthwhile undertaking. Toward this end, alternative neutral condition stimuli such as ‘A + C’ and ‘A x C, and ‘blank + neutral’ and ‘blank x neutral,’ could be employed.

The advancement of the present methodology is important to its use in addressing many of the unresolved issues that have been at the forefront of applied research in the field of cognitive arithmetic. Examples of future studies that have implications for curriculum design and the remediation of disordered number processing skills are now provided (Ashcraft, 1992; Dehaene, 1992; Koshmider & Ashcraft, 1991).

3.7.2 Directions for Applied Research.

A number of applied investigations are possible using the arithmetic priming paradigm. These include studies that relate to both educational practice and the remediation of disordered arithmetic skills.

3.7.2.1 Educational Practice

As a logical progression from the present series of investigations, future research should examine whether automaticity in arithmetic fact retrieval underlies advantages in other mathematical abilities, including measurement, chance and data, space and algebra (Curriculum Council, 2005). The finding that automaticity in
arithmetic fact retrieval underlies other mathematical abilities would be consistent with the notion that basic arithmetic facts are the foundation of number sense (Curriculum Council, 1998, 2005; Resnick, 1989). Moreover, it would reinforce cognitive theories suggesting that the rapid and effortless access to simple arithmetic facts frees attentional resources so that they can be devoted to other more complex cognitive procedures (Campbell, 1987; Koshmider & Ashcraft, 1991; Reed, 1998; Resnick, 1989; Willoughby, 2000).

Further studies would then need to focus on just how automaticity in fact retrieval develops and how to best promote it. Since the 1920s, there has been controversy over whether children learn mathematics better by rote or through meaningful learning (Butterworth, 1999; Resnick, 1989). Rote learning stresses speed and accuracy that is attained through substantial drill on number facts, whilst meaningful learning focuses on the different ways of decomposing and recomposing numbers and hence, principles such as additivity and small number relations (Curriculum Council, 2005; Butterworth, 1999; Resnick, 1989). The priming methodology employed in the present studies should allow for comparisons to be made between the development of automatic and inhibitory processes in groups of children who are exposed to either method or to some combination of these methods. This information could then be usefully applied to curriculum design.

In a similar vein, this new methodology may be employed to determine whether differences in cognitive functioning underlie cross-cultural differences in mathematical performance. Over the past 30 years, East Asian nations have consistently outperformed other nations in mathematics (Geary et al., 1997; Woodward & Montague, 2002). This may result from differences in educational practices (Lee, Graham & Stevenson, 1996; Stigler, Fernandez & Yoshida, 1996).
For example, in comparison to children in Australia and the US, Japanese children are exposed to extra curricular mathematical activities at a very early age (sometimes beginning from as early as 1 year old) and this may continue throughout elementary school (i.e., primary school) (Russell, 1996). One such activity, the Kumon method, is undertaken by 7% of elementary school children, twice a week, after school (Russell, 1996). It employs drill, repetition and over learning of arithmetic facts, a style of learning that generally has negative evaluative connotations associated with it in Western cultures (e.g., see Curriculum Framework, 1998, pp. 197) (Russell, 1996). Additionally, Japanese classes in mathematics are centered on problem solving tasks to a much greater degree than are Australian classes (Stigler et al., 1996). Together these differences may amount to differences in the development of automaticity and inhibitory effects in Japanese and Australian children that may underlie the overall performance differences.

Related to the development of automaticity (and inhibition) is the issue of whether calculators should be introduced at an early age or whether they interfere with development. At present, there is little consensus on this issue. For example, in California, the board of education requires that children rote learn their tables by grade 3 and have banned calculators from tests before grade 6 (Butterworth, 1999). In contrast, the Curriculum Council of Western Australia suggests that calculators should be introduced in early childhood (K – 3 yrs) (see 2005 Curriculum Framework). This is done to enable children to get used to pressing a key, making a specific number appear on the screen, and to checking for accuracy (Curriculum Council, 2005). Later, it enables them to perform basic calculations (Curriculum Council, 2005). However, this practice may encourage children to rely on calculators as a strategy for fact retrieval rather than committing facts to memory. This in turn
may lead to lower levels of automaticity in simple arithmetic processing and consequently, put children at a significant disadvantage in paper and pencil tests in which the use of calculators is generally not allowed (Tsuruda, 1998). A further study would therefore, aim to determine whether the use of calculators is associated with higher or lower levels of automaticity and whether this amounts to overall differences in performance.

3.7.2.2 The Remediation of Disordered Arithmetic Skills

Ideally, another study would determine at what ages automaticity in fact retrieval and the operation of inhibitory mechanisms become evident in a normal student population. These data could then be compared to data obtained from children with math difficulties to determine whether differences exist in cognitive functioning. As noted in the Introduction (see section 1.4.1), the process of retrieving arithmetic facts from memory is particularly error prone due to interrelatedness within and between operations (Ashcraft, 1992; Barrouillet & Lepine, 2005). Consequently, a recent theory suggests that some children may have maths difficulties because they are unable to inhibit interference from incorrect responses (Barrouillet & Lepine, 2005). The priming method employed in the present studies will allow a direct investigation into this possibility.

Due to the fact that students with maths difficulties also often present with reading difficulties, it has been suggested that a general problem with phonological processing may underlie the two types of difficulties (Robinson, Menchetti & Torgesen, 2002). However, the present research has shown that automatic processing can be elicited through visual priming, using Arabic digits, and suggests that phonological processing is only employed when problems are represented in an unusual form (e.g., two + three). A further study should thus explore the utility of
increased visual exposure to arithmetic problems represented as digits in the development of simple arithmetic knowledge in children with math difficulties. This method could possibly work well considering that most maths tests at the primary school level are paper and pencil tests in which problems are visually presented as Arabic digits. As an aside, this study would provide further evidence against the use of verification tasks (in which problems are frequently paired with incorrect solutions) in basic fact learning (Campbell, 1987).

3.8 Conclusion

The present series of investigations was the first in the cognitive arithmetic literature to apply a numerical variant of the single word semantic priming paradigm to the investigation of the cognitive processes underlying simple arithmetic ability. Significant facilitatory and inhibitory effects were identified in skilled arithmetic processing following priming by both multiplication and addition stimuli. Accordingly, the facilitatory effects observed in Arabic digit processing at the short SOA were suggested to result from the operation of an automatic spreading activation mechanism. At the long SOA, the significant increase in facilitation over time indicated the use of a strategic mechanism in performance. In contrast, the inhibitory effects were tentatively explained in terms of the activation of three different mechanisms in processing across the course of the present investigations. The type and nature of the inhibitory mechanisms varied with both the surface form of the arithmetic stimuli, and the SOA. In the first two investigations, it was suggested that the inhibitory effects resulted from the use of a response validity checking mechanism that operated via a comparison of the just presented target to the correct solution activated in memory. When the two did not match, hesitation in responding resulted. In contrast, in the final investigation, the results were more
consistent with the activation of magnitude representations in memory. When there was greater incongruence between the presented target and this representation, the participant was less confident in their response, leading to more inhibition. Finally, in the context of number word primes, it was suggested that the inhibition in processing resulted from the activation of phonological representations of these stimuli, via a reading based mechanism.

Thus, based on the present findings and the number of potential mechanisms identified in this series of investigations, the numerical variant of the single word semantic priming paradigm appears to be a useful methodology for the investigation of simple arithmetic processing. Moreover, as the preceding discussion shows, it has many applications in future research.
4.0 REFERENCES


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