Issues of equity and assessment loom large early in most discussions about the use of graphics calculators in mathematics, often simultaneously. In this paper, we identify and discuss some aspects of these issues, locating them in the broader curriculum framework in which they properly belong. Our discussion is informed by our recent experiences with using graphics calculators at the early undergraduate level (Bradley, Kemp & Kissane 1994), but is not restricted to those experiences.

The issues are of direct importance to mathematics education in the early undergraduate years as well as the senior secondary years, and have implications for the nature of mathematics education throughout secondary education. Of course, issues related to graphics calculators, equity and assessment are not only issues for Australia, but have also appeared in socio-economically comparable countries, such as the USA and the UK. While there is not space in this paper to discuss the overseas experiences in great depth, it is perhaps worth noting at least that The National Council of Teachers of Mathematics' Curriculum and Evaluation Standards (1989, p.124) assumes availability of graphics calculators to all students at all times from the 9th grade onwards, that all the UK examining bodies now allow students to use graphics calculators in external examinations (although they do not all expect them to be used), and that a good deal of curriculum development is taking place in each country to design mathematics curricula accordingly.

Curriculum design

In the best of all possible worlds, mathematics curricula are designed, taking course objectives, learning experiences, teaching methodologies and assessment into account simultaneously. Unfortunately, such design is rather rare in practice, and it is often the case that assessment is attended to after the rest of a course has been designed. Sound curriculum design is especially difficult in the presence of rapidly changing technologies, such as those of graphics calculators and other kinds of computers. It is more likely that curricula happen than that they are designed in such a circumstance. Ideally, all elements of a curriculum should be designed with available technologies in mind, so that there is a coherence between objectives, learning experiences, teaching methodology and assessment. Design of this kind first requires some analysis of the implications for the curriculum of newer technologies, such as that by Kissane (1989).

When graphics calculators are used, there seems to be potential for a lack of coherence between these curriculum elements. Examples of mismatch are easy to generate: students using graphics calculators for learning purposes will not understand why they are not permitted in assessment; teachers using graphics calculators as part of a teaching methodology will be uneasy with objectives that seem not to acknowledge the capabilities of the technology; students will be irritated by using graphics calculators in class, but being prevented from using them in projects; and so on.

There are two ways of thinking about graphics calculators in mathematics curricula, and these have different implications for assessment. The graphics calculator can be regarded either as a means to an end or as an end in itself. In the former case, the calculator is a device that allows
An alternate way of thinking about a graphics calculator focuses on the use of a calculator as an outcome in itself. Rather than being regarded as an optional extra, perhaps available only to a few privileged students, or used by the teacher as a 'teaching aid', the graphics calculator can be seen as a mathematical device that all students in a course should learn to use. It might be expected that students will not only learn how to make the calculator do whatever it is they want to do, but will also develop appropriate discrimination abilities to decide when to use it and when not to use it. If we want evidence about student attainment of outcomes of this kind, there is no logical alternative to finding out what students actually do when they have a graphics calculator with them.

There are two parts to the argument for a mathematics course to incorporate explicitly as an outcome the use of technology such as graphics calculators. Firstly, it is no longer defensible, in the closing days of the second millennium, to design mathematics courses as if such technologies don't exist, thus preventing many students from using them. Secondly, it is unlikely that the necessary technical and discrimination skills associated with graphics calculators will be picked up by students without us paying much explicit attention to them. There is not space here to flesh out these arguments more fully.

Equity issues

Mathematics teachers often raise issues of equity (or, more correctly, inequity) in early deliberations about graphics calculators, particularly when assessment questions are considered. There are at least two potential concerns: inequities between the capabilities of a class of students, and inequities arising from differences between the capabilities of particular graphics calculators. Each of these concerns is likely to be affected by technological developments in the graphics calculator marketplace. In addition, each is also exaggerated when the number of students using a calculator is significantly magnified, notably when a focus is placed on all students at a particular school or in a whole state or nation, rather than on students at a particular school or in a particular undergraduate course.

An obvious 'solution' to all of the equity issues is to make sure that all students are in identical circumstances by prohibiting the use of graphics calculators in mathematics courses. Such a solution raises more problems than it solves, however, as it inhibits (many, but not all) students from having accurate access to powerful learning opportunities, denies (many, but not all) teachers the chance to teach better, prohibits curriculum development to adequately incorporate graphics calculators and, perhaps worst of all, effectively prevents many students of mathematics from gaining access to today's technologies. Official prohibition is unrealistic, because, while it can affect some aspects of a course, most notably formal assessment, it cannot affect all aspects of courses. Better resourced individuals and schools may well be able to take advantage of the learning potential and computational power of graphics calculators, despite the official prohibition. Ironically, this solution to the equity issue might actually increase existing inequities.

Another non-solution to the equity issues involves legislation that some graphics calculators are permitted for student use in assessment situations, but others are not. While this solution may have some appeal at first, the practical difficulties are overwhelming in an area of rapid change, as the case of scientific and programmable calculators has amply demonstrated. There are many models of graphics calculators available (around fifteen to date, but we haven't counted them carefully), each with differing capabilities. Neither in university examination rooms, nor in state and national public examination rooms, is it feasible to require mathematics teachers to be appointed as supervisors, to monitor closely the calculators students bring with them to an examination. A casual glance will not always tell whether or not a calculator is programmable, nor even whether or not it has graphic capabilities. (Some graphics calculators appear to have a single line of display, although others have an evidently larger screen.) Calculators appear in shops at a much faster rate than we are able to monitor, scrutinise and publicise, and the path to legislation of features is necessarily doomed to failure.

A common first step in reducing equity concerns is for an individual school or university to buy class sets of calculators, and make them available to students. This is our own experience, and it is reminiscent of the same initial solution to the same problem of equity when scientific calculators first appeared in the mid-1970's. Differences between calculator capabilities are often resolved by not allowing calculators to be the same. Ideally, access to graphics calculators in all classrooms and laboratories, and as mathematical tools should be individual, so that students can take the calculators with them (to and from home, to a test or to an examination). A useful first step in facilitating this is the storage of some calculators in a library rather than an office, so that they can be routinely borrowed by students in a controlled way.

It is important to have a sense of perspective about these matters. As a mathematics education community, we are moving from a situation in which graphics calculators are still regarded by many as expensive luxuries, to a situation in which they will be taken for granted as standard student equipment in high school and beyond. Eventually, as happened with scientific calculators, it will be neither necessary nor wise for schools to invest resources in class sets of graphics calculators, as students will prefer and need to have their own calculator. But we are at present in a state of transition, in which such a strategy is a good way of getting started. In fact, it will always be necessary for schools to have some calculators available to lend to students in more straightened circumstances, who would otherwise be denied access.

Clearly, the economics of calculator purchase is a major source of potential inequity. Although inexpensive when regarded as personal computers, graphics calculators are still expensive when regarded as calculators. Elsewhere (Bradley, Kemp & Kissane 1994), we observed that a mobile computer laboratory is now available for an investment comparable with that for a single microcomputer, making access to technology much more affordable than even a few years ago. As mathematics begins to be recognised as a laboratory subject, for which appropriate equipment is a necessity rather than a luxury, and departmental budgets are adjusted accordingly, we might expect that inequities between schools might be broken down. This recognition is needed in several places, such as school and university administrations, decision-making groups within schools, and funding bodies such as State, Territory and Federal government departments. In precisely the same way that few schools would attempt to offer courses in photography without cameras, home economics without kitchen equipment, science without laboratories and woodwork without tools, it should be regarded as normal for a mathematics department to argue that appropriate equipment is necessary these days.

At the individual level, it is obviously not possible to completely remove the substantial economic differences between families and thus the resources available to individual students. While educational reform can rely on the argument that educational provision should be free, access and equity, despite the inequitable society we inhabit, One useful approach involves the use of hiring schemes, not unlike book-hiring schemes, in which students pay an annual hire fee for unrestricted use of a graphics calculator. The fee can be surprisingly modest, if students use their graphics calculator over the several years of secondary school, rather than only the last one or two. In addition, calculator costs may be less than expected if students are not required to have a second calculator (a scientific calculator) as well as a graphics calculator, contrary to current practice. In the USA, some schools have reported favourably on schemes of this kind, with students being given options to purchase their own calculators at any point, using any hire fees already paid towards the price. Such schemes demand some initial capital investment by schools to underwrite purchases. In both the USA and the UK, anecdotal reports suggest that many students actually prefer to purchase their own calculators, thus easing the burden on schools to provide them.

It is well to be reminded that the economics of calculator purchase is not a new problem, and was resolved for many students in the mid- to late 1970's in the case of scientific calculators. The resolution was aided by the substantial reductions in price of scientific calculators, as they became
widespread consumer items, and by an increasing community acceptance of the necessity for students to have unrestrained access to a calculator. In fact, it is not widely recognised yet that the cost of a graphics calculator today, in real terms, is comparable with the cost of a scientific calculator then. For example, a typical scientific calculator purchased by Australian students in the late 1970's, when calculators were first permitted in formal assessment situations such as public examinations, cost around $30-$40. Since then, the Consumer Price Index has risen almost 200% (Cassells, 1994), so that an equivalent investment in 1994 is a calculator costing $70 or so, about the price of some, but not the most sophisticated, graphics calculators today. Graphics calculators at this price are certainly adequate for mathematics in the senior secondary school and the lower undergraduate years, and might reasonably be expected to reduce in price in the next few years.

Inequities arising from differences between machines are surprisingly few, despite some substantial differences between machines. One reason for this is that the major uses of graphics calculators are at the lower ends of the machines' capabilities, where the differences between machines are smallest. Another reason is that, if students have substantial ongoing access to their own graphics calculator, they are likely to develop (with some help) efficient ways of using them for particular purposes. For example, recent graphics calculators will allow users to find a point of intersection of two graphs with a single command, whilst earlier models demand that a user undergo a potentially tedious process of repeated zooming to do the same. But the inequity is diminished markedly if the user of the less powerful machine is experienced and skilled at zooming, and they may well be able to locate a point of intersection to fairly high accuracy with a single attempt.

All graphics calculators are programmable to some extent. In many Australian settings, programmable calculators have been prohibited for student use in assessment on grounds of equity, and because of a fear that students might copy programs from calculators to their scripts. This practice is now permitted and increasing. However, it can be difficult to use a program already written and even more difficult to write your own, especially in an examination. Students able to do such things may, in all likelihood, have actually learned more about the relevant mathematics than those who haven't. In an age where students are expected to use mathematics rather than remember it, reflected by the widespread use of table books and drawing templates containing mathematical formulas of various kinds, concern that programmable calculators will be used to import illicit information into an examination seems misplaced.

Indeed, it is interesting in this regard to note recent proposals for the unrestricted use of graphics calculators in the prestigious Advanced Placement Calculus examination in the USA, which is used to allow students to gain college credit while in high school. It was suggested that students be required to clear their calculator memories by removing all the batteries as they left the examination, rather than as they arrived, because concerns for the security of the test questions was greater than concern that students would be advantaged by taking particular programs with them to the examination. The strong version of this point of view would be that questions for which it was possible to have pre-programmed answers should not be part of an examination anyway.

Equity issues adopt a different form across a state or a nation than they do in a single class, in large measure because of the disparate nature of the group concerned. It is simply harder to know what is happening in several places than it is for a single place, and thus harder to be reassured that gross inequities are not being perpetuated. Again, this is not a new problem, since we had identical concerns with scientific calculators. At the least, external examination authorities need to be mindful of the capabilities of graphics calculators when designing external examination papers. A particular mathematical task on an examination paper may be interpreted differently by students who have a graphics calculator with them than by those who do not. Students with a graphics calculator may be able to obtain a numerical answer to a question directly, rather than using some mathematical analysis, or they may have available to them easier ways of verifying that their analytic solutions are correct. It seems important to reduce the likelihood that some students are in effect answering a quite different question from the one intended, because of the mathematics available to them. This will require some care and some detailed graphics calculator knowledge on the part of those responsible for designing individual items.

In summary, equity issues related to the use of graphics calculators in assessment are important, and need to be addressed. It is unrealistic to expect that all such issues can be eliminated altogether, but they can be substantially reduced. We should not use concerns about equity as an argument for doing nothing about graphics calculators in assessment. Rather, we hope that educational funding bodies, administrators, syllabus committees, examination panels, school and university mathematics departments and other members of the mathematics education community will focus appropriate attention on these important issues.

Assessment in practice

In this section, we give some examples of our responses to some of the above issues in the context of a particular undergraduate mathematics course. With limited resources, we were able to provide students with weekly access to graphics calculators in their one-hour tutorial, and provided activities to help them learn to use the calculators while learning mathematics. Some calculators were also available for student use in the library, and were released for overnight loan. More complete details of the course, and the way in which graphics calculators were incorporated into it, are provided in Bradley, Kemp & Kissane (1994).

A calculator test

The course assessment schedule was designed so that one of three in-tutorial tests allowed students to use graphics calculators freely. The scheduling of the weekly tutorial classes over three days made it possible for the class set to be used in the test for this purpose. Under normal circumstances, tests are given simultaneously to all students in a course, but this was not practically possible for a test that required the calculators, since we had many more students than calculators. To minimise the likelihood that students would gain advance knowledge of specific test items, the test papers, which contained both questions and spaces for answers, were collected from students at the end of each session. In addition, three parallel forms of the test were produced, essentially by a process of slightly changing functions and their graphs. In each administration of the test, all three forms were used, with the relevant pair of forms assigned to students so that students were seated next to students attempting different forms. Earlier tests had also been designed in this way to ensure that adjacent students in a large lecture theatre were attempting different questions, and thus would gain no advantage in looking (illicitly) at each other's responses. One of the calculator test forms used in 1994 appears as Appendix 1 to this paper.

As students were familiar with and reminded of the use of several forms of a test, it was hoped that those who took the test later in the week would see little value in trying to encourage those who had taken the test in an early tutorial to recall the questions for their benefit. In fact, there was no anecdotal evidence of this happening, and the test results showed no tendency for students later in the week to outperform those who had taken an earlier test.

In the test itself, students were instructed to clear the calculator memories before the test papers were distributed, and the calculators were then rotated around the class as a further check that this had been successfully completed. This procedure, which took only a few seconds, gave further assurance that student responses were not contaminated by the work of other students, and particularly by the student who had previously used a particular calculator.

Students were given a sample set of test items in the week prior to the test to alert them to the likely level of calculator expertise to be expected. This had the additional effect of encouraging students to pay particular attention to their calculator skills, and gave rise to a noticeable jump in the level of out-of-class use of the calculators.

Final examination

Although arguments, similar to those for in-class tests, can be made for including questions for which graphical calculators can be used on a final examination, practical constraints prevented this in our case. Nonetheless, we were concerned about the integrity of the relationship between the examination and the course, a separate part of the examination paper concentrated on the ability of students to answer mathematical questions that rely on their interpretation of a graph drawn by a computer.
similar to that drawn by a graphics calculator. This part of the examination paper is shown in Appendix 2 to this paper. The tasks concerned evidence of students' abilities to understand the relationships between graphs of functions, equations, roots and inequalities, all of which are important aspects of courses of this kind.

Another possibility in this setting is to distribute the calculators to students during the examination on a rotational basis, so that each student has unrestricted access to a calculator for long enough to deal with specific questions for which it is needed. We suspect that this solution can be implemented with minimum disruptions to the orderliness of the examination and appropriate safeguards, but have not yet tested this suspicion.

Public examinations

Some Australian states still have public, external examinations, often focussed on tertiary entrance. These raise special issues as far as graphics calculators are concerned. The essential difference between these and typical university final examinations is one of scale, with often thousands of students involved, spread over a large geographic area with many individual examination locations. It is more difficult to engage in innovative assessment practice in such settings than in a single school or a single campus, but no less important, because of the very substantial flow-on effects of examination policies and practices. It is especially difficult to institute a system that relies on examination hall monitors checking the nature of calculators that students have with them, for the reasons noted earlier. Keeping legislation about which calculators, or kinds of calculators, are permitted for exam use, up-to-date in a rapidly changing market place is very hard, and probably too hard.

One possible response is to attempt to design examination questions and papers that are 'calculator neutral', so that students using a graphics calculator have no clear advantage over those not doing so, or so that different calculator advantages are minimised. One problem with such a strategy is that it is extremely difficult to write such questions and not to distort the course or the examination as a result. The features of modern graphics calculators are so powerful, and so pervasive as far as senior high school and lower undergraduate curricula are concerned, that skilful users will invariably find a graphics calculator a significant advantage. At the very least, students will often have several ways of checking an analytic solution numerically, which would give an advantage over those without such facilities. Examples include solution of equations, optimisation, curve sketching, matrix operations, integration, differentiation and so on - the backbone of the traditional course progression through algebra to the calculus.

We suspect that 'calculator neutral' questions will often be general rather than specific. For example, rather than finding the turning points of \( f(x) = x^3 - 2x + 1 \), students might be asked to find the turning points of \( f(x) = x^3 + Ax + B \). While such a strategy may work to some extent, it is quite likely to distort seriously the nature and intent of the course concerned. It should be acknowledged, too, that competent students can still use a graphics calculator in such cases, at least to check a particular case. (We reserve for a later day trying to contemplate what to do when calculators will deal with the general cases as well as the numerical ones! Such a discussion will be necessary in only a couple of years, given the current rate of calculator development and the increasing availability of computer algebra systems.)

Another strategy, with similar pitfalls, is to focus on exact answers rather than numerical approximations. In any event, serious thought is needed to make sure that students answers to examination questions are adequately explained, so that the time-honoured emphasis on mathematical argument, justification and expression, rather than just a numerical result, continues to be paramount. We will perhaps need to both find better ways of communicating to students that their working is important, and helping them to decide how much is enough to say. Some students may be inclined to consume too much of their time in an examination describing their work in unnecessary detail (giving key sequences, for example, or showing intermediate numerical results to screens full of significance). These may be even more worrisome to us than students who report too little information about their methods of solution, who may in fact be reflecting a maturity of style, rather than a reluctance to explain.

Perhaps the most serious problem with forms of calculator neutral examinations is the same as the problem of not using graphics calculators at all. It is the failure to acknowledge the now considerable importance of technology to mathematical work. It is no longer acceptable to allow the assessment tail to wag the curriculum dog, and thus deny many students the opportunities to do more mathematics better.

What is assessed

In this section of the paper, we focus attention closely on a few instances of mathematics questions that we have used, to highlight what they are likely to tell us about student abilities.

Our aim with the calculator test was to determine how well students were able to deal with various mathematical situations for which a graphics calculator might be used. Students were permitted to use a calculator, but were not required to do so; for several of the items, alternative solutions using non-calculator techniques were possible. The final forms of the test, a copy of one of which is appended to this paper, provided assessment information about the following mathematical aspects:

Recognising a function from its graph

Question 2 is concerned with using a graphics calculator to see the extent to which graphs intersect. Questions like this require students to zoom out on their graphics calculators to make sure that they find all points of intersection. The default screen on the TI-82™ calculator screen looks like this:

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It is not immediately clear exactly how many solutions the corresponding equation has, especially given the resolution of the calculator screen.
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Finding numerical approximations to solutions of equations and inequalities

Question 3 examines these aspects, and again, demands calculator and mathematical expertise from students. For example, it is clear (to a student who understands the relationships between graphs and equations) that the equation in part (i) has a single solution, but the normal tracing of the curve will not give the solution to the required accuracy. A process of zooming is needed, or an alternative numerical procedure must be employed. In the case of the TI-82™ graphics calculator, students can use an automatic procedure to find points of intersection, but still need to express the result to the appropriate accuracy. An alternative, using a numerical 'solve' procedure is available on this calculator, but it is still necessary for students to deal first with the equation mathematically, and to express the resulting answer in the manner demanded. It is perhaps worthy of note, too, that questions of this kind (solution of a cubic equation) are normally beyond the capabilities of students in courses pitched at this level.
Kissane, Jen Bradley and Marian Kemp

Question 3(ii) illustrates a related point about calculator use for solving equations and inequalities. If the function to the left of the inequality symbol is graphed on a default domain on a graphics calculator, a picture like the following is quite likely, depending on the calculator used:

![Graph of a function](image)

Interpreting calculator screens generally requires that students come to terms with the lack of scales. Interpreting this particular graph requires a significant level of understanding of the nature of vertical asymptotes, and of how machines represent them. While it is rather odd that a graphics calculator would draw only one of the two vertical asymptotes, as above, nonetheless such odd things can happen with this technology. Regardless, students need to interpret the graph, warts and all, with a view to solving the inequality. The most sophisticated interpretations will quickly see that the function has vertical asymptotes at $x = 1$ and $x = -2$, based on a careful study of the function, rather than relying only upon the graphs screen. Less sophisticated calculator users are quite likely to find the question much more difficult and make errors such as locating the vertical asymptotes at the wrong place by trying to read them directly from the screen. Learning to use a calculator well involves thoughtfully considering both the calculator results and the mathematical ideas that gave rise to them.

Solving systems of linear equations

Question 4 gives an example in which the matrix manipulation capacities of the calculator can be exploited, especially when solutions are not integers. Again, mathematical skill must be exercised to formulate the equations using matrices to generate a solution, especially given the ways in which the equations have been written. It is also necessary for students to interpret the screen result, which is quite likely to not look like a conventionally printed matrix, depending on the particular calculator used.

Sketching the graph of a function

Question 5 gives an example for which the calculator can be used to identify the main features of a graph, but which still requires careful interpretation, notably of the scales on the axes. A simple copy of the calculator screen, shown below, would not provide an adequate response to the question:

![Graph of a function](image)

A more suitable scale than the default may be preferable, some suitable scale markings are needed, and a better representation of the asymptotes would be expected for student responses to this item in a course at this level.

The examination question was worth 10% of the total marks for the examination. Our aim was to bring together and test concepts from several stages of the course at once. Students did not have graphics calculators with them, but we expected that their earlier experiences with graphics calculators would help them in dealing with the question.

In part (a), students were asked to identify the general nature of the functions being graphed, given a list of possibilities. It is worth noting that nowhere in the entire question are the specific forms of the functions $f$ and $g$ required, and thus no algebraic manipulation was required or relevant.

Parts (b) and (c) probed student understanding of the concept of a solution (both points and intervals) and the graphical representation of inequalities. They had to decide which specific values ($x$-values, rather than ordered pairs) from the graph answered the questions asked, and then interpret the scales correctly.

Part (f) also demanded that students understand the relationships between graphs and equations. In the second and third parts, students needed to recognise horizontal and vertical translations and imagine them on the existing graph. In part (g), they needed to add a graph by hand before they could determine solutions.

The idea of a relative maximum on an interval is assessed in part (e), although calculus is not involved. Students needed to interpret the maximum as a value of the function, rather than as a point, i.e., the $y$-value, rather than both coordinates of the turning point. Parts (d) and (b) provide information about student understanding of the concept of the derivative of a function and its relationship to the slope of the graph of the function. As noted, it was not expected that students would attempt these (traditional calculus) questions using derivatives of the functions concerned, and no students actually attempted to do so.

These examples, from both the calculator test and the examination question, illustrate the general argument, that care needs to be devoted to analysing what students have to do to respond to a particular assessment item. Both mathematical understanding and skills at using the graphics calculator are involved. We have used these particular examples, because they were relevant to our particular course, but similar arguments apply for other aspects of mathematics (and there are many of them) for which graphics calculators are relevant.

Some results

In general terms, students in our course handled the questions on the calculator tests well, and seemed to have acquired substantial familiarity with the use of the graphics calculators for the mathematical procedures outlined above. They handled the identification of functions from their graphs well, and most students could use their calculators to solve a system of linear equations. It was noticeable that some students were inclined to base their results on graphs on a default domain, rather than zooming to attain a desired level of accuracy, or to make sure that all intersections of graphs were located. Although most students could use the calculator to help them sketch a graph of a function, many others neglected to identify all the necessary information (in this case, intercepts, scales, asymptotes and graphical shape). We concluded that questions of these kinds provide some useful diagnostic information to inform us of student learning, both in respect to the mathematical ideas and procedures and in respect to the effective use of a graphics calculator.

A small proportion of students found the calculator test too difficult, but a somewhat larger proportion successfully answered almost all of the questions, together suggesting that the test was at an appropriate level for this course.
Whilst the students performed adequately on the examination question, they did not do as well as on the calculator test. We thought this could have been due to several reasons, including unfamiliarity with examination questions of this particular kind; this was especially the case with aspects concerned with the derivative. The location of the question was probably a factor as well. As the item was on a separate sheet many students may have left it to the end of the examination period, when they were rushed for time. Observation in the examination room confirmed this, and there were many responses that suggested that students didn’t have time to finish the question, indicated by successive blank responses rather than incorrect responses.

Conclusions

The arrival of the graphics calculator raises many more issues for mathematics education than did either the scientific calculator or the microcomputer. The scientific calculator, which was affordable to many students and which seemed to replace many existing mathematical procedures caused initial concerns about equity and assessment, together with concerns for the content and emphases of mathematics courses. In time, these concerns have abated, although they have not completely disappeared. The appearance of affordable microcomputers has had rather less impact at the level of individual students and courses, in part because they are usually too bulky to move around with students, and in part because they have continued to be beyond the financial means of many students, especially when software is taken into account.

But if one regards the graphics calculator as an almost affordable, genuinely portable computer with inbuilt mathematical software, then it becomes necessary to consider carefully the resultant implications for equity and course design, particularly course assessment. In this paper, we have identified some of these issues, with a focus on a particular lower undergraduate course, much of the content of which is also routinely taught in the upper secondary years. In our view and our experience to date, the issues are important, but are resolvable, provided proper attention is given to them, and we have offered some ways of considering them further. The resolution of issues of equity and assessment may require us to reconsider some of our standard assessment practices, to ensure that we are concentrating on the mathematical things that matter most if students have a new level of technological power at their fingertips.

References


2. How many solutions are there to
   (i) \[ 3x^2 + 4x - 10 = 3x - 12 \]
   (ii) \[ x^3 - 4x^2 + 3x + 3 = 6x - 12 \]

3. Solve the following to 1 d.p. accuracy.
   (i) \[ x^3 - 4x^2 - 5x + 2 = 0.7x + 5 \]
   (ii) \[ \frac{(x + 4)}{(x - 1)(x + 2)} \leq 0 \]
   (iii) \[ e^x - 2 = -2x + 5 \]

4. Solve the following system of linear equations.
   \[ \begin{align*}
   2x + 3y + z &= 3 \\
   2x + y - 6z &= 4 \\
   8x + 5y &= 1
   \end{align*} \]
   \[ x = \ldots \quad y = \ldots \quad z = \ldots \]

5. Use the calculator to help you sketch (on the axes below) the graph of
   \[ f(x) = \frac{(x^3 + 3)}{(x + 1)} \]

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Appendix Two: An examination question

13. This question relates to the graphs, shown below, of the two functions \( f \) and \( g \).
   (a) Indicate the type of functions \( f \) and \( g \) could be.
      Choose from: linear, quadratic, cubic, quartic, rational or trigonometric.
      \( f \) is : \( g \) is :
   (b) Find (1 d.p.) the value(s) of \( x \) when \( f(x) = g(x) \):
   (c) For what value(s) of \( x \) is \( f(x) > g(x) ? \)
   (d) Give the sign of \( f'(0.8) \) and \( g'(-3) \):
   (e) Find, to nearest whole no, the maximum of \( g \) on \([-3, 3]\):
   (f) How many solutions are there to each of the following?
      \[ f(x) = -8 \quad f(x) = g(x) + 5 \quad f(x - 8) = g(x) \]
   (g) Use the graph below to solve \( f(x) = 3x - 5 \):
   (h) Give the approximate interval for which \( g'(-x) < 0 \):