Level Identification Using Input Data Mining for Hierarchical Fuzzy System

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Fuzzy rule based systems have been very popular in many control applications. However, when fuzzy control systems are used in real problems, many rules may be required. Hierarchical fuzzy system that partitions a problem for more efficient computation may be the answer. In the stages of creating a hierarchical fuzzy system, level identification stage is a crucial and time consuming one. This has the direct effect on how efficient is the hierarchical fuzzy system. This paper has reported the use of input data mining technique to efficiently perform the level identification stage. Without the use of the input data mining, $k^r(k-1)$ ways of building the hierarchical fuzzy system have to be tried.

1. Introduction

The tendency and main motivation of various Artificial Intelligence approaches has been to cope with very complex and often analytically unknown systems, in the sense of identifying approximate models, controlling or generating decision support for them. The first major step was symbolic rule based expert systems, which had the disadvantage of not utilising the internal structure of the problem space (such as ordering / partial ordering and metrics or similarity within the space). Subsymbolic approaches built in these specific mathematical features, for example by the fuzzy membership function. These approaches still have not successfully tackled problems with reasonably high numbers of input variables because of the high computational cost involved.

In classical fuzzy approaches from Zadeh [1] and Mamdani [2], the basic idea is to calculate the conclusion by evaluating the degree of matches from the observation that triggered one or several rules in the model. In most fuzzy modelling or fuzzy control systems, experiments and simulations are set up to generate a set of data that best describe all the possible outcomes. After this, a human expert will create the set of fuzzy rules that best perform control or modelling. Normally, fuzzy rules generated in this manner will cover the whole universe of discourse by taking all the possibilities into account. However, a serious problem may occur due to the high computational time and space complexity of rule bases used to describe the model with multiple input variables with proper accuracy. The "exponential explosion" allows little general systems application or real time application of the classical fuzzy algorithms where the number of input variables becomes large [3]. For $k$ input variables, and where $T$ fuzzy linguistic terms are required in every dimension of $X$ for all the $a$-covers, the number of fuzzy rules covering $X$ at least to $a$ is

$$|R| = O(T^k)$$

which could be very high, unless $k$ is very small. This "exponential explosion" in the number of rules is a major problem hindering the application of fuzzy techniques beyond the area of fuzzy control systems. Besides, the time required in searching the large rule base in order to perform inference might become impractical for any potential applications.

There are basically three possible solutions in general to overcome this problem. First is to reduce the value of $T$, this can be performed by fuzzy rule interpolation techniques [4, 5, 6, 7]. Secondly, reducing the value of $k$; which is more effective. In 1991, Sugeno et al. [8] had proposed the use of the hierarchical structured rule bases, through little subsequent application of this method has been done as their method required significant manual crafting. The third possible solution is to decrease both $T$ and $k$ simultaneously has been examined in Kóczy et al. [3].

In [9], we have presented a hierarchical fuzzy rule base algorithm that extracts its rules directly from the measured or simulated data. The hierarchical fuzzy system is used to perform classification with the purpose of reducing the complexity of computation for inference. However, in the constructing of that hierarchical fuzzy system, it is difficult identified which input variables should be located in which level of the hierarchy. This paper has looked at the use of input data mining technique [10] to efficiently identify the level.
2. Hierarchical Fuzzy System

This section gives a brief summary of the steps in constructing a hierarchical fuzzy system for classification [9]. There are basically three main steps.

A. Converting Numerical Values to Fuzzy Rules

This first step is to translate all the available data into linguistic fuzzy rules using linguistic labels. The number of linguistics terms \( T \) required and the distribution of the data in each dimension of the domain are determined.

After the fuzzy regions and membership functions have been distributed, the available input-output pairs will be mapped. If the value cuts on more than one membership function, the one with the maximum membership grade will be assigned to the value. After all the input-output values have been assigned a fuzzy linguistic label, Mamdani type fuzzy rules are then formed.

After fuzzy rules have been generated from each data point, repetitive rules are removed. Depending on the quality of the simulation or experiment set up, noise may create some conflicting fuzzy rules. In this case, the number of repetitions of the fuzzy rules and the firing strengths of the rules will be examined to resolve the conflict. Normally, with appropriate preprocessing, the total number of rules generated in this stage will follow the formulae presented in equation (1).

B. Level Identification

After obtaining the fuzzy rules that can represent the set of available data, the next step is to build the hierarchy levels in the hierarchical fuzzy system. The notion in this stage is to allow one input state variable to be used in one level of the hierarchy, which structures the search space for the subsequent steps. The problem arises in which input variable should be placed at the top and each level of the hierarchy. This question will be investigated in this paper.

The hierarchical fuzzy system will start by building from the top of the hierarchy with input \( x_c \). Let us assume that this input \( x_c \) has \( T \) linguistics terms, therefore this input can has \( T \) branches of sub rules that can be formed in the next level down the hierarchy. If we have two input variables and one output variable, and all variables have \( T \) linguistic terms, our hierarchical fuzzy rule base will have the following levels of rules:

- **R1:**
  - If \( x_1 \) is \( A_{11} \) then use \( R21 \)
  - If \( x_1 \) is \( A_{12} \) then use \( R22 \)
  - ...  
  - If \( x_1 \) is \( A_{1T} \) then use \( R2T \)

- **R21:**
  - If \( x_2 \) is \( A_{21} \) then \( y \) is \( B_{211} \)
  - If \( x_2 \) is \( A_{22} \) then \( y \) is \( B_{212} \)
  - ...  
  - If \( x_2 \) is \( A_{2T} \) then \( y \) is \( B_{21T} \)

- **R2T:**
  - If \( x_2 \) is \( A_{2T} \) then \( y \) is \( B_{2T1} \)
  - If \( x_2 \) is \( A_{2T} \) then \( y \) is \( B_{2T2} \)
  - ...  
  - If \( x_2 \) is \( A_{2T} \) then \( y \) is \( B_{2TT} \)

C. Pruning Stage

After the hierarchical fuzzy rule base has been constructed, pruning is carried up to reduce the complexity of the fuzzy rule base. This stage is very much dependent on the level identification stage, i.e. depending on which input variable should be at the top level.

Pruning is performed by examining the hierarchy from the bottom to the top of the hierarchical fuzzy system using two measures:

1. if the output classification is the same for the whole sub-rule base, it can be pruned by moving the classification label up the hierarchy, or
2. if the output classification can be interpolated by two neighbouring fuzzy rules, it is also removed from the sub-rule base.

3. Data Mining of Inputs and Level Identification

The main purpose of the input data mining is to identify the significant of inputs in predicting the output. Gedeon [10] has suggested that the significance of the inputs for some cases has a direct relationship with their correlation to the output. He has also shown that by pruning some input variables (irrelevant inputs), the results can be improved.

Using knowledge from the input data mining, we can translate that if we can make use of an input data mining technique in the level identification stage, the hierarchical fuzzy system could have a better prune system. This will in term reduce the number of the fuzzy rules and thus reduce the computational complexity.
In this paper, the Regularity Criterion (RC) is used to perform the input data mining [11]. The whole data set is first divided into two sets, A and B. The RC used is defined as:

\[ RC = \frac{\sum_{i=1}^{k_A} (y_i^A - y_i^{AB})^2 / k_A + \sum_{i=1}^{k_B} (y_i^B - y_i^{AB})^2 / k_B}{2} \]

where

- \( k_A \) and \( k_B \) = number of data in group A and B
- \( y_i^A \) and \( y_i^B \) = the output data of group A and B
- \( y_i^{AB} \) = the model output for the group A input estimated by the model identified using the group B data.
- \( y_i^{AB} \) = the model output for the group B input estimated by the model identified using the group A data.

The smaller the RC value suggests that the input is more significant in the data analysis model. Thus, in order to remove as much sub-rule bases in the hierarchical fuzzy system, it is advisable to have the input that is most correlated to the output at the top level. Our proposition here is that the probability of having the input with the lowest RC value at the top of the hierarchy to the input with the highest RC value at the bottom will effectively allow more sub-rule bases to be pruned. The contribution of the inputs for predicting the classification output is a measure of how well the output classification correlates to the input space. The more correlated the output classification to the input variable, the better chance that the classification can be determined without moving down the sub-rules in the hierarchical fuzzy system.

Without the use of the input data mining method, we would have to try \( kn^2 \) ways of building the hierarchical fuzzy system. Then, we would have to decide which one would give us the best hierarchical fuzzy rule base depending on how much it can be pruned. This is a time consuming step.

4. Case Study and Results

A case study in determining the salary category based on Age, Experience and Contacts are used to demonstrate the use of the proposed hierarchical fuzzy system. A simulated case is used to generate a total of 200 data points, to be used as the set of input-output pairs to construct the rule base.

After the first stage of converting all the numerical values to fuzzy rules, we have generated the following fuzzy rules.

- **Age (Age) = {Young (Y), Middle (M), Old (O)}**:
  - Experience (Exp) = {Little (L), Some (S), Good (G)};
  - Contacts (Con) = {Poor (P), Normal (N), Quality (Q)};
  - Salary (Sal) = {Basic (B), Fair (F), High (H)}

  - R1: If Age is Y and Exp is L and Con is P then Sal is B
  - R2: If Age is Y and Exp is L and Con is N then Sal is B
  - R3: If Age is Y and Exp is L and Con is Q then Sal is B
  - R4: If Age is Y and Exp is S and Con is P then Sal is B
  - R5: If Age is Y and Exp is S and Con is N then Sal is B
  - R6: If Age is Y and Exp is S and Con is Q then Sal is F
  - R7: If Age is Y and Exp is G and Con is P then Sal is H
  - R8: If Age is Y and Exp is G and Con is N then Sal is H
  - R9: If Age is Y and Exp is G and Con is Q then Sal is H
  - R10: If Age is M and Exp is L and Con is P then Sal is B
  - R11: If Age is M and Exp is L and Con is N then Sal is B
  - R12: If Age is M and Exp is L and Con is Q then Sal is F
  - R13: If Age is M and Exp is S and Con is P then Sal is F
  - R14: If Age is M and Exp is S and Con is N then Sal is F
  - R15: If Age is M and Exp is S and Con is Q then Sal is F
  - R16: If Age is M and Exp is G and Con is P then Sal is H
  - R17: If Age is M and Exp is G and Con is N then Sal is H
  - R18: If Age is M and Exp is G and Con is Q then Sal is F
  - R19: If Age is O and Exp is L and Con is P then Sal is B
  - R20: If Age is O and Exp is L and Con is N then Sal is F
  - R21: If Age is O and Exp is L and Con is Q then Sal is H
  - R22: If Age is O and Exp is S and Con is P then Sal is F
  - R23: If Age is O and Exp is S and Con is N then Sal is F
  - R24: If Age is O and Exp is S and Con is Q then Sal is H
  - R25: If Age is O and Exp is G and Con is P then Sal is H
  - R26: If Age is O and Exp is G and Con is N then Sal is H
  - R27: If Age is O and Exp is G and Con is Q then Sal is H

Without the use of an input data mining technique, we have to test all the possibilities. Figure 1 shows the hierarchical fuzzy rule system using Contacts at the top follows by Age and then Experience. Figure 2 shows the hierarchical fuzzy system from Figure 1 after pruning.

![Figure 1: Hierarchical Fuzzy System using Contacts – Age – Experience.](image)

Now we repeat the same process by testing another combination, using Experience at the top, follows by Age and then Contact. The results are shown in Figure 3 and Figure 4. Figure 5 and Figure 6 show...
the hierarchical fuzzy system using Age – Experience – Contact. The cycle can keep on until all the combinations have been tried. Finally the best prune hierarchical fuzzy system are then selected. This is very time consuming and inefficient.

Figure 2: Hierarchical Fuzzy System using Contacts – Age – Experience after pruning.

Figure 3: Hierarchical Fuzzy System using Experience – Age – Contacts.

Figure 4: Hierarchical Fuzzy System using Experience – Age – Contacts after pruning.

Figure 5: Hierarchical Fuzzy System using Age – Experience – Contacts.

Figure 6: Hierarchical Fuzzy System using Age – Experience – Contacts after pruning.

In using input data mining for level identification, RC measure is now performed on the data set to determine the level of each input in the hierarchy. The RC values of the three input variables are Age (338.584), Experience (149.102), and Contacts (495.492). Following the values of the RC measure, the arrangement of the level of hierarchy is Experience at the top, followed by Age and then Contacts. This matches our trial approach in selecting the best prune hierarchical fuzzy system. The pruning is performed based on the two factors mentioned earlier. For this case, the total number of fuzzy rules in this hierarchical fuzzy rule base is 15, which is a notable reduction from the original 27 fuzzy rules.

5. Conclusion

In this paper, we have proposed the use of input data mining in identifying the level in a hierarchical fuzzy system that used for classification. Although RC method is used as the input data mining technique for this paper, any other reliable input data mining technique can also be used. Without the use of the input data mining method, \( k^*(k-1) \) ways of building the hierarchical fuzzy system have to be tried. This is a time consuming steps. By using an input data mining technique to examine how the inputs correlate and relate to the output, the process of level identification can be done easier and more efficiently.
6. References


